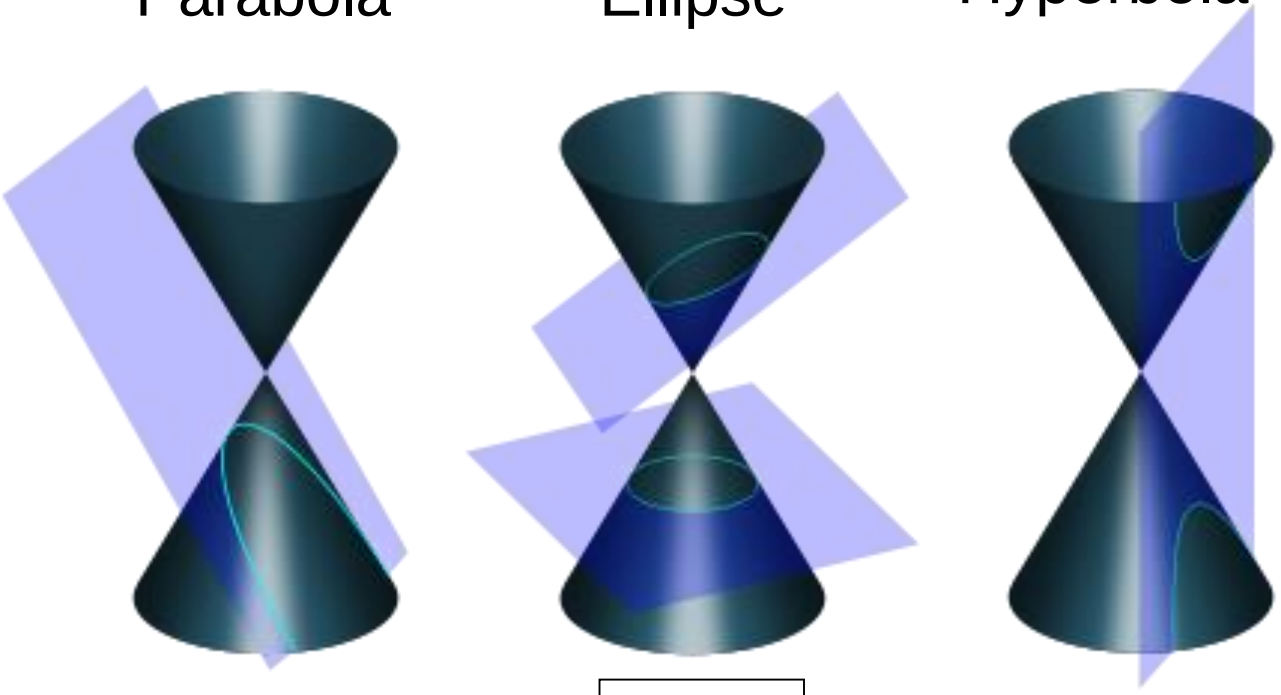
# Math-2
# Lesson 8-1
# Circles

## Conic Sections

Parabola

Ellipse

Hyperbola



Circle

# Transformations of the Square function.

$f(x) = x^2$ $\qquad$ $y = (-1)a(x-h)^2 + k$

Reflection across x-axis

VSF

left/right

up/down

$y = -2(x-3)^2 + 4$ Reflected across x-axis, VSF = 2, right 3, up 4

$y = 3(x+5)^2 - 6$ VSF = 3, left 5, down 6

To convert the equation into a graph:

1) Start with the "parent function"

2) Move the vertex left/right and up/down

3) The shape of the graph depends upon VSF and x-axis reflection.

# Finding the equation of a circle:

What is the radius of the circle?

