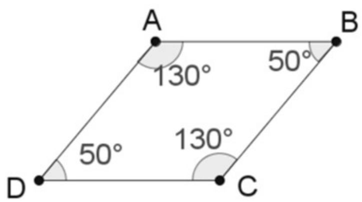
SM2 HANDOUT 7-4 (Properties of Parallelograms and Isosceles Triangles



Parallelogram Properties :

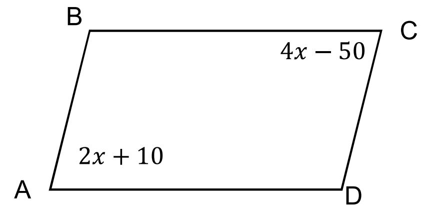
1. Opposite Angles are congruent.

$$m\angle A = m\angle C$$
$$m\angle B = m\angle D$$

2. Consecutive Interior Angles are supplementary.

$$m\angle A + m\angle B = 180$$

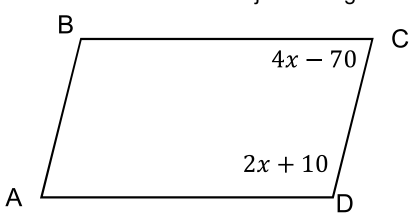
Math Problems from "Opposite Angles of Parallelograms are Congruent"



$x = ?$

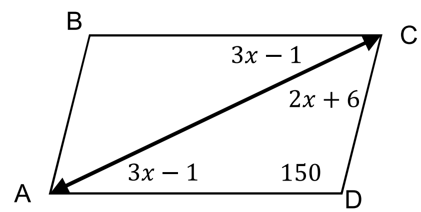
$m\angle A = ?$

Math Problems from "Adjacent Angles of Parallelograms are Supplementary"



$x = ?$

Math Problems from "Adjacent Angles of Parallelograms are Supplementary"



Segment AC is a ***diagonal***.

$x = ?$

$m\angle BCA + m\angle DCA = m\angle BCD$

Angle Addition Postulate

$m\angle ADC + m\angle BCD = 180$

Adjacent Angles of Parallelograms

$3x - 1 + 2x + 6 + 150 = 180$

$5x + 155 = 180$ $\boxed{x = 5}$

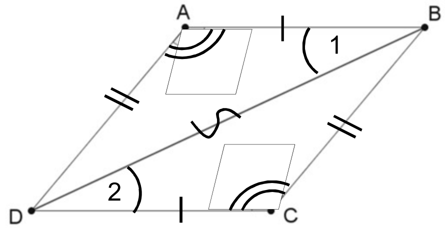
$\angle BCA \cong \angle DAC$ Alternate Interior Angles

$m\angle CAD + m\angle DCA + m\angle D = 180$ Triangle Angle Sum Theorem (we'll prove this later).

$3x - 1 + 2x + 6 + 150 = 180$

$5x + 155 = 180$ $\boxed{x = 5}$

If we could prove the diagonal forms two congruent triangles, we could use CPCTC to prove more properties of Parallelograms.



$m\angle A = m\angle C$

Opposite Angles are congruent.

$\angle 1 \cong \angle 2$

Alternate Interior Angles

$BD = DB$

Same segment → same length

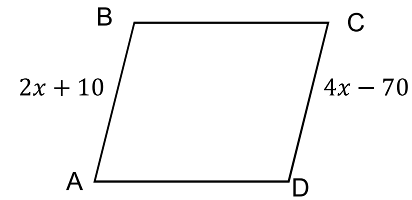
$\Delta ABC \cong \Delta CDB$

AAS Theorem

$AD = BC$ CPCTC

$AB = CD$ CPCTC

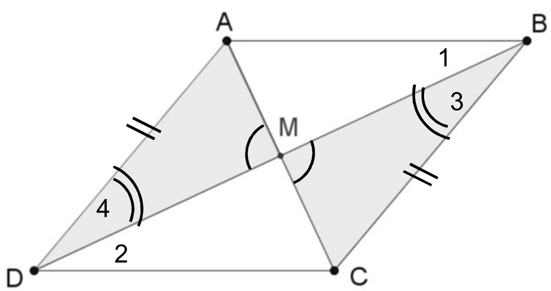
Math Problems from "Opposite Sides of Parallelograms are congruent"



$x = ?$

$AB = ?$

Can we prove that diagonals form two pairs of congruent triangles?



