Math-2

Lesson 7-5

Triangle Similarity

Vocabulary

Proportion: An equation where a fraction equals a fraction.

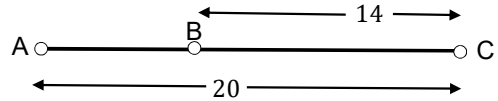
$$\frac{3}{6}=\frac{1}{2}$$

Proportional: to be related by a constant ratio. We say sides are proportional if the ratios of corresponding sides equals the same number.

$$\frac{AE}{AC}=\frac{AD}{AB}=\frac{DE}{BC}=\frac{10}{5}=2$$

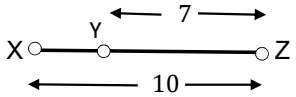
Ratio: a fraction

Compare BC to AC with a ratio.



$$\frac{BC}{AC}=?\quad=\frac{14}{20}$$

Compare YZ to XZ with a ratio.



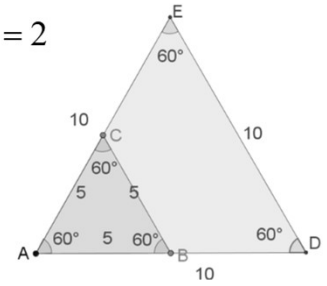
$$\frac{YZ}{XZ}=?\quad=\frac{7}{10}$$

Proportional: to be related by a constant ratio. We say lengths are proportional if the ratios of corresponding lengths equals the same number.

Proportional: to be related by a constant ratio. We say sides are proportional if the ratios of corresponding sides equals the same number.

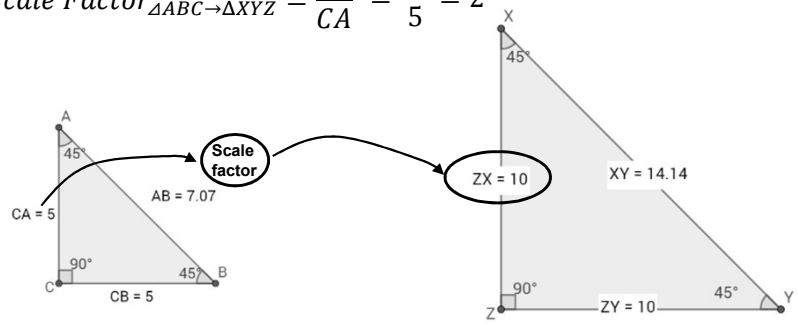
$$\frac{AE}{AC} = \frac{AD}{AB} = \frac{DE}{BC} = \frac{10}{5} = 2$$

The side lengths of ΔADE are twice as long as the side lengths in ΔABC



Scale Factor: the number that is multiplied by the length of each side of one triangle to equal the lengths of the sides of the other similar triangle.

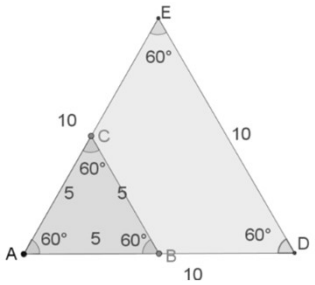
$$Scale\ Factor_{\Delta ABC \to \Delta XYZ} = \frac{ZX}{CA} = \frac{10}{5} = 2$$



Vocabulary

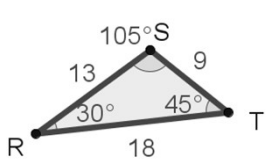
Similar: Same shape but not necessarily the same size.

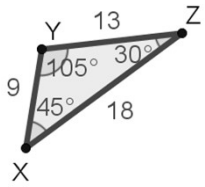
Similar Symbol: ~



Review: Triangle Congruence

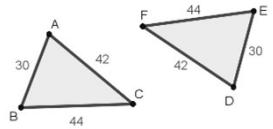
All 3 pairs of corresponding angles and all 3 pairs of corresponding sides are congruent (CPCTC)



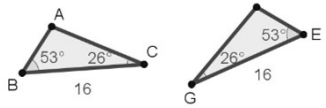


We can prove Triangle Congruence using congruence of only three pairs of corresponding parts.

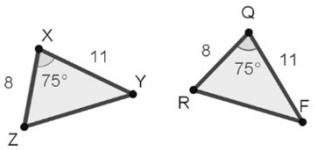
Side-Side-Side (SSS)



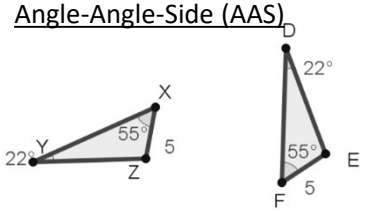
Angle-Side-Angle (ASA)



Side-Angle-Side (SAS)



Angle-Angle-Side (AAS)



Triangle Similarity: IF all corresponding angles are congruent and all corresponding sides are proportional THEN the triangles are similar.