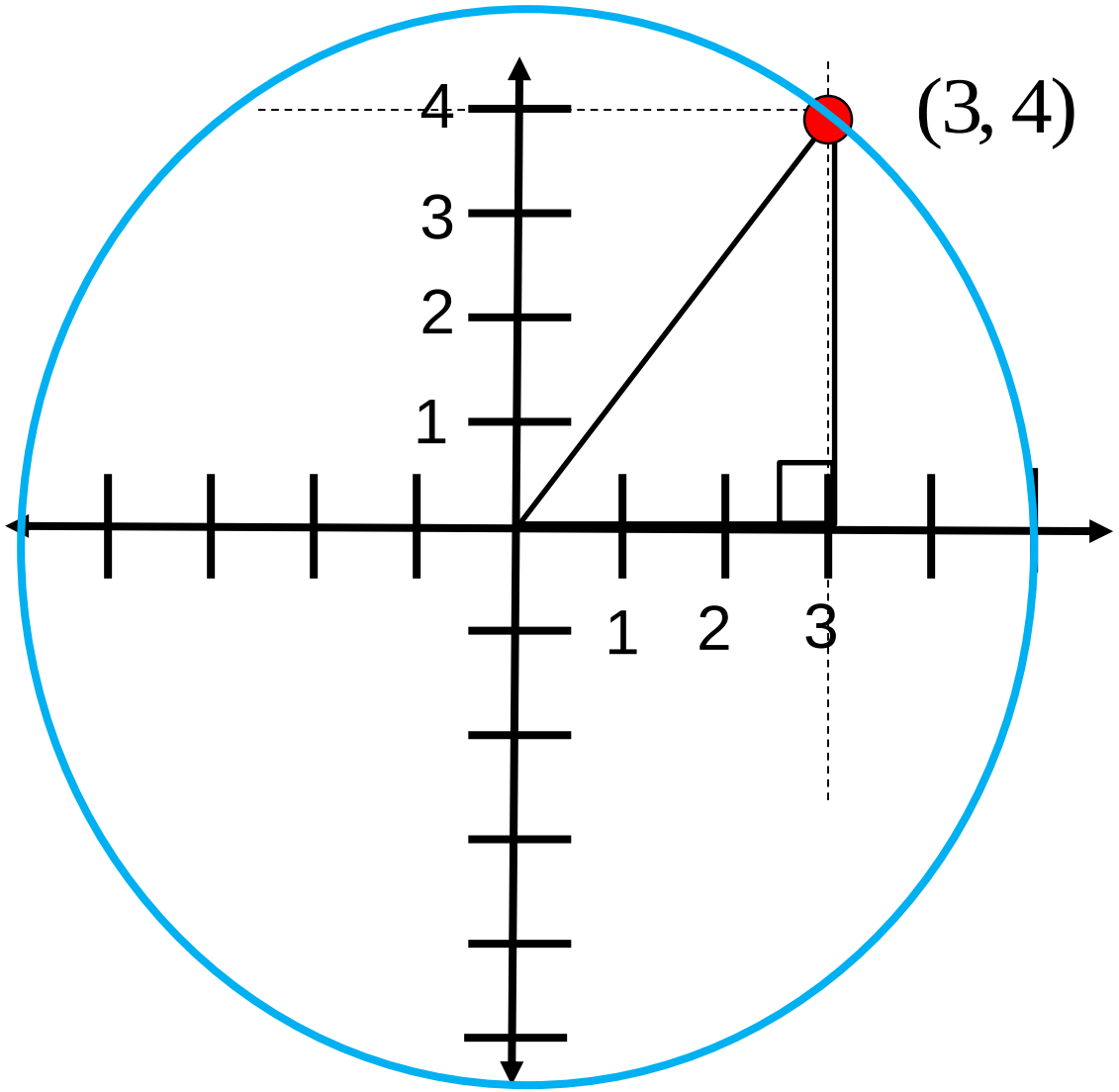
$$a^2 + b^2 = c^2$$

$$3^2 + 4^2 = r^2$$

$$9 + 16 = r^2$$

$$25 = r^2$$

$$r = 5$$

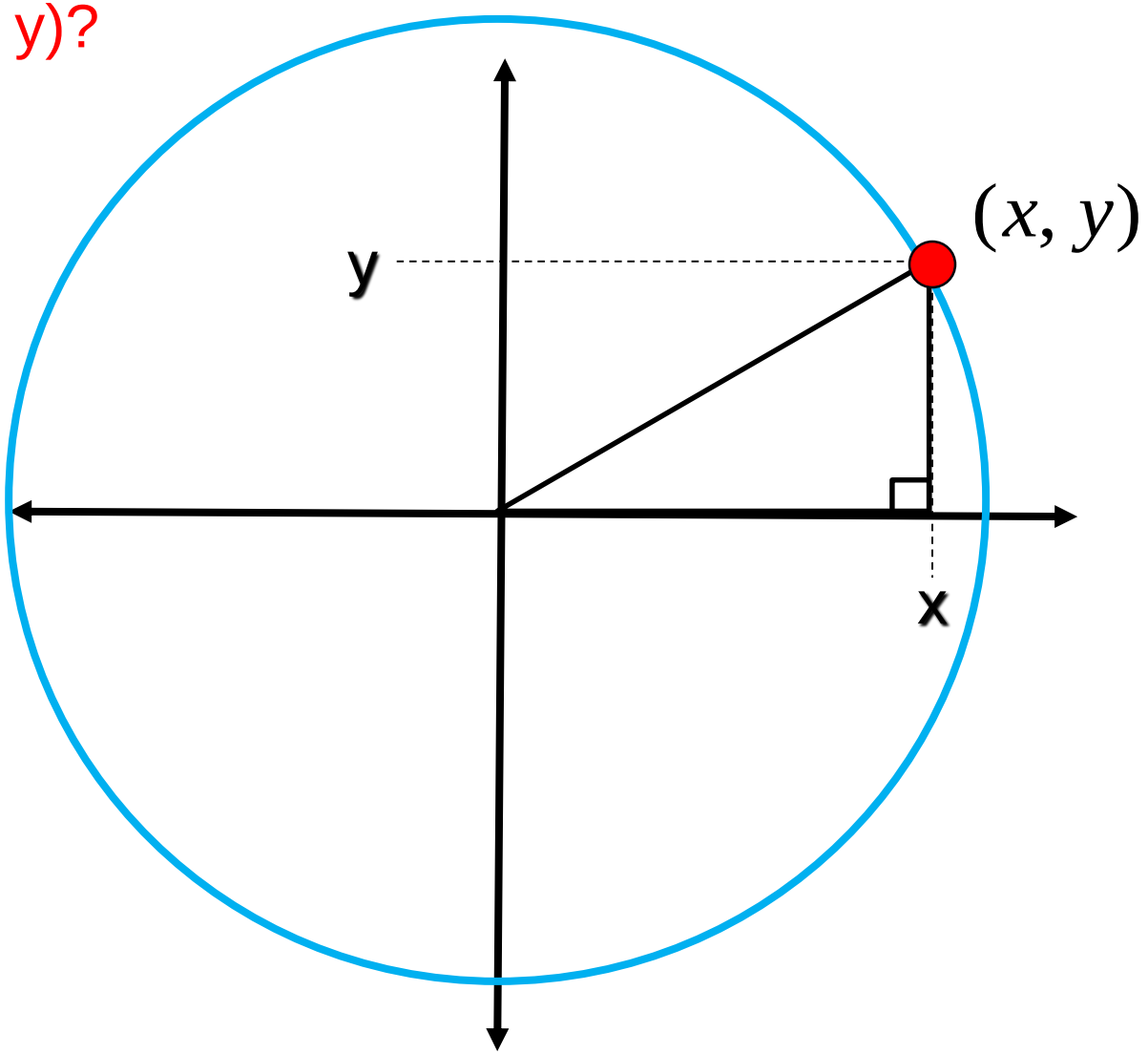


Now I will pick a random point on the circle.

What is the equation for the radius of the circle when the point is (x, y)?

$$x^2 + y^2 = r^2$$

This is the general equation for a circle centered at (0, 0)!!!

