Math-2
Lesson 6-4

Solving Systems of Equations by Elimination

Elimination Method Add or subtract multiples of one equation to the other equation to eliminate one of the variables.

What Property allows adding equations?

$$3x - 2 = -2x + 8$$
$$+2x \qquad +2x$$

$$3x - 2 = -2x + 8$$
$$2x \quad = \quad 2x$$

Adding two equations means "adding equivalent values to the left and right sides of an equation".

The property of equality!!

Slide #3: Easiest Problem

$$\begin{aligned} x - 3y &= 5 \\ -x + 5y &= 3 \\ \hline 2y &= 8 \\ y &= 4 \end{aligned}$$

$x - 3(4) = 5$

$x = 17$

Solution: (17, 4)

Even though the equation that we add does not look the same left and right of the '=' sign,

the "=" sign guarantees left side is equivalent to the right side of the equation.

Replace 'y' with 4 in either of the original equations, then solve for 'x'.

Slide #4: Requires some work

$2x - 2y = 6$
$-x + 6y = 7$

If we just add these two equations, no variables will be eliminated.

Add or subtract *multiples of one equation* to/from the other to eliminate one of the variables

$$\begin{aligned} 2x - 2y &= 6 \\ (2)\,[-x + 6y] &= [7]\,(2) \\ \hline 2x - 2y &= 6 \\ -2x + 12y &= 14 \\ \hline 10y &= 20 \\ y &= 2 \end{aligned}$$

x = ?

Substitute y = 2 into either of the original equations, then solve for 'x'.

$2x - 2(2) = 6$

$2x - 4 = 6$

$2x = 10$

$x = 5$

Solution: (5, 2)

Slide #5: The Hardest problem

(8)[9x – 5y]= [18](8)

(5)[-10x + 8y]=[2] (5)

$72x - 40y = 144$

$-50x + 40y = 10$

$22x = 154$

$x = 7$

Requires the most work: you must multiply both equations equation different numbers to obtain same coefficient but opposite sign on one of the variables. (In this case let's eliminate the y-variable first.

Substitute x = 7 into either of the original equations, then solve for 'y'.

$9(7) - 5y = 18$

$63 - 5y = 18$

$-5y = -45$

$y = 9$

Solution: (7, 9)

In summary,
there are 3 Levels of Difficulty for Elimination Problems

x – 3y = 5
-x + 5y = 3

Easy: (1st example on slide #3)) → same coefficient but opposite sign on one of the variables. If you just add the equations, one of the variables is eliminated.

2x – 2y = 6
(2)[-x + 6y]=[7](2)

Requires some work: (2nd example on slide #4) you must multiply one equation by a number to obtain same coefficient but opposite sign on one of the variables.

(8)[9x – 5y]= [18](8)
(5)[-10x + 8y]=[2] (5)

Requires the most work: (3rd example on slide #5) you must multiply both equations equation different numbers to obtain same coefficient but opposite sign on one of the variables.

Solve using elimination.

$$\begin{aligned} 2x - y &= 2 \\ 4x + 2y &= 8 \end{aligned}$$

$$\begin{aligned} -6x - 3y &= 12 \\ 12x + 4y &= -8 \end{aligned}$$

$$\begin{aligned} -6x - 10y &= 2 \\ -12x - 20y &= 4 \end{aligned}$$

$$\begin{aligned} -3x + 8y &= -6 \\ 2x + 6y &= 4 \end{aligned}$$

How do you know how many solutions there are using the elimination method (1, 0, or infinitely many) ?

When you perform the elimination step and both variables disappears and you get a number equal to another number:

a. and it's true:
(3 = 3 or 0 = 0) → Infinitely many solutions (same line)

b. and it's false:
(-2 = 3 or 10 = 0) → No solution (parallel lines)