

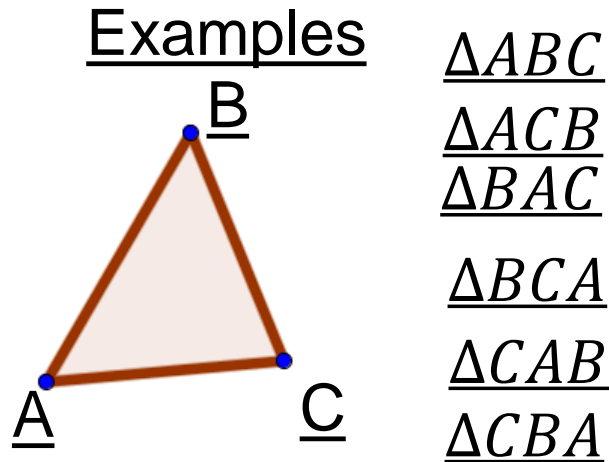
Math-2A
Lesson 8-3

Triangle Congruence

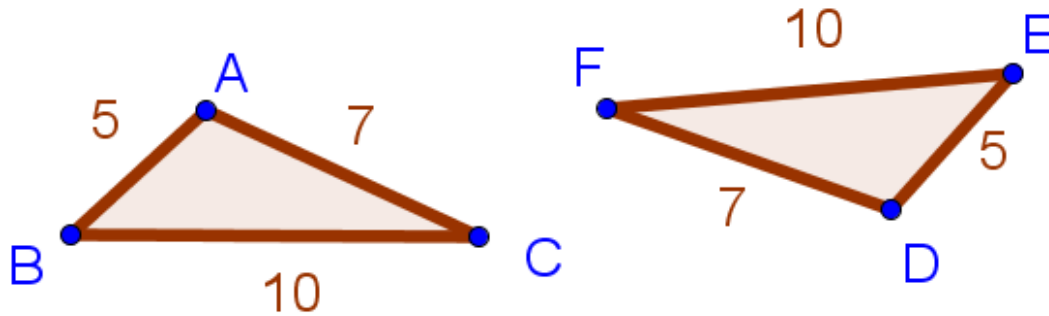
Naming Triangles

Triangles are named using a small triangle symbol and the three vertices of the triangles.

The order of the vertices does **not** matter for **NAMING** a triangle



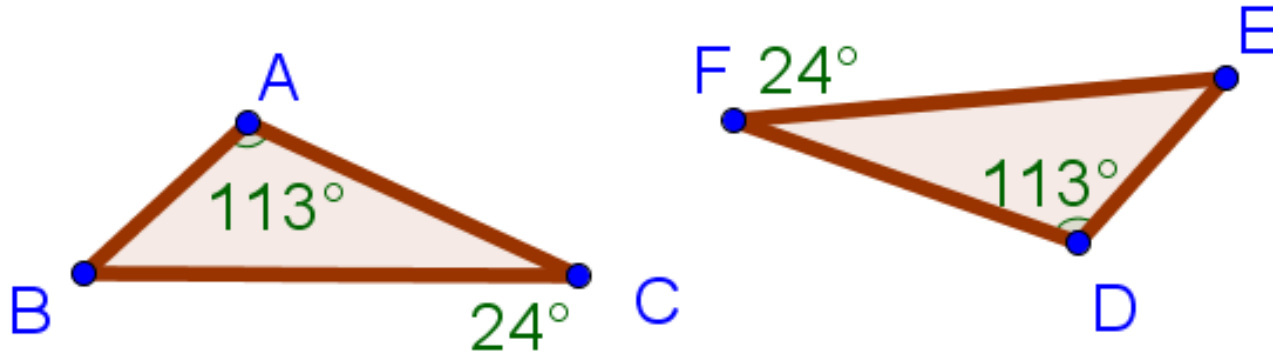
Corresponding Angles of Triangles: an angle in one triangle that has the same position (relative to its sides) as an angle in another triangle (relative to its sides).



What angle corresponds to $\angle A$?

