SM2: HANDOUT 2-6 (Analyzing the Graphs of Functions)
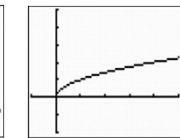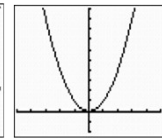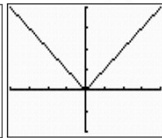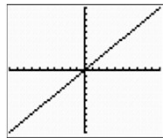
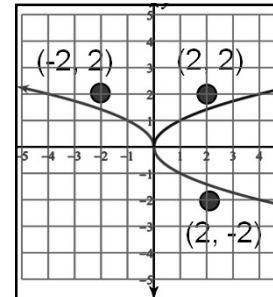Which of the functions are <u>symmetric across the y-axis?</u>

$f(x) = x$ $f(x) = |x|$ $f(x) = x^2$ $f(x) = \sqrt{x}$

<u>Even Function:</u> _____

What are the coordinates of this point if it is "reflected across the <u>y-axis</u>?

$(-2, 2)$ $(2, 2)$

$(2, -2)$

To reflect the graph of an equation across the y-axis we just <u>replace 'x' with</u> _____

$f(x) = \sqrt{x}$ $g(x) =$ [ ]

What are the coordinates of this point if it is "reflected across the <u>x-axis</u>?

To reflect the graph of an equation across the x-axis we just <u>replace 'y' with</u> _____ '

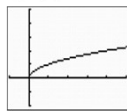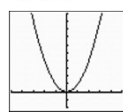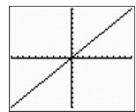$f(x) = \sqrt{x}$ $h(x) =$ [ ]

For which function does a reflection across the <u>y-axis</u> look exactly like a reflection across the <u>x-axis</u>?

$f(x) = x$ $f(x) = |x|$ $f(x) = x^2$ $f(x) = \sqrt{x}$
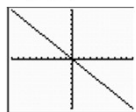
Reflection (y-axis)   Reflection (x-axis)
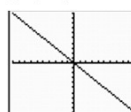$-f(x) = -x$ $f(-x) = -x$

<u>Odd Function:</u> a function whose reflection across the y-axis looks exactly like its reflection across the x-axis.

<u>Odd function:</u> If you reflect it across the x-axis, it looks exactly the same as if you reflect it across the y-axis.
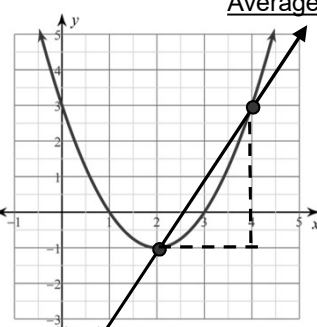
$f(x) = x^3$

$f(-x) = (-x)^3$ Means: _____

$-f(x) = -x^3$ Means: _____

$f(-x) = -f(x)$

Average Rate of Change

What is the "average rate of change" between x = 2 and 4?

Means "what is the slope of the graph between the two points

$(2, y_1)$ and $(4, y_2)$?

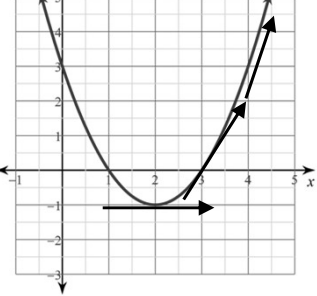$$slope = m = \frac{\Delta y}{\Delta x} = \frac{4}{2} = 2$$

What if you don't have the graph?

$$f(x) = (x - 2)^2 - 1$$

What is the "average rate of change" between x = 2 and 4?

Means "what is the slope of the graph between the two points $(2, f(2))$ and $(4, f(4))$?

$$f(2) = (2 - 2)^2 - 1 \qquad f(4) = (4 - 2)^2 - 1$$
$$f(2) = -1 \qquad\qquad f(4) = 3$$

$$slope = m = \frac{\Delta y}{\Delta x} \qquad m = \frac{3 - (-1)}{4 - 2} \qquad = \frac{4}{2} = 2$$

The Function is Increasing

→ if you draw a tangent line at a point on the graph and it has a positive slope, the function is increasing at that point.
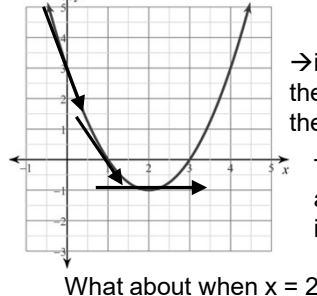
The slope of a tangent line at any point on the graph for the interval x = (2, ∞) is positive.