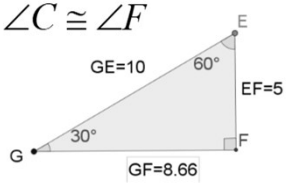
$\angle A \cong \angle G$
$\angle B \cong \angle E$
$\angle C \cong \angle F$

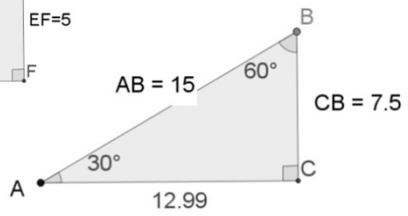
$\frac{AB}{GE} = \frac{15}{10} = \frac{3}{2}$ $\frac{BC}{EF} = \frac{7.5}{5} = \frac{3}{2}$

$\frac{AC}{GF} = \frac{12.99}{8.66} = \frac{3}{2}$

$\Delta ABC \sim \Delta GEF$

Similarity statement.



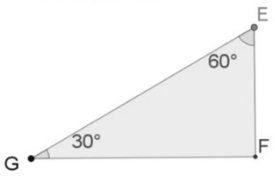


Triangle Similarity: But we don't need all corresponding angles are congruent and all corresponding sides are proportional.

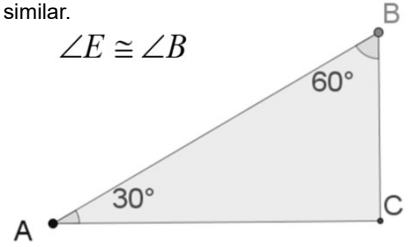
We can get by with the following patterns: AA, SSS, and SAS

Angle-Angle (AA) Triangle Similarity: IF two pairs of corresponding angles are congruent THEN the triangles are similar.

$\angle G \cong \angle A$



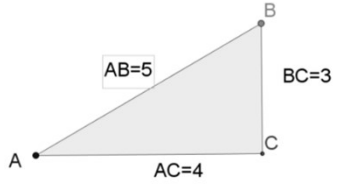
$\angle E \cong \angle B$

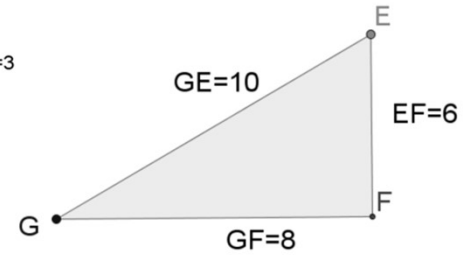


Why don't we need AAA?

Side-Side-Side (SSS) Triangle Similarity: IF all three pairs of corresponding sides are proportional THEN the triangles are similar.

$\frac{AB}{GE} = \frac{BC}{EF} = \frac{AC}{GF} = \frac{5}{10} = \frac{1}{2}$

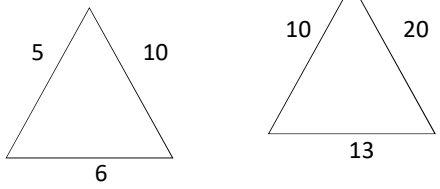




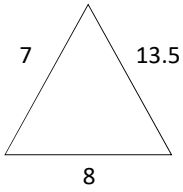
Examples of SSS Triangle similarity

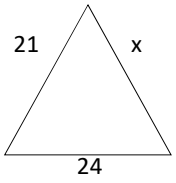
$$\frac{side_{Tri-1}}{side_{Tri-2}} = \frac{10}{5} = \frac{20}{10} \neq \frac{13}{6}$$

NOT similar

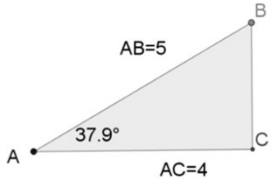


If the triangles to the right are similar, what must be the value of 'x'?





Side-Angle-Side (SAS) Triangle Similarity: IF two pairs of corresponding sides are proportional and the included angles are congruent THEN the triangles are similar.