$\overline{AD} \cong \overline{CB}$

Opposite Sides are congruent.

$\angle AMD \cong \angle CMB$

Vertical Angles

$\angle 3 \cong \angle 4$

Alternate Interior Angles

$\Delta AMD \cong \Delta CMB$

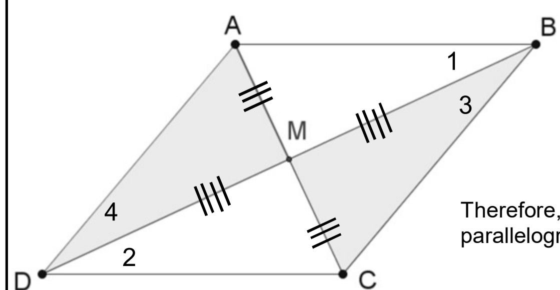
AAS Theorem

Using the other pairs of:

1) Opposite sides
2) Vertical angles
3) Alternate Interior Angles

$\Delta CMD \cong \Delta AMB$

AAS Theorem



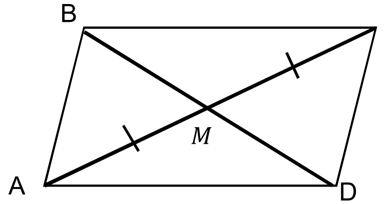
By CPCTC

$\overline{DM} \cong \overline{MB}$

$\overline{AM} \cong \overline{CM}$

Therefore, diagonals of parallelograms bisect each other.

Math Problems from “Diagonals of Parallelograms BISECT each other.”



$AC = 26$

$AM = 3x - 5$

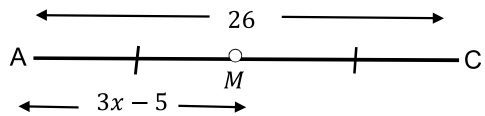
x = ?

1. Draw a picture of the diagonal and label the known measurements.

2. Write an equation that relates the lengths in the problem.

$2 * AM = AC$

$2(3x - 5) = 26$



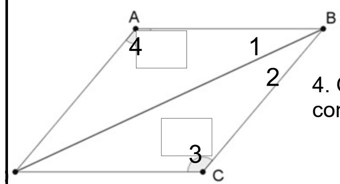
3. Solve for ‘x’. $3x - 5 = 13$

$3x = 18$

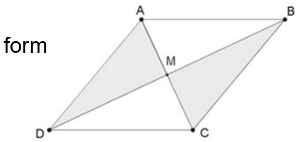
$x = 6$

Parallelogram Properties :

1. Opposite Angles are congruent. $m\angle 3 = m\angle 4$
2. Consecutive Interior Angles are supplementary. $m\angle 1 + m\angle 2 + m\angle 3 = 180$



3. A diagonal of a parallelogram forms two congruent triangles. $\triangle DAB \cong \triangle CBD$
4. Opposite Sides of parallelograms are congruent. $AB = CD$
5. Opposite triangles formed by the diagonals (plural) form congruent triangles. $\triangle AMD \cong \triangle CMB$

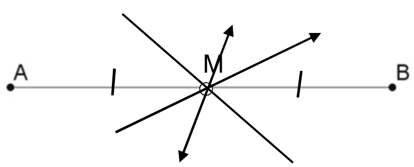


6. Diagonals of parallelograms bisect each other.

$AM = MC$ $\quad AC = 2 * MC$

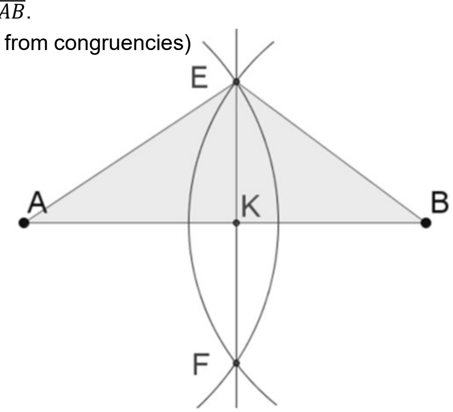
Segment Bisector: if a line segment is intersected by a ray, segment or line at the midpoint of the segment, then the ray, segment line is a segment bisector.

a) Another segment b) A ray c) A line.

