SM2 HANDOUT 2-6 Rational Exponents

$\sqrt[4]{x^{13}}$ We can simplify this in two ways

1. $\sqrt[4]{x*x*x*x*x*x*x*x*x*x*x*x*x}$

x

x

x

$\sqrt[4]{x^{13}} \rightarrow x*x*x*\sqrt[4]{x} \rightarrow x^3\sqrt[4]{x}$

2. $\sqrt[4]{x^{12}*x^1} \rightarrow x^{12/4}*\sqrt[4]{x} \rightarrow x^3\sqrt[4]{x}$

We can write radical as powers!! $\sqrt[4]{x^{13}} \rightarrow x^{13/4}$

Radicals CAN be written as Powers

$6\sqrt[5]{x}$: **Index**, **Radicand**, **Coefficient**

$6(x)^{1/5}$: Exponent, Base, Coefficient

Coefficient → Coefficient

Radicand → Base

Index → Denominator of the Exponent

The index number is the denominator of the exponent.

Your turn:

Write the following in "radical form"

5th Root of 18 $= \sqrt[5]{18}$

4th Root of 25 $= \sqrt[4]{25}$

What type of number does 5th sound like?

$1/5$

Are radicals related to powers?

$3^{1/2} = \sqrt[2]{3}$

$5^{1/3} = \sqrt[3]{5}$

$\sqrt[2]{x} = x^{1/2}$

$\sqrt[3]{7} = 7^{1/3}$

None of these have coefficients!

$3\sqrt[2]{y} = 3y^{1/2}$

$5\sqrt[3]{7} = 5(7)^{1/3}$

Multiplication (by a coefficient) is "repeated addition." This explains why coefficients of radicals become coefficients of powers.

$3\sqrt[2]{y} = \sqrt{y} + \sqrt{y} + \sqrt{y}$

$\sqrt{y} = y^{1/2}$

$3y^{1/2} = y^{1/2} + y^{1/2} + y^{1/2}$

What happens if there is a product under the radical?

$\sqrt[2]{xy}$ →

$5\sqrt[3]{3x}$ →

$2\sqrt[4]{21mn}$ →

How did we show that the index number applied to the entire product (radicand) when re-written in "power form"?

Power of a product → product inside parentheses with an exponent.

What happens if there is a power under the radical?

$\sqrt[5]{x^2 y}$ →

$6\sqrt[3]{3m^2}$ →

How did we show that the index number applied to the entire product (including the power) when re-written in "power form"?

Power of a product → product inside parentheses with an exponent.

"Exponential Form" that has both a numerator and denominator

The exponent can be written as a rational number.

$$x^{5/2} = \sqrt[2]{x^5}$$

Numerator: Exponent of the base.

Denominator: Root of the base.

$\sqrt[3]{2^2}$ →

Radical Form Exponential Form

Write the following radicals as powers.

$\sqrt[2]{3m}$ →

$4\sqrt[3]{5y}$ →

$3m\sqrt[4]{6n}$ →

$\sqrt[5]{x^3 y^2}$ →

$5\sqrt[4]{3m^2}$ →

Rewrite in "radical form"

$m^{1/5}$ →

$3nm^{1/4}$ →

$2(18n^2)^{1/6}$ →

$5(4x^2y^6)^{1/3}$ →

Multiply Powers Property $y^2 * y^3 = ? \quad = y^{2+3} \quad = y^5$

When multiplying "same based powers" add the exponents.

$x^{\frac{2}{3}} * x^{\frac{3}{4}} \quad \rightarrow x^{\frac{2}{3}+\frac{3}{4}}$ Yes, you must be able to add fractions

Working with just the exponent → $\frac{2}{3}+\frac{3}{4}$

Multiply by "1" in the form of…→ $\frac{2}{3}+\frac{3}{4} \quad \rightarrow \frac{8}{12}+\frac{9}{12} \rightarrow \frac{17}{12}$

Rewrite the power → $\rightarrow x^{\frac{17}{12}}$

Exponent of a Power Property $(y^2)^3 = ? \quad = y^{2*3} \quad = y^6$

When multiplying "same based powers" add the exponents.

$(y^{1/2})^{2/3} \quad = y^{\frac{1}{2}*\frac{2}{3}} \quad = y^{\frac{2}{6}} \quad = y^{\frac{1}{3}}$

$\left(\frac{x^2}{y^{3/2}}\right)^{2/3}$ →

$3x\left(y^{1/5}\right)^{2/3}$ →

$\left(x^{\frac{3}{4}}y^5\right)^{\frac{1}{3}}$ →

Negative Exponent Property

Grab and drag same-based powers to be next to each other.

$$\frac{x^2y^{2/3}}{y^{-1/2}} = x^2y^{2/3}y^{1/2} = x^2y^{\frac{2}{3}+\frac{1}{2}} = x^2y^{\frac{4}{6}+\frac{3}{6}} = x^2y^{\frac{7}{6}}$$

$\frac{2x^{\frac{1}{3}}}{x^{\frac{2}{3}}} \rightarrow \frac{2}{x^{\frac{2}{3}}x^{-\frac{1}{3}}} \rightarrow \frac{2}{x^{\frac{1}{3}}}$ Not allowed to have rational exponents in the denominator

$\rightarrow 2x^{-1/3}$ Not allowed to have negative exponents.

Rational Exponents in the Denominator

$$\frac{1}{y^{1/2}} = \frac{1}{\sqrt{y}}$$

Rational exponent in the denominator means irrational denominator, which we rationalize

$$\frac{1}{y^{1/2}} = \frac{1}{\sqrt{y}} * \frac{\sqrt{y}}{\sqrt{y}} = \frac{\sqrt{y}}{y}$$

Rational exponent in the denominator → what is the next bigger natural number from ½ ?

1

What number do you add to ½ to get 1?

$$\frac{1}{y^{1/2}} * \frac{y^{1/2}}{y^{1/2}} = \frac{y^{1/2}}{y}$$

In order to add a number to an exponent you have to multiply by a same-based power with the exponent you are trying to add.

Negative Exponent Property

$$\frac{2x^{\frac{1}{3}}}{x^{\frac{2}{3}}} \rightarrow \frac{2}{x^{\frac{2}{3}}x^{-\frac{1}{3}}} \rightarrow \frac{2}{x^{\frac{1}{3}}}$$

What is the next bigger whole number than 1/3 ?

1

What number do you add to 1/3 to get 1?

2/3

$$\rightarrow \frac{2}{x^{\frac{1}{3}}} * \frac{x^{\frac{2}{3}}}{x^{\frac{2}{3}}} \rightarrow \frac{2x^{\frac{2}{3}}}{x^{\frac{1}{3}+\frac{2}{3}}} \rightarrow \frac{2x^{\frac{2}{3}}}{x}$$

Multiply by one "in the form of" a same-base power whose exponent is 2/3 (both numerator and denominator)