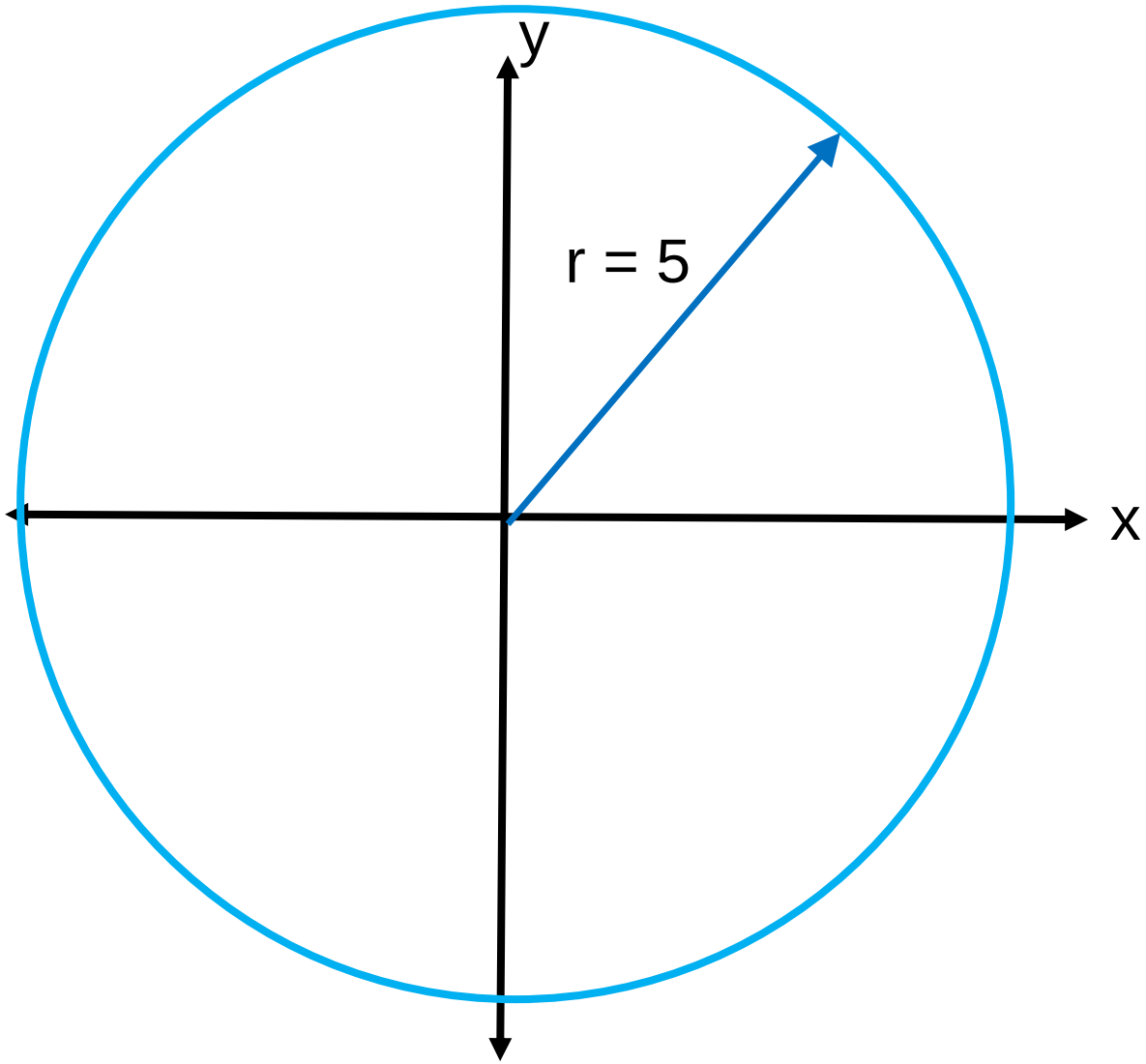


This is the equation of a circle centered at the origin whose radius is 5.

$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = 5^2$$

$$x^2 + y^2 = 25$$



What is the equation of this circle?

$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = 3^2$$

$$x^2 + y^2 = 9$$

What is the y-value when x = 3?

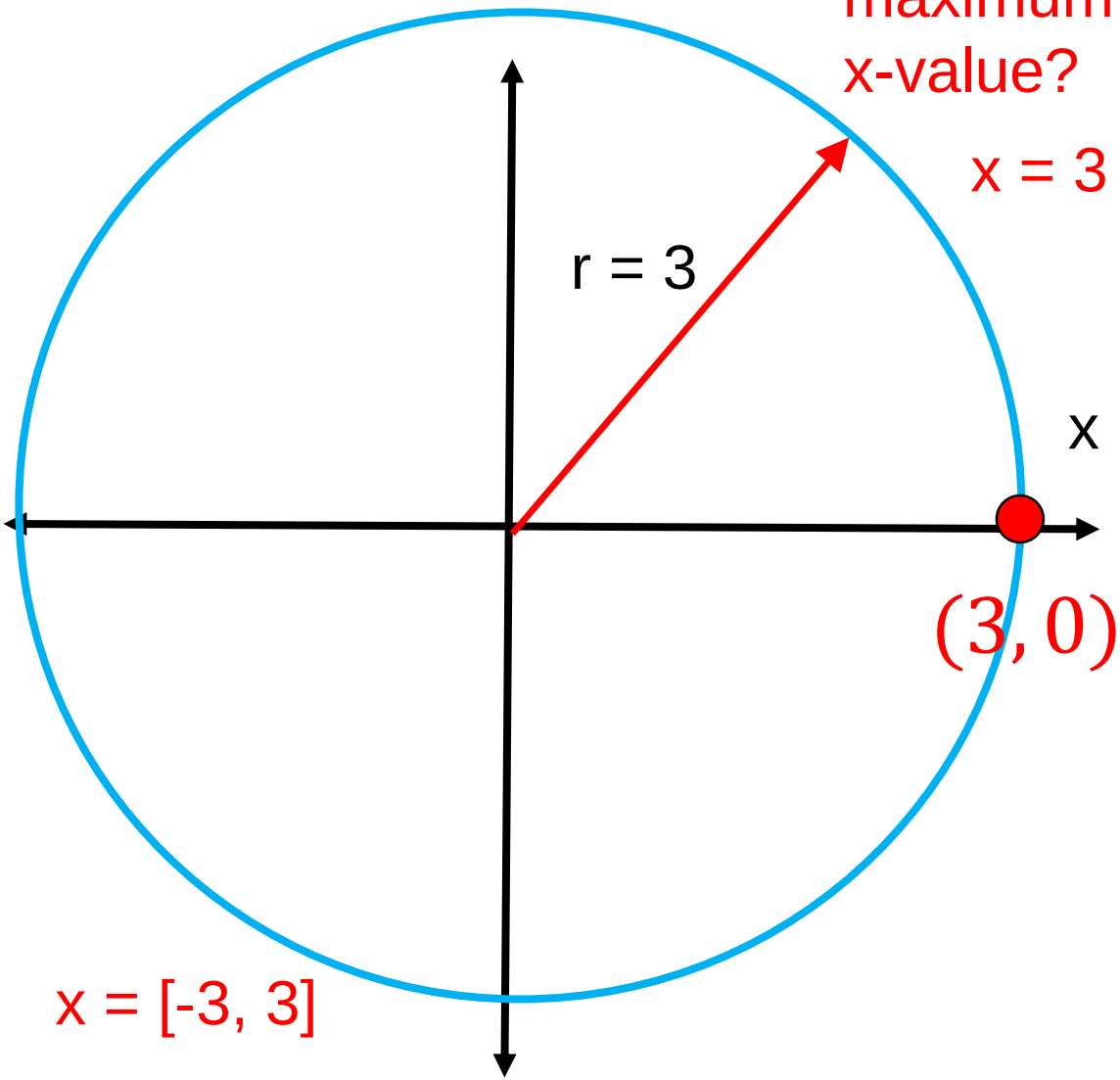
$$(3)^2 + y^2 = 9$$

$$9 + y^2 = 9$$

$$y = 0$$

What is the maximum x-value?

x = 3




What is the domain? x = [-3, 3]

What is the range? y = [-3, 3]

What are the y-values when x = 2?

