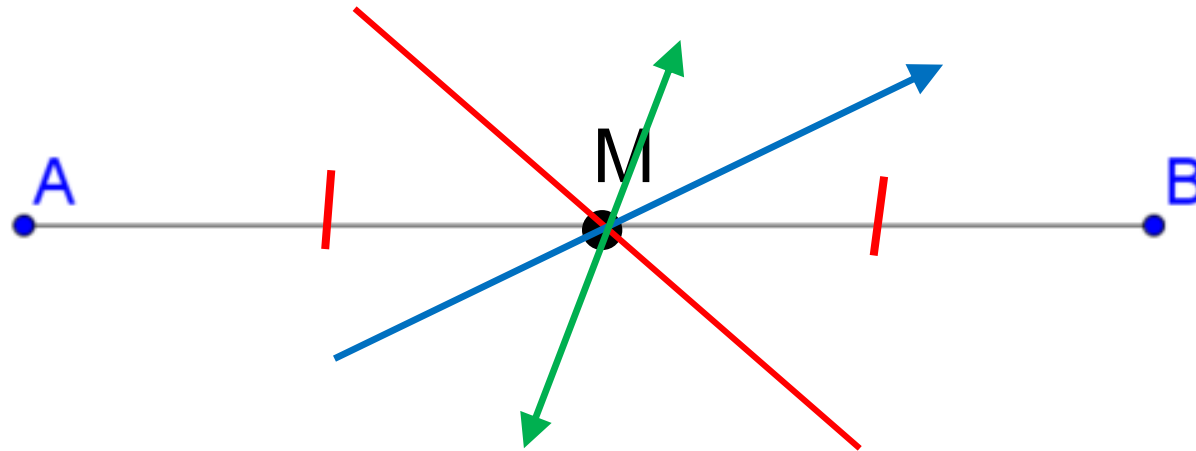# Math-2A

Lesson 8-7
Properties of Isosceles Triangles

Segment Bisector: if a <u>line segment</u> is intersected by a <u>ray</u>, <u>segment</u> or <u>line</u> at the midpoint of the segment, then the ray, segment line is a segment bisector.

a) Another segment   b) A ray   c) A line.

