Math-2 HANDOUT 2-4 (Radicals)

$\sqrt{3}$ What number is equivalent to the square root of 3?

$x = \sqrt{3}$ Square both sides of the equation

$(x)^2 = \left(\sqrt{3}\right)^2 \quad x^2 = 3$

$x = \sqrt{3}$ is an equivalent statement to $x^2 = 3$

$\sqrt{3} \approx 1.732$ There is no equivalent number

≈ 1.7321

≈ 1.73205

≈ 1.732051

$\approx 1.7320508...$

The decimal, is just an ____________

Radicals

$\sqrt{3} \rightarrow \sqrt[2]{3}$

$x = \sqrt[2]{3}$

$x^2 = 3$

The "square root of 3" means: "what ________________?"

$x = \sqrt[3]{4}$

$x^3 = 4$

The "3rd root of 4" means: "what ________________?"

$x = \sqrt[5]{2}$

$x^5 = 2$

The "5th root of 2" means: "what number ____________________?"

Adding and subtracting radicals

Can these two terms be combined using addition? $3x + 2x$

Write 3x as repeated addition $x + x + x$

Write 2x as repeated addition $x + x$

$3x + 2x \rightarrow x + x + x + x + x$ → ____

When multiplication is written as repeated addition, "like terms" look exactly alike.

$3\sqrt{x} + 2\sqrt{x} \rightarrow \sqrt{x} + \sqrt{x} + \sqrt{x} + \sqrt{x} + \sqrt{x}$ → ____

$3\sqrt{6} + 2\sqrt{6} \rightarrow \sqrt{6} + \sqrt{6} + \sqrt{6} + \sqrt{6} + \sqrt{6}$ → ____

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Define "like powers" "Same _____, same _______".

$3x^4 + 2x^4 \rightarrow 5x^4$

Define "like radicals" "Same ______, same ____________".

$3\sqrt{6} + 2\sqrt{6} \rightarrow 5\sqrt{6}$

Circle the "like radicals" that can be added.

$\sqrt{2} + \sqrt{3}$ $\qquad \sqrt[4]{5} + \sqrt[4]{5}$

$2\sqrt{3} + 3\sqrt{2}$ $\qquad 3\sqrt[5]{2} + 4\sqrt[5]{2}$

$\sqrt[4]{2} + \sqrt[3]{2}$ $\qquad 6\sqrt[3]{4} + 6\sqrt[4]{4}$

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$\sqrt{3}+\sqrt{2}$ Are they equivalent? $\rightarrow \sqrt{3+2}=\sqrt{5}$

$\sqrt{3}\approx 1.7321\ldots$ $\sqrt{2}\approx 1.4142\ldots$

$\sqrt{3}+\sqrt{2}\approx 3.1462\ldots$ $\sqrt{5}\approx 2.2630\ldots$

$\sqrt{3}+\sqrt{2}\neq\sqrt{5}$

$\sqrt{a}+\sqrt{b}\neq\sqrt{a+b}$ This is NOT a property of radicals. *NEVER DO THIS*!!!!

Another counter-example to show that this is not a property of radicals!!!

$\sqrt{4}+\sqrt{9} \rightarrow 2+3\rightarrow 5$

$\sqrt{4}+\sqrt{9} \neq \sqrt{13}$

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Simplify the following:

$3\sqrt{2}+5\sqrt{2}$ → ____

$5\sqrt{3}-4\sqrt{3}$ → ____

$\sqrt{5}+3\sqrt{5}$ → ____

$7\sqrt{6x}+2\sqrt{6x}$ → ____

$3\sqrt{x}+2\sqrt{x}$ → ____

$5\sqrt{2x}-\sqrt{5x}+3\sqrt{5x}$ → ____________

$7\sqrt{6}+2\sqrt{24}$ Not "like terms" (in their present form), We'll talk about how to add these later.