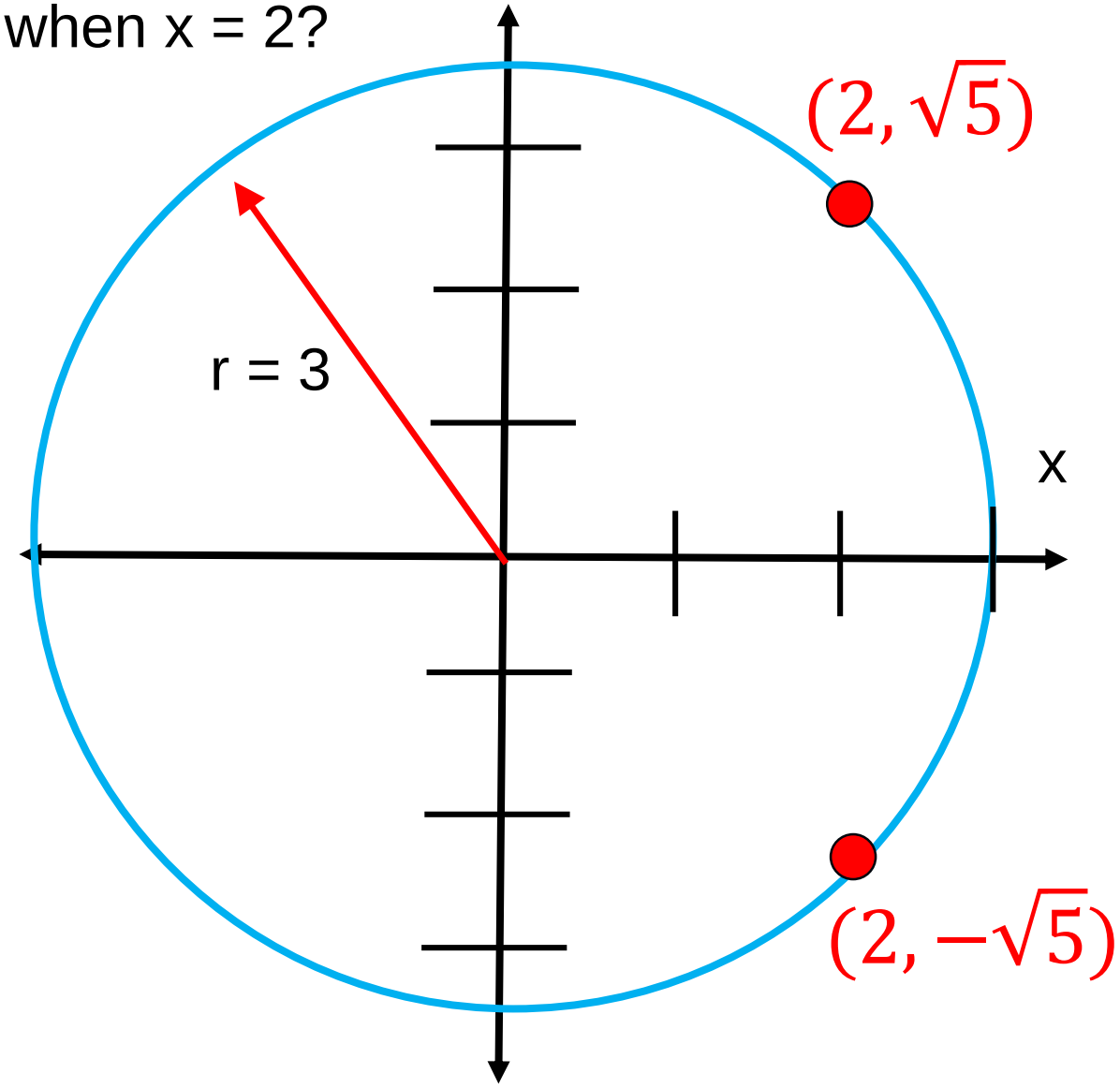
$$x^2 + y^2 = 9$$

$$(2)^2 + y^2 = 9$$

$$4 + y^2 = 9$$

$$y^2 = 5$$

$$y = \pm\sqrt{5}$$



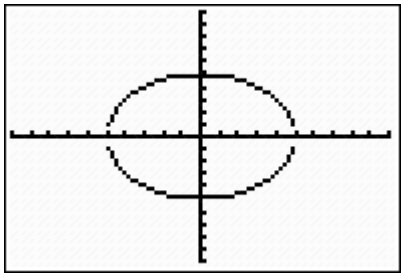
# How an Equation Makes a Circle.

$x^2 + y^2 = 25$ How do you graph this on your calculator??!!!

$y^2 = 25 - x^2$ Your turn, solve for 'y'

$y = \sqrt{25 - x^2}$ Graph this equation on your calculator.
Why do you only get the top half?

$y = -\sqrt{25 - x^2}$ Also Graph this equation on your calculator.



Why is the circle "squished"?

The physical width of the calculator window is wider than the calculator window height so you can't use the same "window" numbers for both.

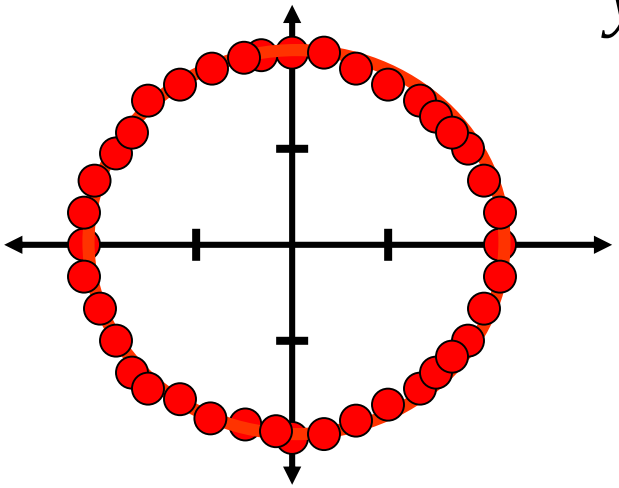
To reflect the physical dimensions of the calculator window, use "Zoom Square".

Now we just plug in number to <u>both equations.</u>

$$x^2 + y^2 = 4$$

$$y = \sqrt{4 - x^2}$$

$$y = -\sqrt{4 - x^2}$$



More inputs result in more outputs.

$$y = -\sqrt{4 - 0^2} = -2$$

$$y = \sqrt{4 - 0^2} = 2$$

$$y = \sqrt{4 - 1^2} \approx 1.7$$

$$y = -\sqrt{4 - 1^2} \approx -1.7$$

Each *<u>input</u>* will give a "+' and a "-" output.

$$y = \sqrt{4 - \left(\tfrac{1}{3}\right)^2} \approx 1.97$$

$$y = -\sqrt{4 - \left(\tfrac{1}{3}\right)^2} \approx -1.97$$

(except for the input 2 and -2)