M

A   B

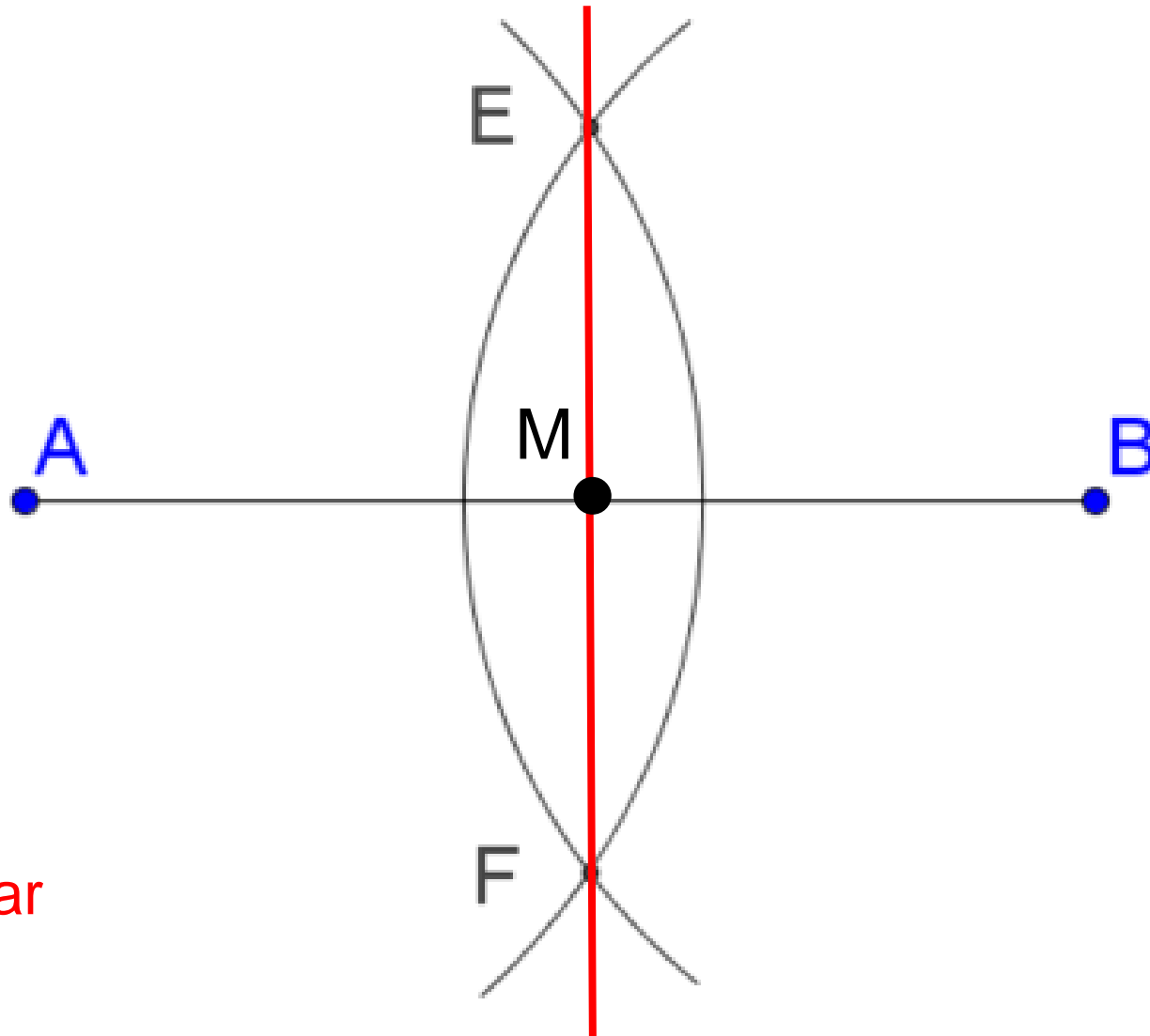# Constructing a Perpendicular Bisector

Given a line segment AB

1) Using a compass draw two arcs of equal radius using the endpoints as the center of each are.

2) Construct a point where the two arcs intersect.

3) Construct a line through these two points.

4) $\overline{EF}$ Is the perpendicular bisector of $\overline{AB}$

E

A   M   B

F

$\overline{EF}$ is a <u>perpendicular bisector of</u> $\overline{AB}$.

Are there any equations (that come from congruencies) that we can write from this result?

$$\mathrm{m}\angle AKE = m\angle BKE = 90$$
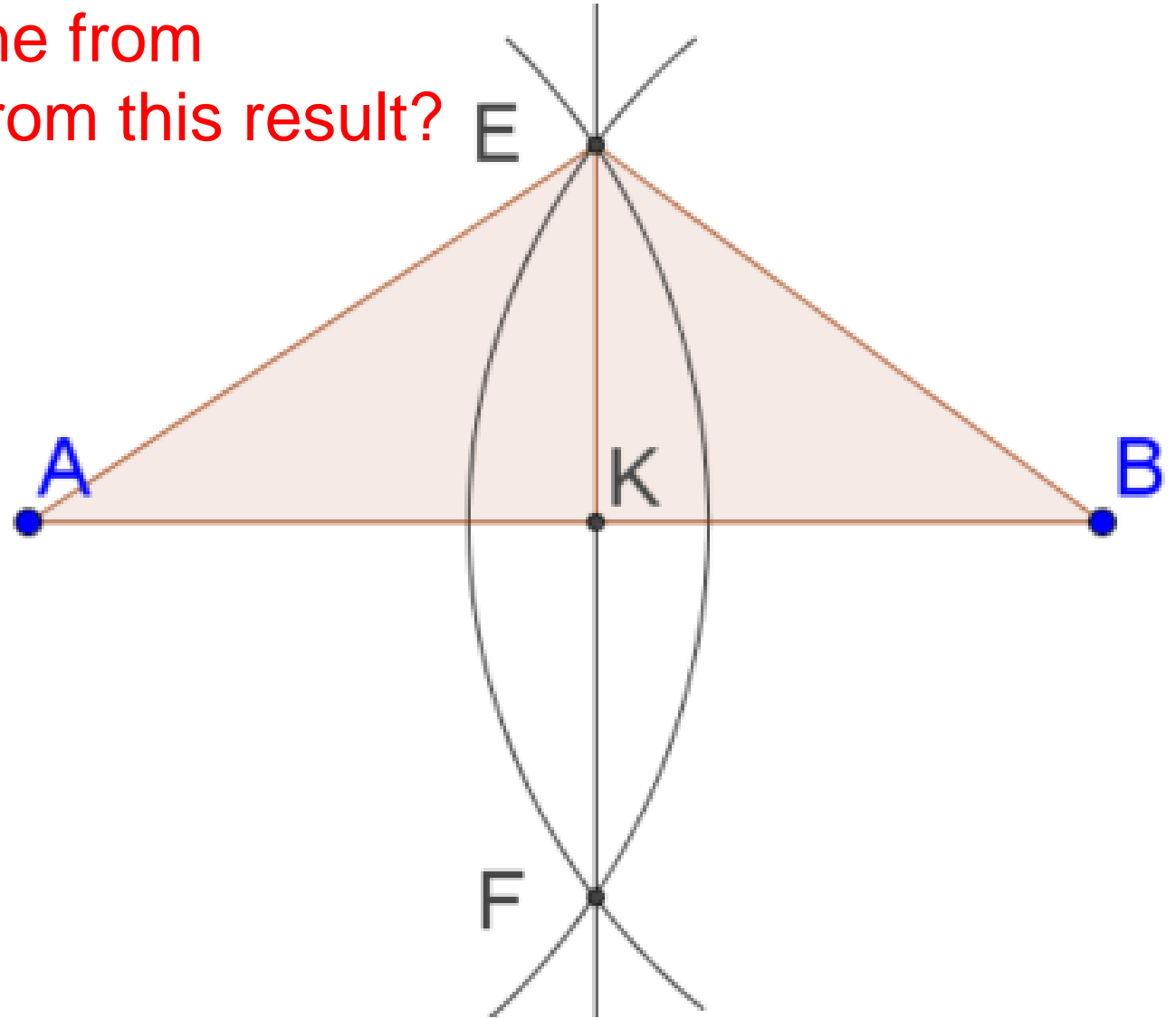
<u>perpendicular</u> bisector
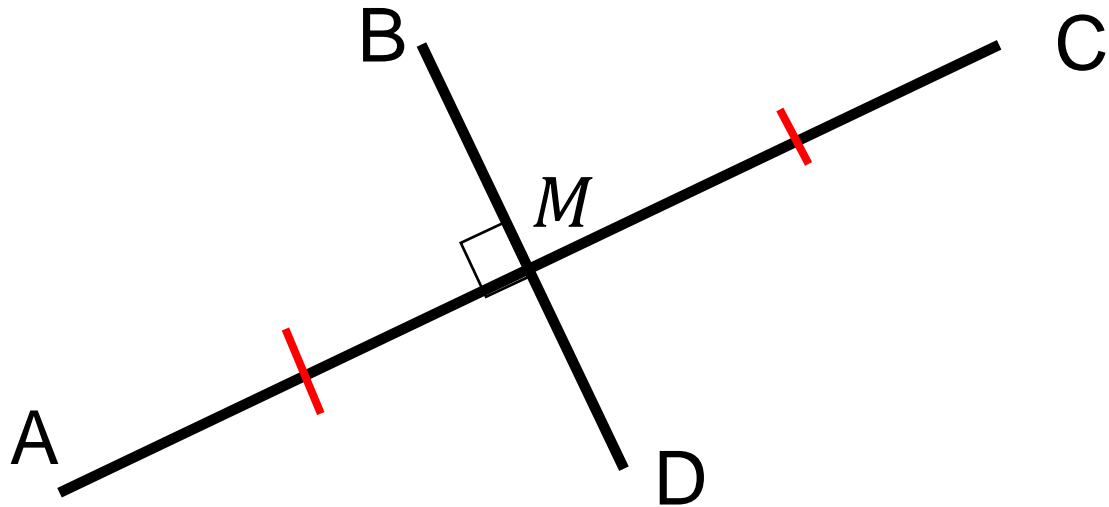
$$AK = BK$$