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$\sqrt{3}*\sqrt{2}$ $\sqrt{3}\approx 1.7321\ldots$ $\sqrt{2}\approx 1.4142\ldots$

$\sqrt{3*2}\rightarrow\sqrt{6}$ $\sqrt{3}*\sqrt{2}\approx 2.4495$

Will this work? $\sqrt{6}\approx 2.4495\ldots$

Product of Radicals Property

$\sqrt{a}*\sqrt{b}\rightarrow\sqrt{a*b}$ $\sqrt{5}*\sqrt{2}=\sqrt{10}$

$\sqrt{4}*\sqrt{9}\rightarrow\sqrt{4*9}$ Are these equivalent?

$2*3\rightarrow\sqrt{36}$ $\sqrt{a}*\sqrt{b}=\sqrt{ab}$

$2*3\rightarrow 6$

$6=6$

Although I only gave two examples, it actually DOES WORK for any real-number radicand.

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$$\sqrt{a}*\sqrt{b}=\sqrt{ab}$$

Simplify the following:

$3\sqrt{8}*5\sqrt{2}$

$3*\sqrt{8}*5*\sqrt{2}$

$3*5*\sqrt{8}*\sqrt{2}$

$15*\sqrt{8}*\sqrt{2}$

$15*\sqrt{16}$

$15*4=60$

$2\sqrt{3}*3\sqrt{5}$ → ____

$7\sqrt{6}*2\sqrt{5}$ → ____

$\sqrt{5}+3\sqrt{5}$ → ____

$7\sqrt{6}+2\sqrt{6}$ → ____

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Simplify radicals: use the Product of Radicals to "break apart" the radical into a "perfect square" times a number.

$$\sqrt{a} * \sqrt{b} = \sqrt{ab}$$

$\sqrt{18} \rightarrow \sqrt{9} * \sqrt{2} \rightarrow 3 * \sqrt{2} \rightarrow 3\sqrt{2}$

Simplify

$\sqrt{24}$ → ____

$3\sqrt{32x^2}$ → ____

$-2\sqrt{56x^3y}$ → ________

Can we add "unlike" radicals? $\sqrt{a} * \sqrt{b} = \sqrt{ab}$

$7\sqrt{6} + 2\sqrt{24}$ Simplify

$\rightarrow 7\sqrt{6} + (2 * \sqrt{4} * \sqrt{6})$

$\rightarrow 7\sqrt{6} + (2 * 2 * \sqrt{6})$

$\rightarrow 7\sqrt{6} + 4\sqrt{6}$

$\rightarrow 11\sqrt{6}$

$-3\sqrt{32} + 2\sqrt{8}$ → ____

Simplify radicals: use the Product of Radicals to "break apart" the radical into a "powers of exponent 'm' " times a number.

$$\sqrt[m]{a} * \sqrt[m]{b} = \sqrt[m]{ab}$$

$\sqrt[3]{x^4} \rightarrow \sqrt[3]{x^3} * \sqrt[3]{x} \rightarrow x\sqrt[3]{x}$

Simplify

$\sqrt[4]{3x^5y}$ → ____

$3\sqrt[3]{16x^2y^5}$ → ____

Another way to Simplify Radicals Factor, factor, factor!!!

$\sqrt{54} \rightarrow \sqrt[2]{54} \rightarrow \sqrt[2]{2*27} \rightarrow \sqrt[2]{2*3*9} \rightarrow \sqrt[2]{2*3*3*3}$

What is the factor that is used '2' times under the radical?

Bring that out factor (that is used 2 times).

$\rightarrow 3\sqrt[2]{2*3} \rightarrow 3\sqrt{6}$

Using Properties of Exponents to reduce the writing:

$\sqrt[4]{32x^6}$ → ____

factor the numerator

$$\frac{\sqrt{12}}{\sqrt{2}} = \frac{\sqrt{2}*\sqrt{6}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}}*\frac{\sqrt{6}}{1} = 1*\frac{\sqrt{6}}{1} = \sqrt{6}$$

Any number divided by itself = ?

$$\frac{\sqrt{48x^3}}{\sqrt{16x}}$$ → ____

$$\frac{\sqrt{56x^3y}}{\sqrt{8xy}}$$ → ____

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Simplify

$$\sqrt{\frac{32}{9x^2}}$$ → ____

$$\frac{\sqrt{50y^2}}{\sqrt{2y}}$$ → ____

$$\frac{\sqrt{49}}{\sqrt{7}}$$ → ____

$$\sqrt{\frac{48}{49}}$$ → ____

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Rationalizing the denominator: using mathematical properties to change an irrational number (or imaginary) in the denominator into a rational number.