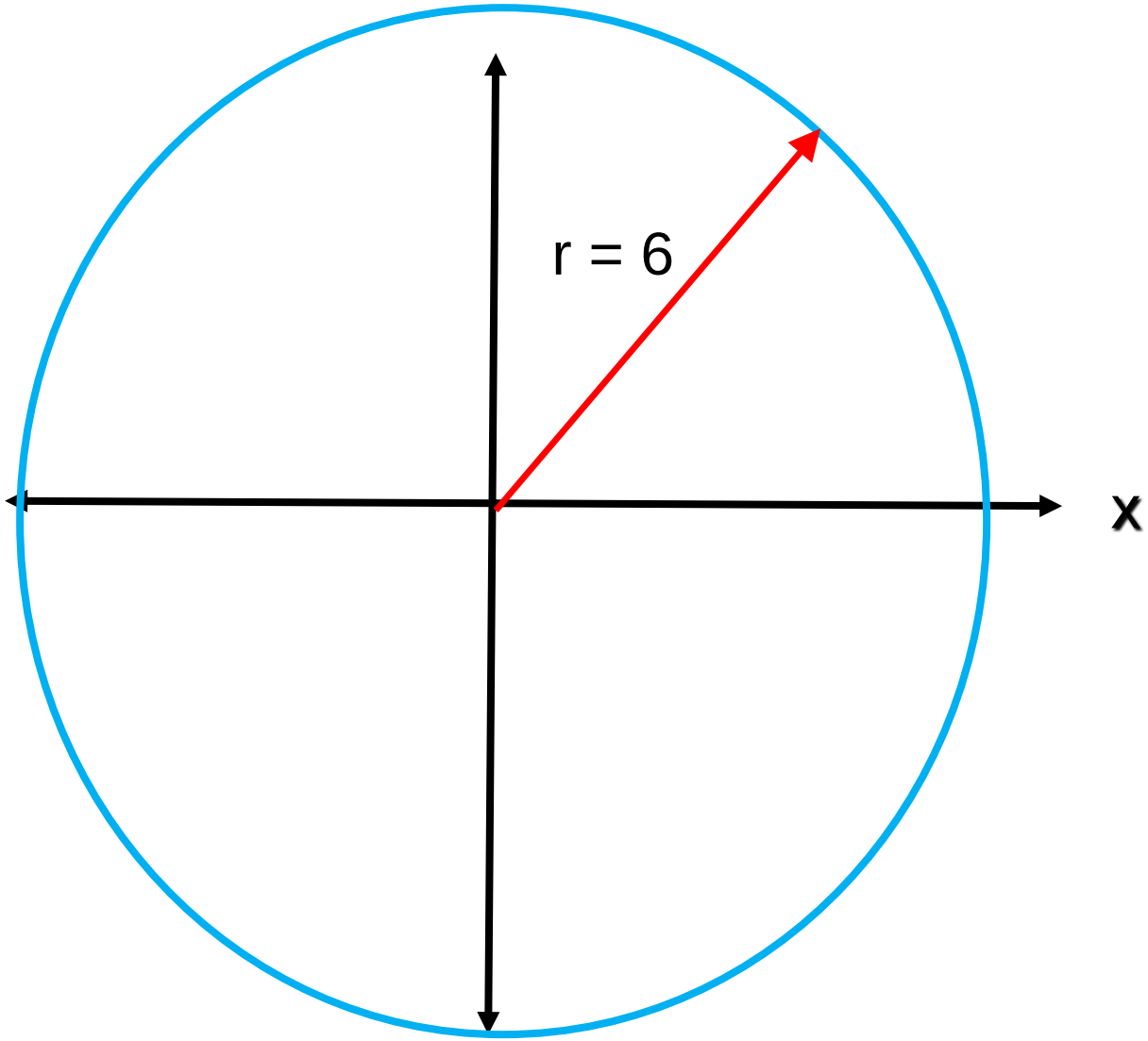
| x | y |
|---|---|
| 0 | -2 |
| 0 | 2 |
| 1 | $\sqrt{3} \approx 1.7$ |
| 1 | $-\sqrt{3} \approx -1.7$ |
| ⅓ | $\approx 1.97$ |
| ⅓ | $\approx -1.97$ |
| 2 | 0 |
| -2 | 0 |

What is the equation of the circle?

$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = 6^2$$

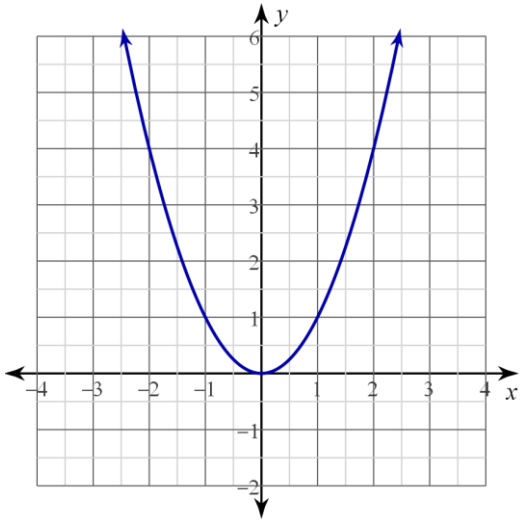
$$x^2 + y^2 = 36$$



# Graphical Transformations

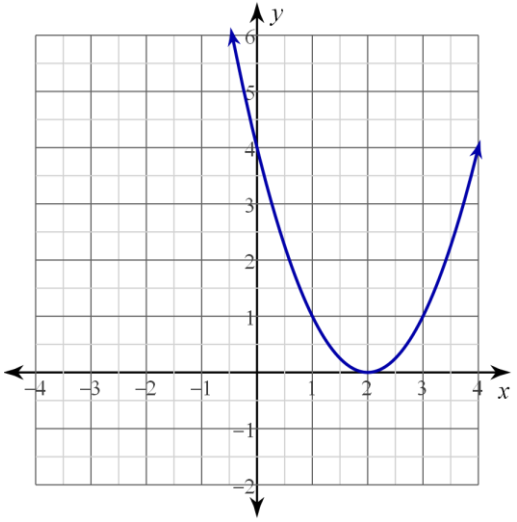
Parent Function: The simplest function in a family of functions (lines, parabolas, cubic functions, etc.)

$$f(x) = x^2$$



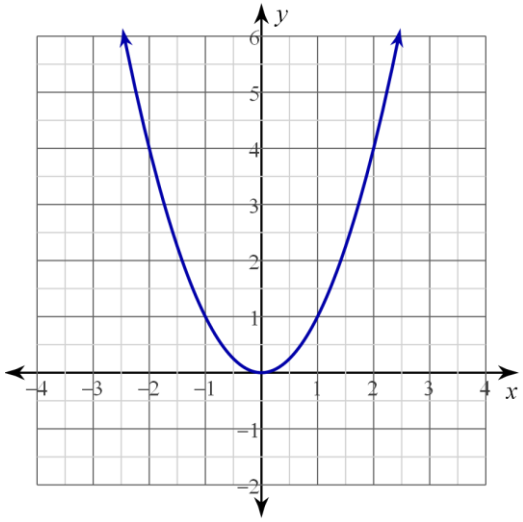

How is g(x) a transformation of f(x)?

$$g(x) = (x-2)^2$$




Replacing ‘x’ with ‘x – 2’ translates the parent function right by 2.

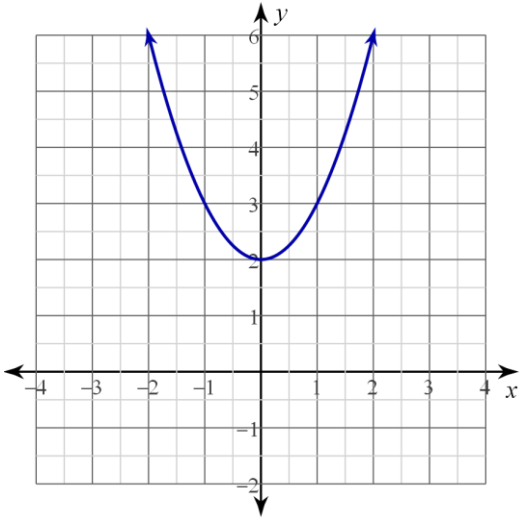
$f(x) = x^2$



How is h(x) a transformation of f(x)?