<u>perpendicular</u> bisector

$$AB = 2 * AK$$

<u>segment</u> addition

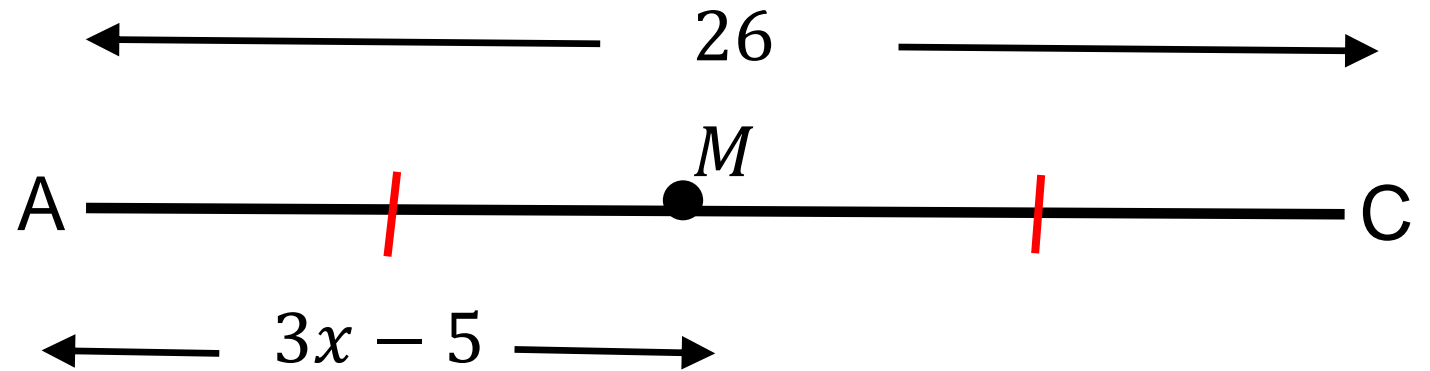# Math Problems from "Perpendicular Bisectors"