What about when x = 2?

The slope of a tangent line at x = 2 is zero (not increasing at that point).

We say: "the function _____

$$f(x) \uparrow on\ x = \rule{2cm}{0.4pt}$$

The function is decreasing

→if you draw a tangent line at a point on the graph, and it has a negative slope, the function is decreasing at that point.
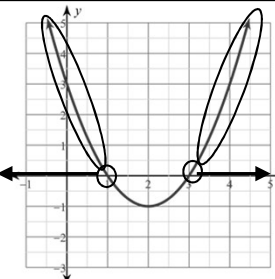
The slope of a tangent line at any point on the graph for the interval x = (-∞, 2) is negative.

What about when x = 2?

The slope of a tangent line at x = 2 is zero (not decreasing at that point).

We say the function is _____

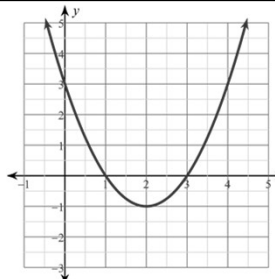$$f(x) \downarrow on\ x = \rule{2cm}{0.4pt}$$

Where is the function positive?

means: "What x-values have corresponding y-values that are positive"?
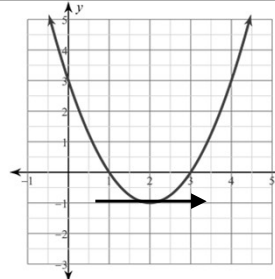
Or, "The graph is above the x-axis for what x-values"?

To see what these x-values are, shade the portion of the x-axis where the graph is above the x-axis.

We say: $f(x) > 0 \; for \; x = $ _____

Is the Function "even"?

→ means, "is the graph symmetrical about the y-axis"?

NO.

Is the Function "odd"?

→ means, "if the graph is reflected across the y-axis, would it look exactly the as if it were reflected across the x-axis"?

NO.

"Extrema"

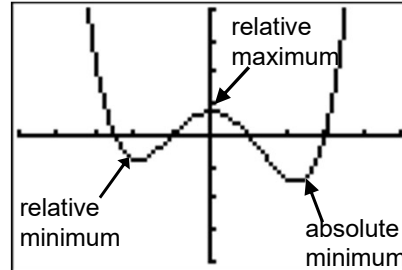Extrema: a point on a graph whose tangent line has a slope of zero.

Does the graph have any extrema?

Yes. The slope of a tangent line at (2, 0) is zero (slope changes from negative to positive at x = 2).

You can think of extrema an points on the graph that are "_____" or "_____".

Extrema: the y-value of points that are extrema are either
(1) the maximum or minimum y-value on the graph, OR
(2) compared the points adjacent to them, are either the maximum or minimum y-value.

Extrema: a point on a graph whose tangent line has a zero slope.

relative maximum
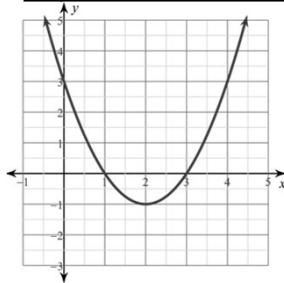
relative minimum

absolute minimum

We classify extrema by their y-values.

Absolute minimum (maximum): an extrema whose y-value is the smallest (largest) y-value for the entire function.
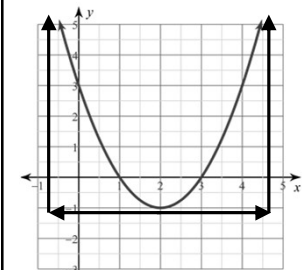
relative maximum (minimum): an extrema whose y-value is the greater than (less than) the y-value of points near it.

Graphical transformations of the parent

How does it relate
to its "parent function"?