Up 2

$h(x) = x^2 + 2$



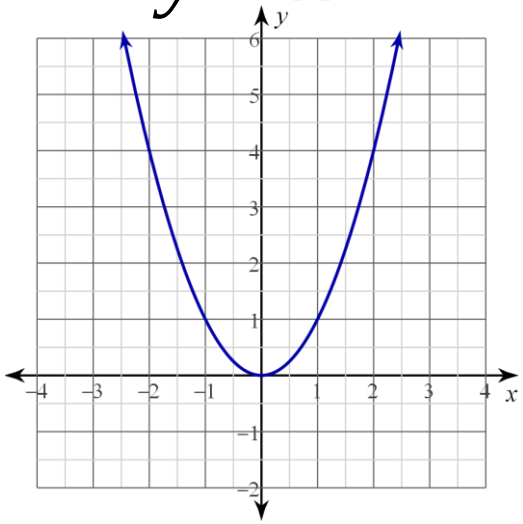
Notice: Subtract 2 from both sides yields:

$$y - 2 = x^2$$

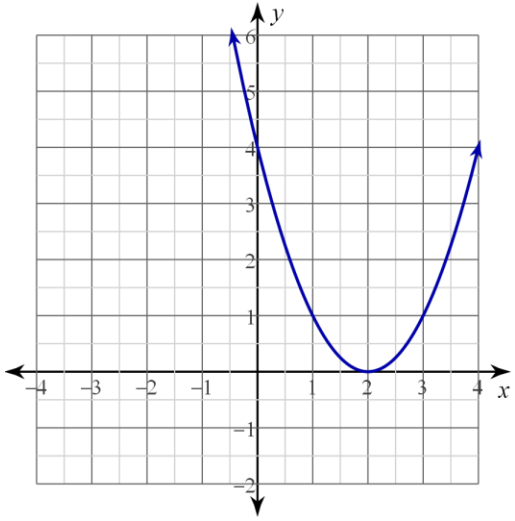
$$(y - 2) = x^2$$

Replace “y” in the parent function with (y – 2) moves the graph up 2.

$y = x^2$

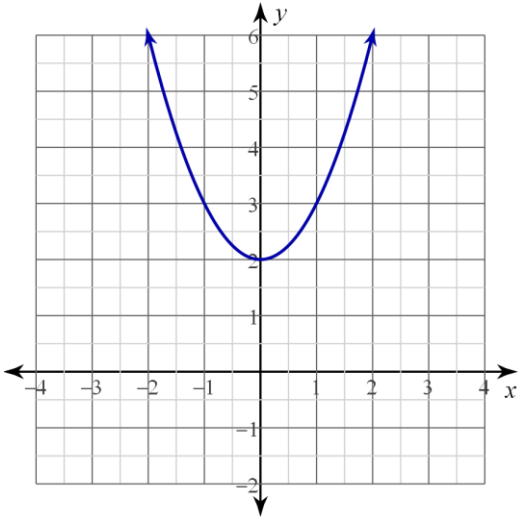



$y = (x-2)^2$




Replacing ‘x’ with (x – 2) moves the parent function right by 2.

$(y-2) = x^2$




Replace “y” with (y – 2) moves the parent function up by 2.

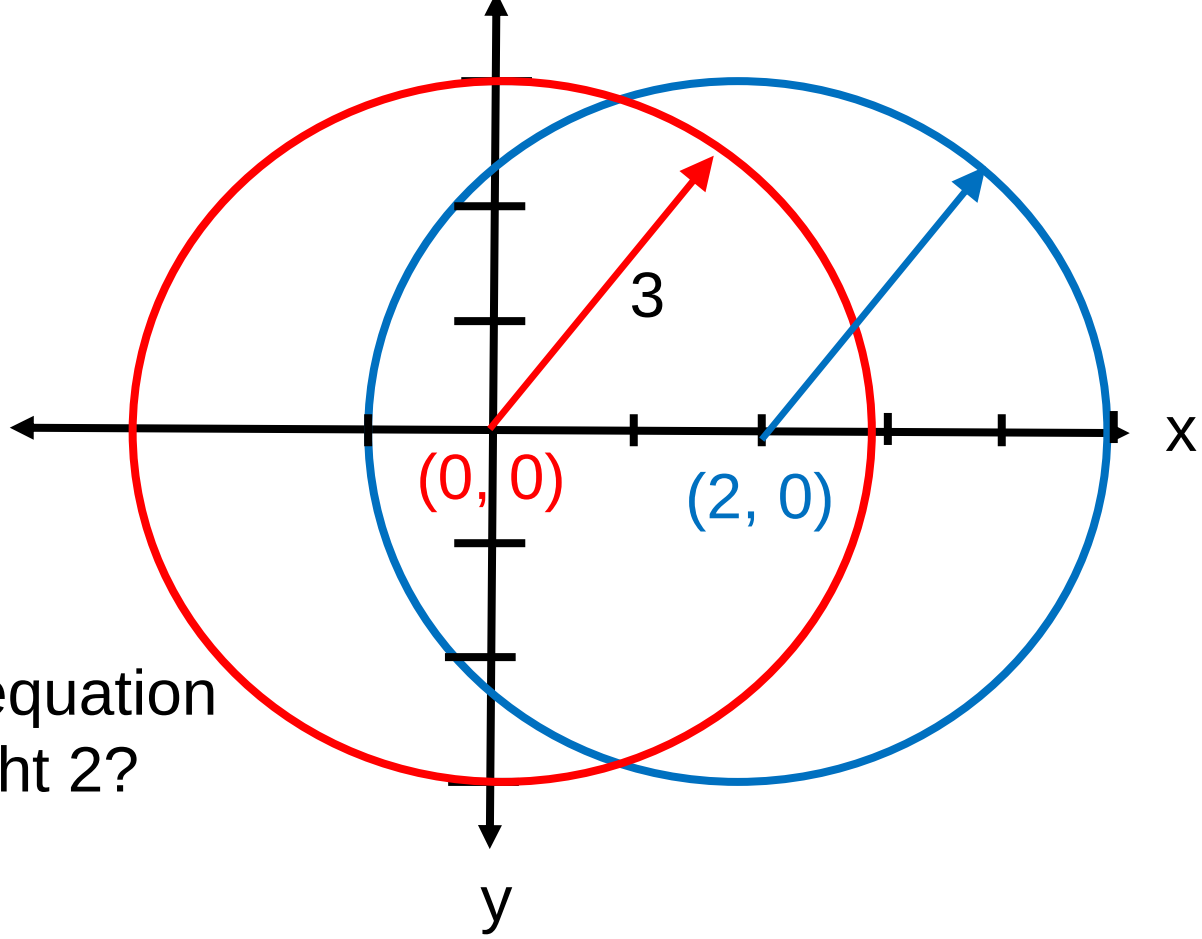
$x^2 + y^2 = 9$

What is the domain?

x = [-3, 3]

What is the range?

y = [-3, 3]




How do we change the equation to translate the graph right 2?