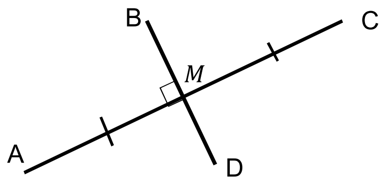


$\overline{EF}$ is a perpendicular bisector of $\overline{AB}$.

Are there any equations (that come from congruencies) that we can write from this result?



Math Problems from “Perpendicular Bisectors”

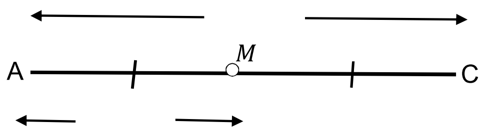


$AC = 26$

$AM = 3x - 5$

x = ?

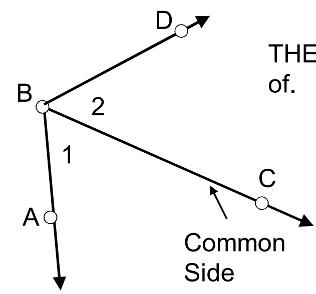
1. Draw a picture of the segment and label the known measurements.



2. Write an equation that relates the lengths in the problem. $2 * AM = AC$

3. Solve for ‘x’.

Angle Bisector: a common side of two adjacent angles that divides the angle into two angles of equal measure.



If $m\angle 1 = m\angle 2$

THEN $\overline{BC}$ is an angle bisector of. $\angle ABD$

Are there any equations that we can write from this result?

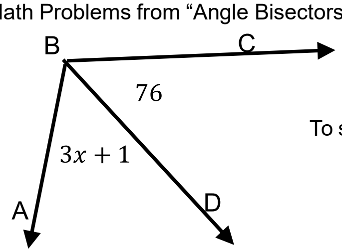
$m\angle ABC = m\angle DBC$

angle bisector

$m\angle ABD = 2 * m\angle DBC$

angle bisector

Math Problems from “Angle Bisectors”



$\overrightarrow{BD}$ is an angle bisector of $\angle ABC$

$x = ?$

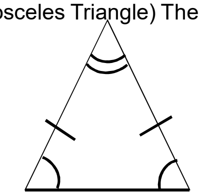
To solve for an unknown value, you need an equation.

$m\angle ABD = m\angle CBD$

Isosceles Triangle: A triangle with two congruent sides.

Legs: (Of an Isosceles Triangle) The two congruent sides.

Vertex Angle: (Of an Isosceles Triangle) The included angle of the legs.

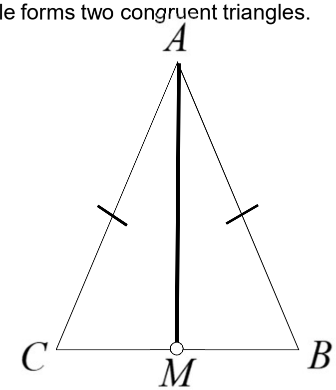


Base: (Of an Isosceles Triangle) The opposite the vertex angle.

Base Angles: (Of an Isosceles Triangle) The angles that include the base.

Given: ΔABC is an Isosceles Triangle and $\overline{AM}$ is an angle bisector of vertex angle A.

Prove that an angle bisector of an Isosceles Triangle forms two congruent triangles.



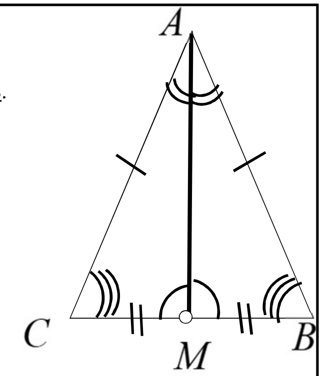
$\Delta CAM \cong \Delta BAM$

Congruent triangles give us SIX Pairs of congruencies.