The graph of $f(x) = (x - 2)^2 - 1$
is the parent function $f(x) = x^2$
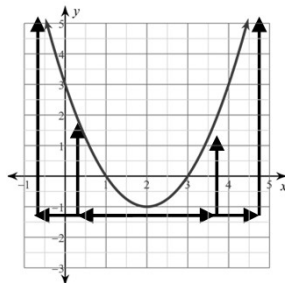transformed:

_____

"End Behavior"
is a description of which direction
the graph is going (up or down) on
the right and left ends of the graph.

In English we could say:
"up on right, up on left"

As 'x' gets bigger (right end) 'y' gets bigger (goes upward)

As 'x' gets smaller (left end), 'y' gets bigger (goes upward)

"Infinity Notation"
What is the "end behavior" of
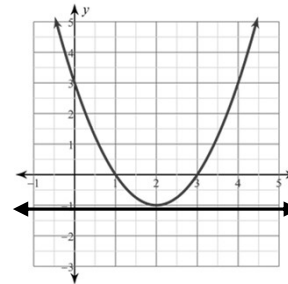the graph?

"Up on right".

As 'x' gets bigger, 'y' gets bigger.

$$\text{as } x \rightarrow +\infty, \; y \rightarrow +\infty$$

"Up on left"

As 'x' gets smaller, 'y' gets bigger.

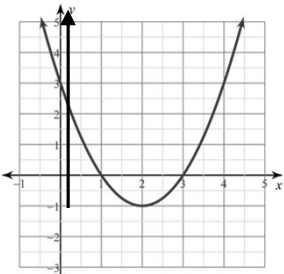$$\text{as } x \rightarrow -\infty, \; y \rightarrow +\infty$$

What is the "domain" of the graph?

Means "what values of "x" are
found in the graph?"

Which function is the only one
which does NOT have "all real
numbers" as the domain?

$$y = \sqrt{x}$$

For non-vertical lines, the square function, and the absolute
value function, we say the domain is "all real numbers."

Panel 1 (top left)

What is the "range" of the graph?

Means "what values of "y" are found in the graph?"

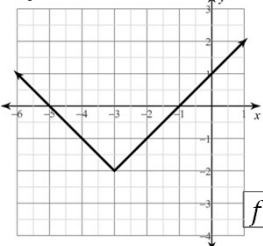The smallest y-value of this graph is zero, and it goes upward from there.

In "inequality notation" we say the range is: $y \geq 0$

In "interval notation" we say the range is: $y = [0, \infty)$

Panel 2 (top right)
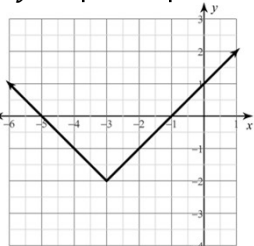
Analyzing the graph

1. Where is the function increasing?
2. Where is the function decreasing?
3. Where is the function positive?
4. Is the function even/odd or neither?
5. Are there any "extrema"? If so, what type are they?
6. How does it relate to its "parent function"?
7. What is the "end behavior" of the graph?

   as $x \to -\infty$, $y \to$?   as $x \to +\infty$, $y \to$?

8. What is the "domain" of the graph?
9. What is the "range" of the graph?
10. What is the "average rate of change" between two given values of 'x'?

Panel 3 (bottom left)

$y = |x + 3| - 2$

1. Where is the function increasing?

   $f(x) \uparrow on\ x =$ _____

2. Where is the function decreasing?

   $f(x) \downarrow on\ x =$ _____

3. Where is the function positive?

   $f(x) > 0\ for\ x =$ _____

4. Is the function even, odd or neither?

   _____

5. What are the "extrema"?

   _____

Panel 4 (bottom right)

$y = |x + 3| - 2$

6. How does it relate to its "parent function"?

   Parent translated _____

7. What is the "end behavior" of the graph?

   $x \to$ _____, $y \to$ _____

   $x \to$ _____, $y \to$ _____

8. What is the "domain" of the function?   _____

9. What is the "range" of the function? $y =$ _____

10. What is the "average rate of change" between x = -3 and x = -1 ?

   $m =$ _____