Replace ‘x’ with (x – 2)

$(x - 2)^2 + y^2 = 9$

What is the domain?

x = [-1, 5]

What is the range?

y = [-3, 3]

How do we change the equation to translate the graph down 2?

$x^2 + y^2 = 9$

Replace ‘y’ with (y + 2)

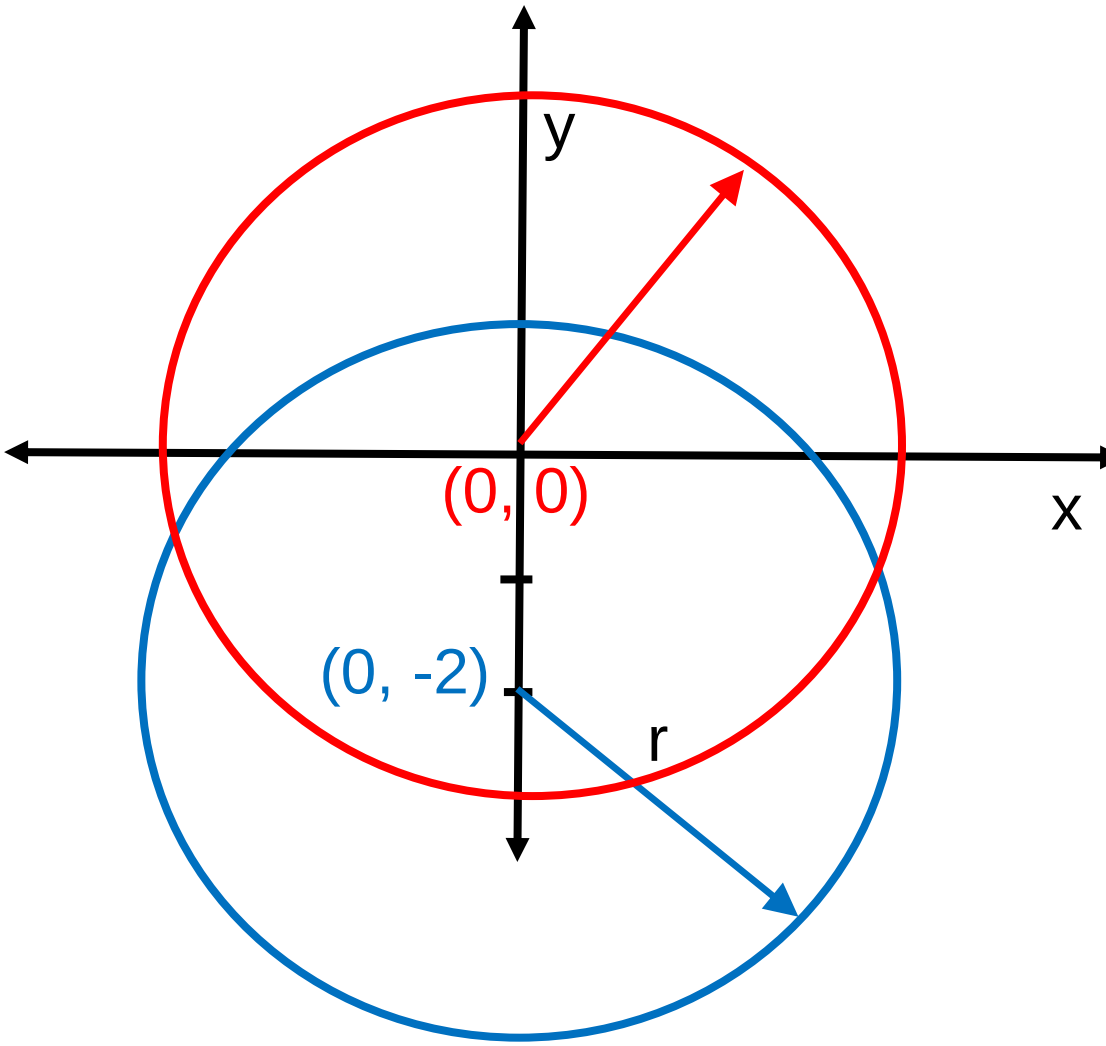
$x^2 + (y + 2)^2 = 9$

What is the domain?

x = [-3, 3]

What is the range?

y = [-5, 1]



# What is the radius and center of the circle?

$x^2 + y^2 = 25$ radius is 5 No left/right or up/down shift → center is (0, 0).

$(x+3)^2 + y^2 = 25$ Left 3 shift → center is (-3, 0)

$(x-5)^2 + (y+2)^2 = 25$ Right 5, down 2 shift → center is (5, -2)

$(x-7)^2 + y^2 = 49$ radius is 7
Right 7 shift → center is (7, 0)

$(x+3)^2 + (y-4)^2 = 5$ $radius = \sqrt{5}$

Left 3, up 4 shift → center is (-3, 4)

# Equations of Circles

$$r^2 = (x-h)^2 + (y-k)^2$$

'r' is the radius of the circle.

(h, k) is the center of the circle.

$9 = (x-3)^2 + (y+4)^2$ 'r' = ? (h, k) = ?

r = 3 center is (3, -4)

$3 = (x+2)^2 + (y-1)^2$ 'r' = ? (h, k) = ?

$r = \sqrt{3}$ center is (-2, 1)

Equations of Circles

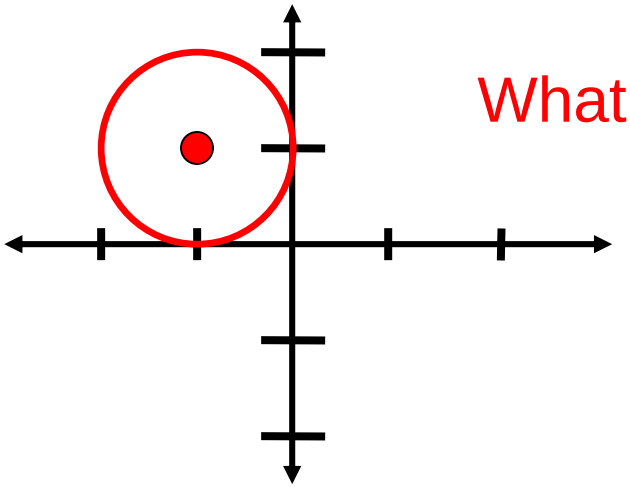
$$r^2 = (x-h)^2 + (y-k)^2$$

'r' is the radius of the circle.

(h, k) is the center of the circle.