$CM = BM$

$m\angle CMA = m\angle BMA$

$m\angle ACM = m\angle ABM$



Properties of Isosceles Triangles

1. The vertex and bisector forms two congruent triangles. $\Delta CAM \cong \Delta BAM$

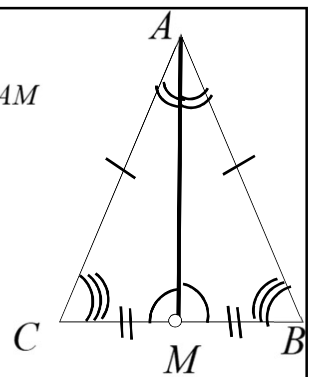
2. The vertex angle bisector is a perpendicular bisector of the base.

$m\angle CMA = m\angle BMA = 90$

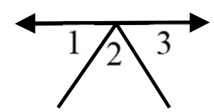
$CM = BM$

3. Base Angles are congruent.

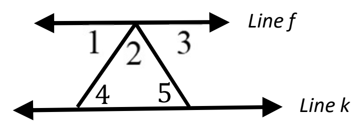
$m\angle ACM = m\angle ABM$



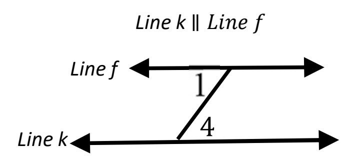
Triangle Sum Theorem: If $\angle A, \angle B, and \angle C$ are the interior angles of a triangle, then their measures add up to $180°$.



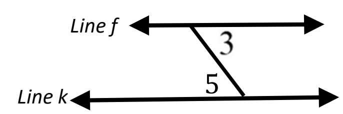
$m\angle 1 + m\angle 2 + m\angle 3 =$ $\underline{180°}$



$m\angle 4 + m\angle 2 + m\angle 5 =$ $\underline{180°}$



$m\angle 1$ __=__ $m\angle 4$



$m\angle 3$ __=__ $m\angle 5$

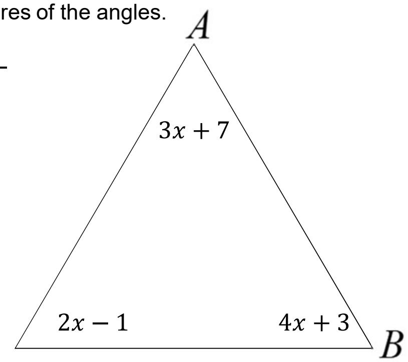
Math Problems from "The Triangle Sum Theorem."

1. Write an equation that relates the measures of the angles.

$m\angle A + m\angle B + m\angle C = \underline{\ 180^\circ\ }$

2. Substitute the measures of the angles into the equation.

3. Solve for 'x'.



Constructing a Perpendicular Bisector

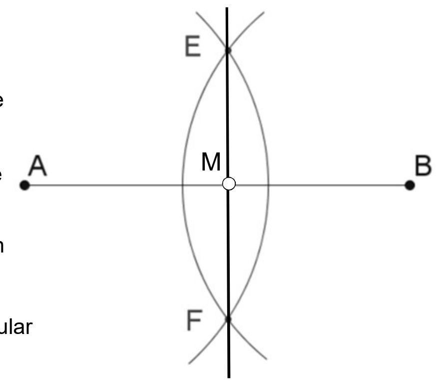
Given a line segment AB

1) Using a compass draw two arcs of equal radius using the endpoints as the center of each are.

2) Construct a point where the two arcs intersect.

3) Construct a line through these two points.

4) $\underline{\overline{EF}}$ Is the perpendicular bisector of $\underline{AB}$



Constructing an Angle Bisector

Given $\angle B$

1) Using a compass draw an arc using point B as the center.

2) Construct two points (points A and C) where the arc intersects the side of the angles

3) Construct $\overline{AC}$

4) Construct a perpendicular bisector of $\overline{AC}$

5) $\underline{\overline{BM}}$ is the angle bisector of $\underline{\angle ABC}$