$$AC = 26$$
$$AM = 3x - 5$$

$$x = ?$$

1. Draw a picture of the segment and label the known measurements.

2. Write an equation that relates the lengths in the problem.
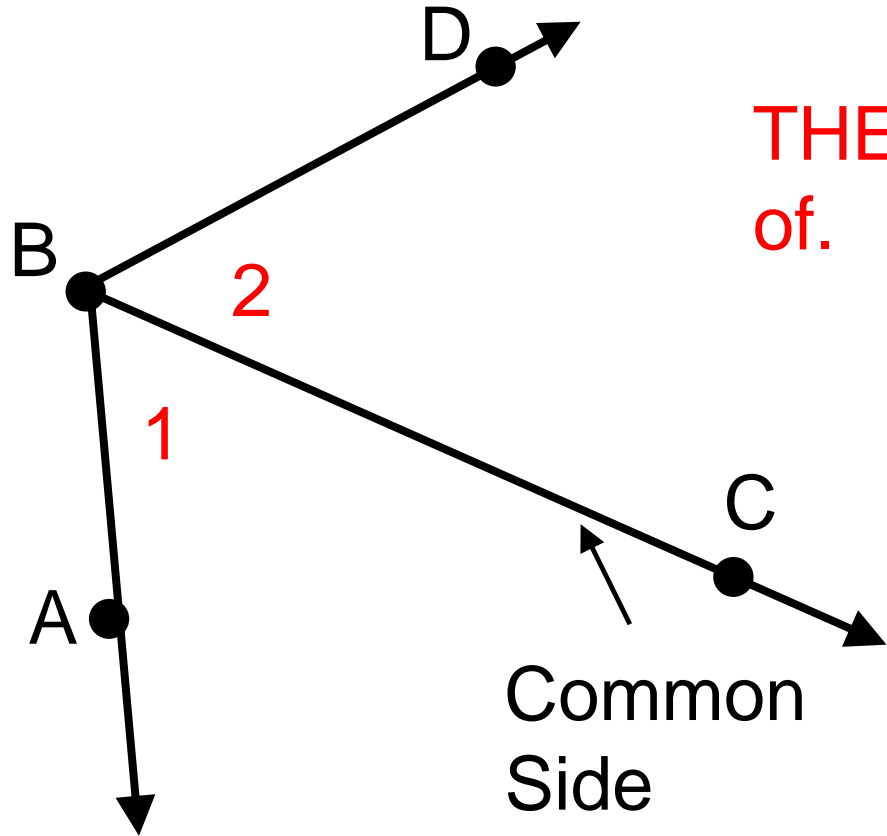$$2 * AM = AC$$
$$2(3x - 5) = 26$$

3. Solve for 'x'.
$$3x - 5 = 13$$
$$3x = 18$$
$$x = 6$$

Angle Bisector: a common side of two adjacent angles that divides the angle into two angles of equal measure.

If $m\angle 1 = m\angle 2$

THEN $\overline{BC}$ is an angle bisector

of. $\angle ABD$

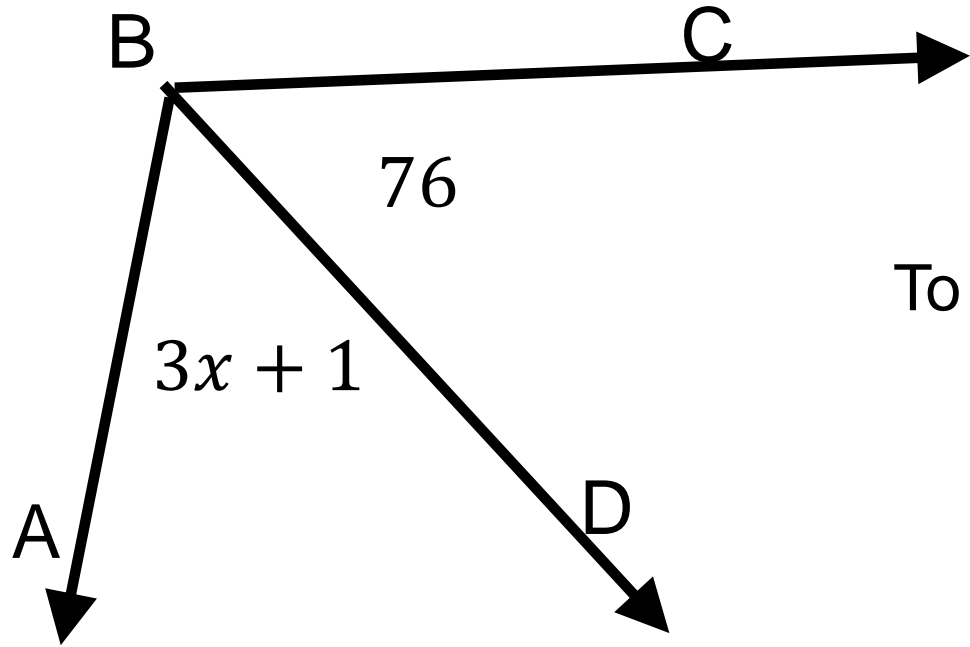Are there any equations that we can write from this result?