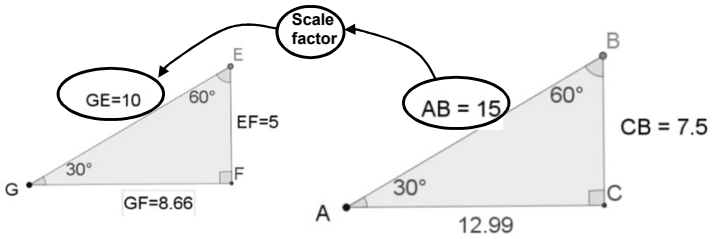
$$\frac{AB}{GE} = \frac{AC}{GF} = \frac{5}{10} = \frac{4}{8} = \frac{1}{2}$$

$$\angle G \cong \angle A$$

Scale Factor: the number that is multiplied by the length of each side of one triangle to equal the lengths of the sides of the other similar triangle.

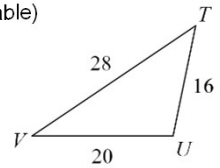
$$AB(\text{scale factor}) = GE$$

$$\text{scale factor}_{\Delta ABC \to \Delta GEF} = \frac{GE}{AB} = \frac{10}{15} = \frac{2}{3}$$



If the triangles are similar:
a) Show that the triangles are similar using ratios (if applicable)
b) give the similarity theorem
c) write the similarity statement.
d) write the scale factor (small Δ to large Δ)

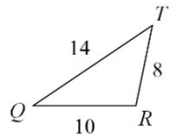
$$\frac{VT}{QT} = \frac{28}{14} = 2 \qquad \frac{TU}{TR} = \frac{16}{8} = 2 \qquad \frac{VU}{QR} = \frac{20}{10} = 2$$



SSS Triangle Similarity

$\Delta TUV \sim \Delta TRQ$

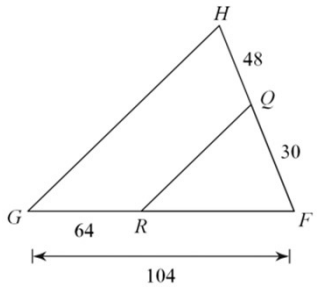
$$\text{scale factor}_{\Delta TRQ \to \Delta TUV} = 2$$

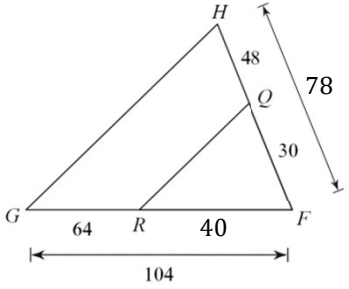


Name the two triangles. ΔFGH and ΔFRQ

Name the angle pair congruencies: $\angle F \cong \angle F$ $\angle HFG \cong \angle QFR$

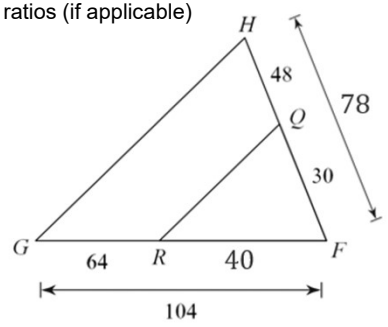
List the missing side lengths:





$RF =$ 104 − 64 = 40 $HF =$ 30 + 48 = 78

If the triangles are similar:
a) Show that the triangles are similar using ratios (if applicable)
b) give the similarity theorem
c) write the similarity statement.
d) write the scale factor (small Δ to large Δ)



$\frac{FG}{FR} = \frac{104}{40} = 2.60$ $\frac{FH}{FQ} = \frac{78}{30} = 2.60$

$\angle F \cong \angle F$

SAS Triangle Similarity

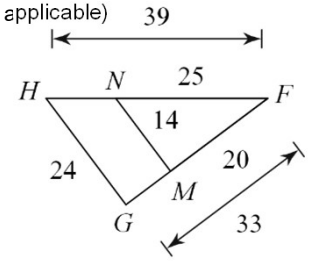
$\Delta FGH \sim \Delta FRQ$

scale factor$_{\Delta FRQ \to \Delta FGH}$ = 2.6

If the triangles are similar:
a) Show that the triangles are similar using ratios (if applicable)
b) give the similarity theorem
c) write the similarity statement.
d) write the scale factor (small Δ to large Δ)

$\frac{FG}{FM} = \frac{33}{20} = 1.65$

$\frac{FH}{FN} = \frac{39}{25} = 1.56$ NOT Similar



If the triangles are similar:
a) Show that the triangles are similar using ratios (if applicable)
b) give the similarity theorem
c) write the similarity statement.
d) write the scale factor (small Δ to large Δ)

$\angle HTU \cong \angle HGF$ (corresponding angles)

$\angle H \cong \angle H$

AA Triangle Similarity

$\Delta HGF \sim \Delta HTU$

scale factor = ??