'r' = ?  'r' = 1

(h, k) = ?  (h, k) = (-1, 1)



What is the equation of the circle?

$$1^2 = (x-(-1))^2 + (y-1)^2$$

$$1 = (x+1)^2 + (y-1)^2$$

Write the equation of a circle centered at (0, 3) with a radius of 4.

$$x^2 + (y-3)^2 = 16$$

Write the equation of a circle centered at (3, -2) with a radius of 6.

$$(x-3)^2 + (y+2)^2 = 36$$

Write the equation of a circle centered at (-2, -4) with a radius of SQRT(5).

$$(x+2)^2 + (y+4)^2 = 5$$

Prove that a point is on a circle:

The circle below is the graph of: $(x-1)^2+(y-2)^2=4$

Is the point $(2, 2+\sqrt{3})$ on the circle?

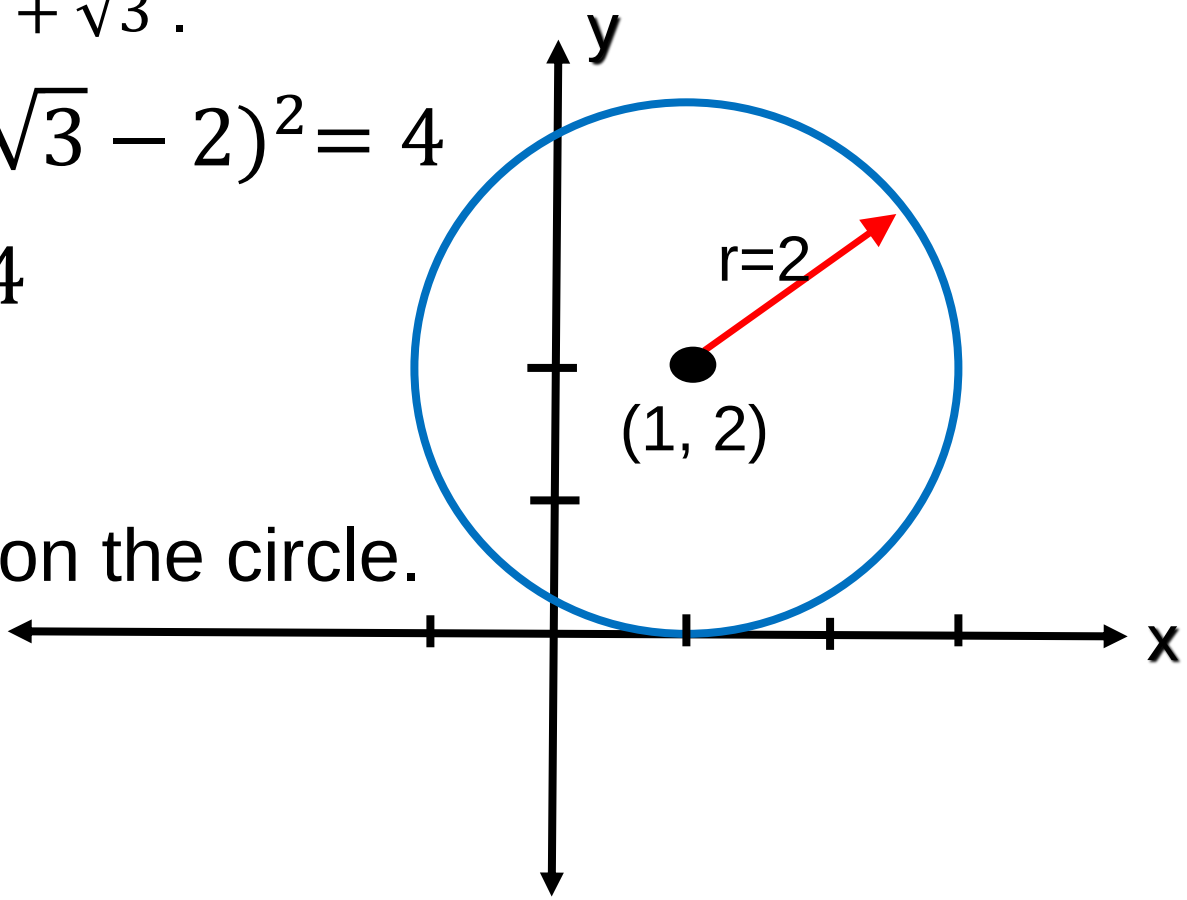
Plug in x = 2, y = $2+\sqrt{3}$ .

$$(2-1)^2+(2+\sqrt{3}-2)^2=4$$

$$(1)^2+(\sqrt{3})^2=4$$

$$1+3=4$$

Yes $(2, 2+\sqrt{3})$ is on the circle.



$(x-5)^2+(y-7)^2=25$

Is the point $(9, 4)$ on the circle?

Plug in x = 9, y = 4

$(9-5)^2+(4-7)^2=25$

$(4)^2+(-3)^2=25$

$16+9=25$

Yes $(9, 4)$ is on the circle.

Review

$$y = (x-1)^2 = (x-1)(x-1) = x^2 - 2x + 1$$

$$y = (x-2)^2 = (x-2)(x-2) = x^2 - 4x + 4$$

$$x = -\frac{b}{2a} \qquad x = -\frac{-4}{2(1)} \qquad x = 2$$

$y = (x-3)^2$ $\qquad$ $y = x^2 - 6x + 9$

$y = (x-4)^2$ $\qquad$ $y = x^2 - 8x + \underline{16}$

What number is missing?

$y = (x-5)^2$ $\qquad$ $y = x^2 - 10x + \underline{25}$

What number is missing?

This is called the *"number required to complete the square".*

Conic Section equation of a circle: $x^2 + y^2 - 2x - 4y = -1$

Notice that standard circles are written as squares of binomials.

$$(x-h)^2 + (y-k)^2 = r^2$$ **Complete the square!**

Separate into 'x' and 'y parts: $x^2 - 2x + y^2 - 4y = -1$

What do you add to $x^2 - 2x$ to complete the square?

$$x^2 - 2x + 1 + y^2 - 4y = -1 + 1$$

$$x^2 - 2x + 1 + y^2 - 4y = 0$$

$$(x-1)^2 + y^2 - 4y = 0$$

What do you add to $y^2 - 4y$ to complete the square?

$$(x-1)^2 + y^2 - 4y + 4 = 0 + 4$$

$$(x-1)^2 + (y-2)^2 = 4$$

What is the center and radius of the circle?:

**Complete the square!** $x^2 + y^2 - 6x + 8y = 0$

Separate into 'x' and 'y parts: $x^2 - 6x + y^2 + 8y = 0$

What do you add to $x^2 - 6x$ to complete the square?

$$x^2 - 6x + 9 + y^2 + 8y = 0 + 9$$

$$x^2 - 6x + 9 + y^2 + 8y = 9$$

$$(x - 3)^2 + y^2 + 8y = 9$$

What do you add to $y^2 + 8y$ to complete the square?

$$(x - 3)^2 + y^2 + 8y + 16 = 9 + 16$$

$$(x-3)^2 + (y+4)^2 = 25$$

$$(h, k) = (3, -4) \quad \mathrm{r} = 5$$