$m\angle ABC = m\angle DBC$

angle bisector

$m\angle ABD = 2 * m\angle DBC$

angle bisector

D

B

2

1

A

C

Common Side

Math Problems from "Angle Bisectors"

B

C

76

$3x + 1$

A

D

$\overrightarrow{BD}$ is an angle bisector of $\angle ABC$

$x = ?$

To solve for an unknown value, you need an equation.

$m\angle ABD = m\angle CBD$

3x + 1 =76

$3x$ = 75

x = 25

Constructing an Angle Bisector
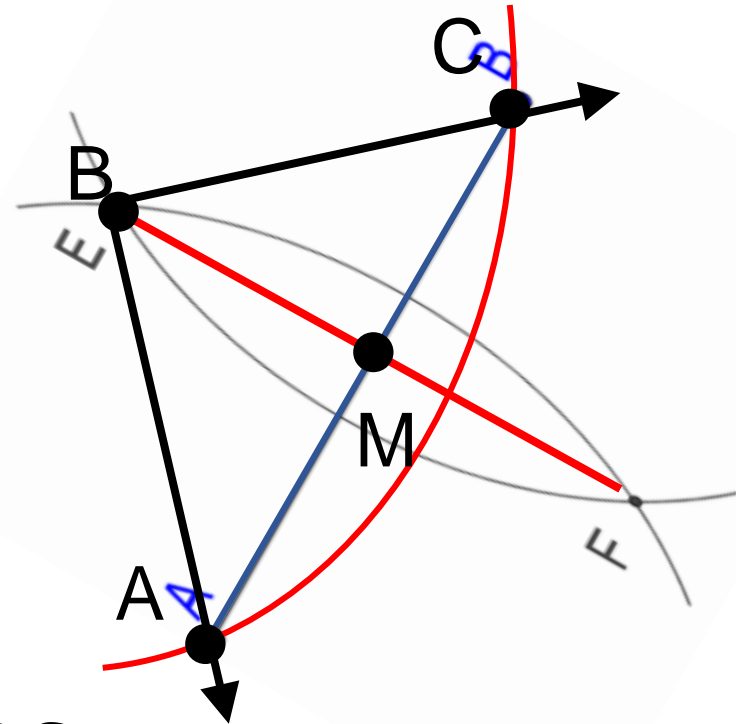
Given $\angle B$

1) Using a compass draw an arc using point B as the center.

2) Construct two points (points A and C) where the arc intersects the side of the angles

3) Construct $\overline{AC}$
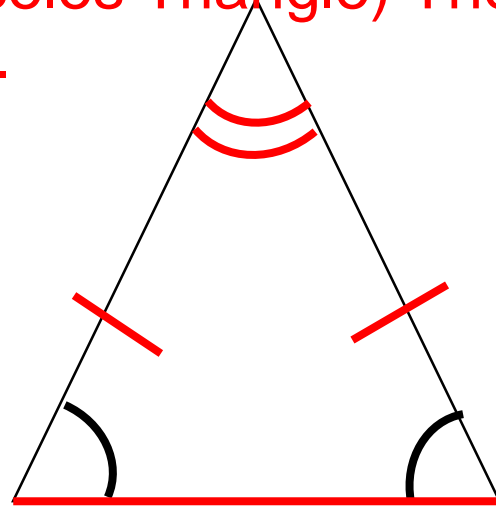
4) Construct a perpendicular bisector of $\overline{AC}$

5) $\overline{BM}$ is the angle bisector of $\angle ABC$

Isosceles Triangle: A triangle with two congruent sides.

Legs: (Of an Isosceles Triangle) The two congruent sides.

Vertex Angle: (Of an Isosceles Triangle) The included angle of the legs.

Base: (Of an Isosceles Triangle) The opposite the vertex angle.

Base Angles: (Of an Isosceles Triangle) The angles that include the base.