We take advantage of the idea:

$$\sqrt{2}*\sqrt{2} = \sqrt{2*2} = \sqrt{4} = 2$$
$$\sqrt{3}*\sqrt{3} = \sqrt{3*3} = \sqrt{9} = 3$$

$$\frac{1}{\sqrt{2}}*\frac{\sqrt{2}}{\sqrt{2}} \rightarrow \frac{\sqrt{2}}{2}$$

Identity Property of Multiplication

multiplying by '1' doesn't change the number.

$$\frac{1}{\sqrt{6}}*\frac{\sqrt{6}}{\sqrt{6}} \rightarrow \frac{\sqrt{6}}{6}$$

$$\frac{2}{\sqrt{6}}$$ → ____

$$\frac{25}{\sqrt{15}}$$ → ____

$$\frac{14}{3\sqrt{21}}$$ → ____

In all of the previous examples we just multiplied by "one in the form of" the denominator radical over the denominator radical.

$$\frac{3\sqrt{7}}{\sqrt{8}} \rightarrow \frac{3\sqrt{7}}{\sqrt{8}} * \frac{\sqrt{8}}{\sqrt{8}} \rightarrow \frac{3\sqrt{7*2*2*2}}{8} \rightarrow \frac{3*2\sqrt{7*2}}{8} \rightarrow \frac{3*2\sqrt{14}}{2*4}$$

It is always easier to simplify (by factoring) ***BEFORE*** you multiply

$$\frac{3\sqrt{7}}{\sqrt{8}} \rightarrow \frac{3\sqrt{7}}{\sqrt{4}*\sqrt{2}} \rightarrow \frac{3\sqrt{7}}{2\sqrt{2}} * \frac{\sqrt{2}}{\sqrt{2}} \rightarrow \frac{3\sqrt{14}}{2*2} \rightarrow \frac{3\sqrt{14}}{4}$$

$$\frac{6\sqrt{5}}{3\sqrt{12}} \rightarrow \frac{3*2*\sqrt{5}}{3*\sqrt{4}*\sqrt{3}} \rightarrow \frac{2*\sqrt{5}}{2*\sqrt{3}} \rightarrow \frac{\sqrt{5}}{\sqrt{3}} * \frac{\sqrt{3}}{\sqrt{3}} \rightarrow \frac{\sqrt{15}}{3}$$

What about variables?

$$\frac{3}{\sqrt{5x}} * \frac{\sqrt{5x}}{\sqrt{5x}} \rightarrow \frac{3\sqrt{5x}}{5x}$$

$$\frac{\sqrt{15}}{\sqrt{5x}}$$

→ _____

What about higher index numbers?

$$\frac{1}{\sqrt[3]{x}} * \frac{\sqrt[3]{x*x}}{\sqrt[3]{x*x}} \rightarrow \frac{\sqrt[3]{x*x}}{\sqrt[3]{x*x*x}} \rightarrow \frac{\sqrt[3]{x^2}}{x}$$

How many more 'x's are needed in the denominator radicand?

Remember: the cubed root of x-cubed equals x. $\sqrt[3]{x^3} = x$

We need two more x's under the denominator radical.

Using the multiply powers property we don't have to write out all the individual x's.

$$\frac{1}{\sqrt[3]{x}} * \frac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}} \rightarrow \frac{\sqrt[3]{x^2}}{x}$$

Rationalize the denominator

$$\frac{2x}{\sqrt{3x}}$$

$$\frac{3}{\sqrt[4]{x^2}}$$

$$\frac{2\sqrt{3y^3}}{\sqrt{5y}}$$

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