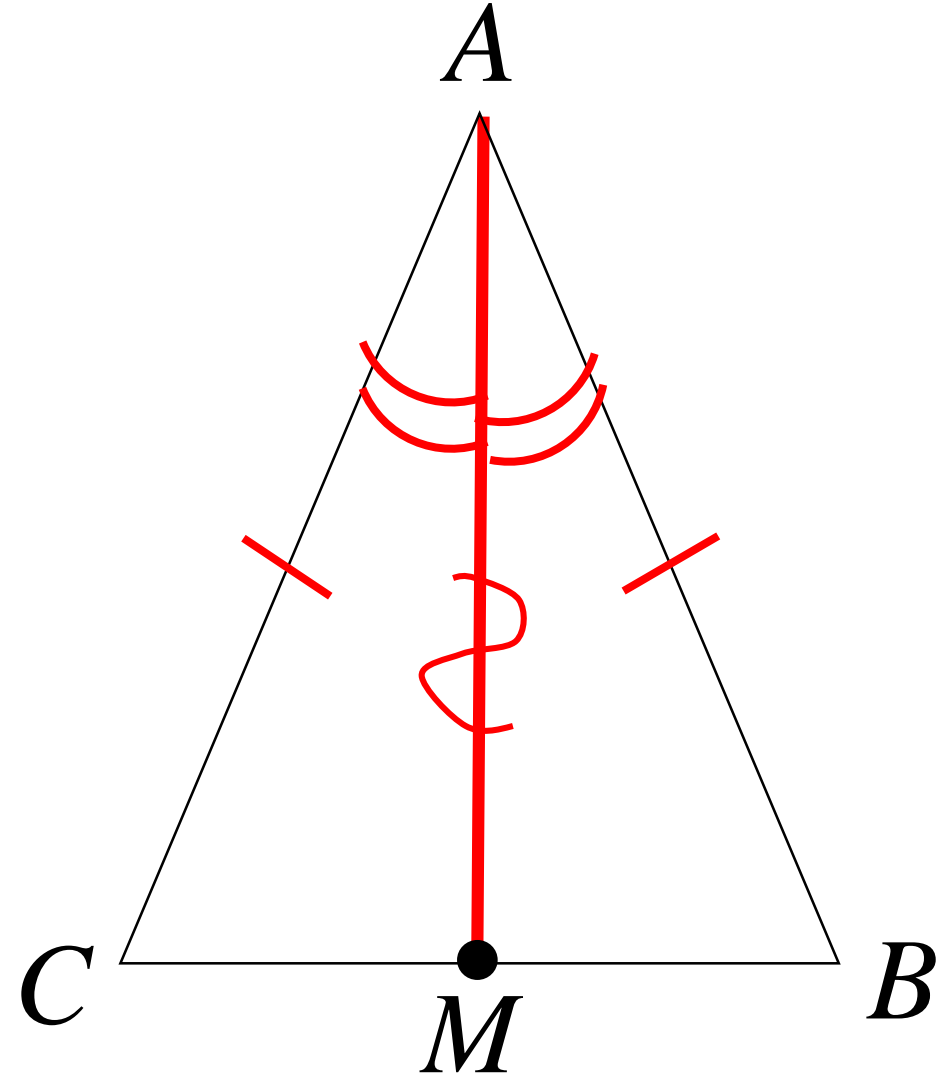Given: ΔABC is an Isosceles Triangle and $\overline{AM}$ is an angle bisector of vertex angle A.

What other congruencies result from this statement?

$\angle CAM \cong \angle BAM$ ($\overline{AM}$ bisects $\angle A$)

$AC = AB$ (Isosceles Triangle)

$AM = AM$ (congruent to itself)
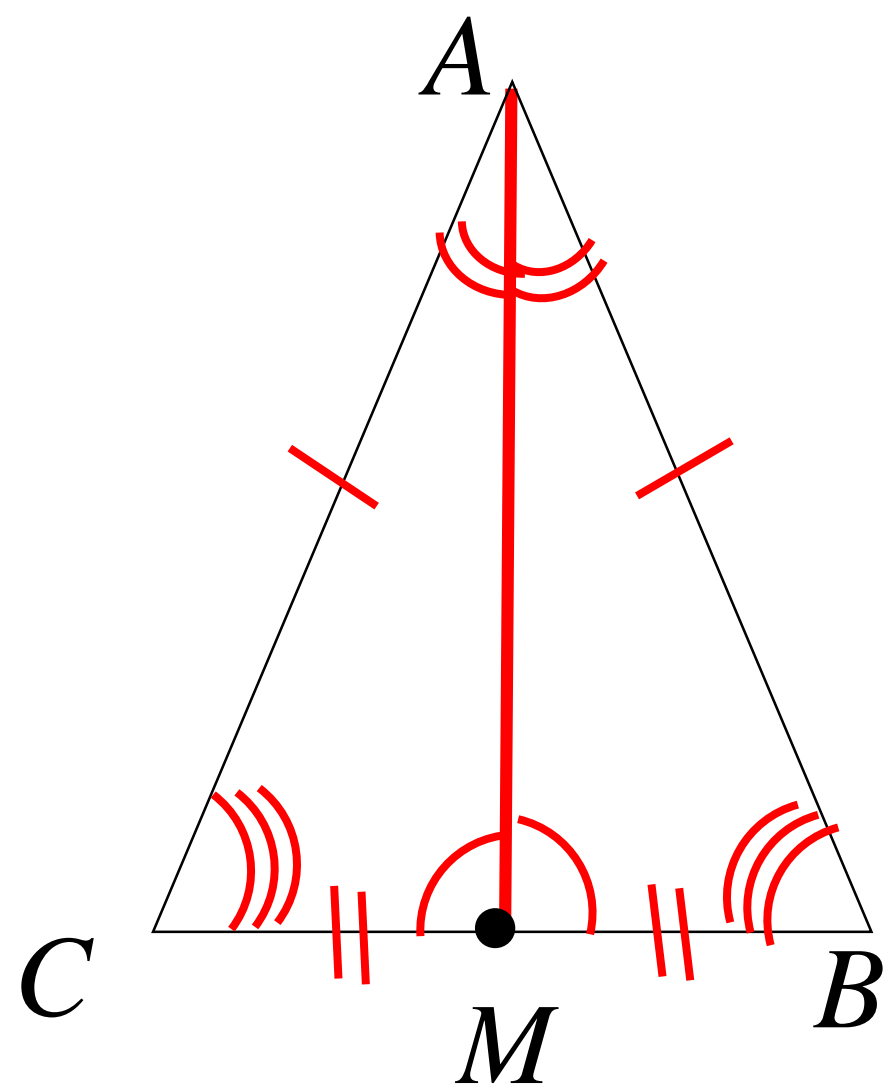
$\Delta CAM \cong \Delta BAM$ (SAS)

$$\Delta CAM \cong \Delta BAM$$

Congruent triangles give us <u>SIX Pairs of congruencies</u>.

$$CM = BM$$

$$m\angle CMA = m\angle BMA$$

$$m\angle ACM = m\angle ABM$$

Properties of Isosceles Triangles

1. The vertex and bisector forms two congruent triangles.
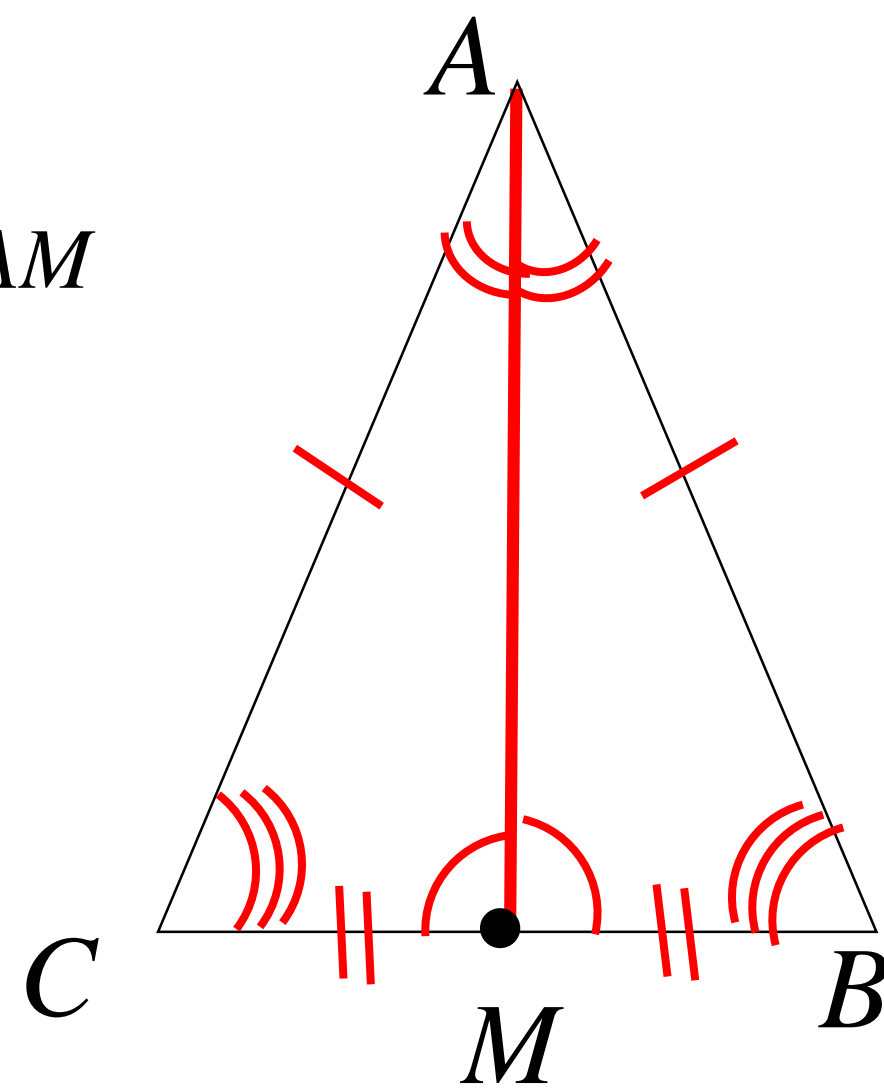
$\Delta CAM \cong \Delta BAM$

2. The vertex angle bisector is a perpendicular bisector of the base.
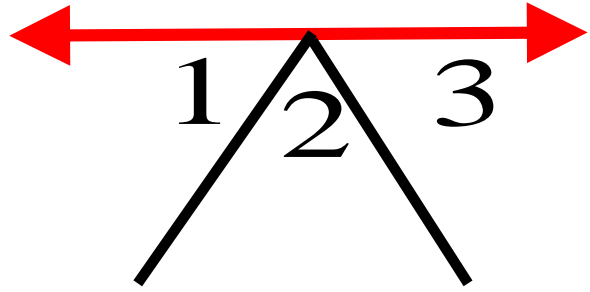
$$m\angle CMA = m\angle BMA = 90$$
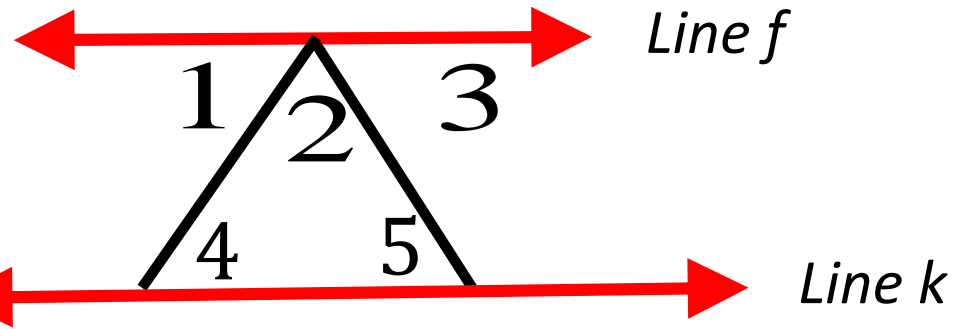$$CM = BM$$

3. Base Angles are congruent.
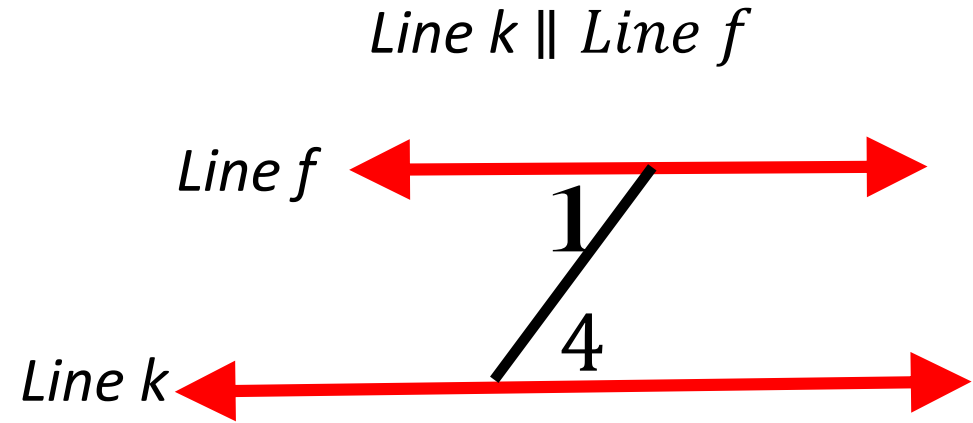
$$m\angle ACM = m\angle ABM$$

Triangle Sum Theorem: If $\angle A, \angle B, and \angle C$ are the interior angles of a triangle, then their measures add up to 180°.

$$m\angle 1 + m\angle 2 + m\angle 3 = \underline{180°}$$

Line $k \parallel$ Line f

Line f

1

4

Line k

$$m\angle 1 \underline{\;=\;} m\angle 4$$

Line f

1   2   3

4   5

Line k

$$m\angle 4 + m\angle 2 + m\angle 5 = \underline{180°}$$

Line f

3

5

Line k
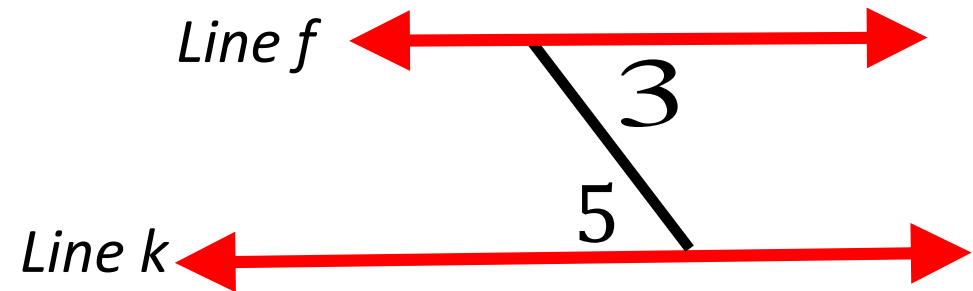
$$m\angle 3 \underline{\;=\;} m\angle 5$$

Math Problems from "The Triangle Sum Theorem."

1. Write an equation that relates the measures of the angles.

$$m\angle A + m\angle B + m\angle C = \underline{\quad 180° \quad}$$

2. Substitute the measures of the angles into the equation.

$$2x - 1 + 3x + 7 + 4x + 3 = 180°$$

3. Solve for 'x'.

$$9x + 9 = 180°$$
$$9x = 171°$$
$$x = 19$$

$$m\angle B = ?$$
$$m\angle B = 4x + 3$$
$$m\angle B = 4(19) + 3$$
$$m\angle B = 79°$$

(Triangle diagram: vertex A at top, C at bottom left, B at bottom right. Side near A labeled $3x + 7$, angle at C labeled $2x - 1$, angle at B labeled $4x + 3$.)