$\angle A$ corresponds to $\angle D$ since they are opposite the longest side of their triangles

Corresponding Sides of Triangles: a side in one triangle that has the same position (relative to its angles) as a side in another triangle (relative to its angles).

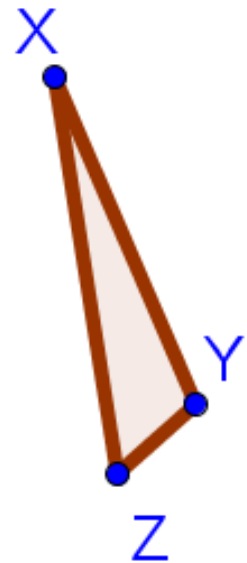
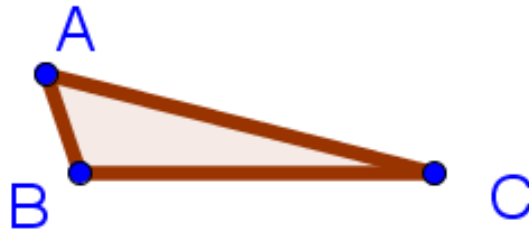


What side corresponds to \overline{BC} ?

\overline{BC} corresponds to \overline{FE} since they are opposite the largest angle of their triangles

Your Turn:

- 1) What angle does $\angle A$ correspond to? $\underline{\underline{\angle Z}}$
- 2) What angle does $\angle X$ correspond to? $\underline{\underline{\angle C}}$
- 3) What side does \overline{XY} correspond to? $\underline{\underline{\overline{CB}}}$
- 4) What side does \overline{AC} correspond to? $\underline{\underline{\overline{ZX}}}$

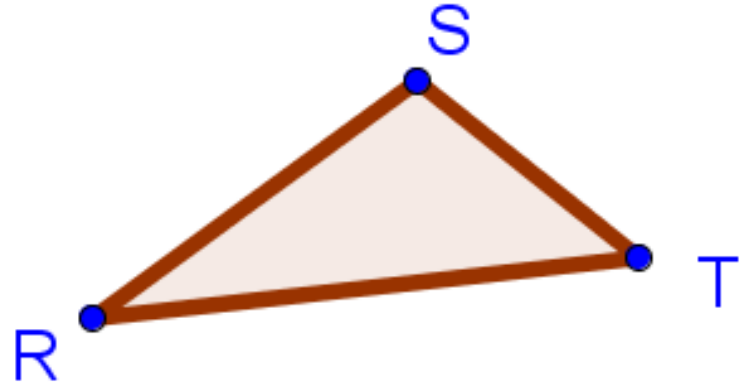


- Included side: If two angles in a triangle are given, the included side is the side that is between the two angles or side that both of the angles have in common.

– \overline{RS} is the included side of $\angle R$ and $\angle S$

– What is the included Side for $\angle S$ and $\angle T$?

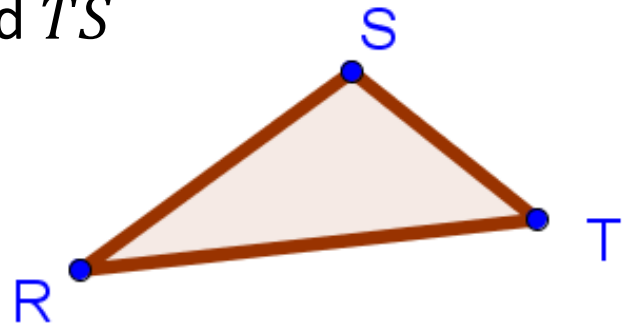
\overline{ST} is the included side of $\angle S$ and $\angle T$



- Included angle: If two sides of a triangle are given, the included angle is the angle formed by those two sides.

– $\angle T$ is the included angle of \overline{RT} and \overline{TS}

– What is the included angle of \overline{SR} and \overline{RT} ?



$\angle R$ is the included angle of \overline{SR} and \overline{RT}

1) $\angle D$ is the included angle of which two sides?

\overline{DF} and \overline{DE}

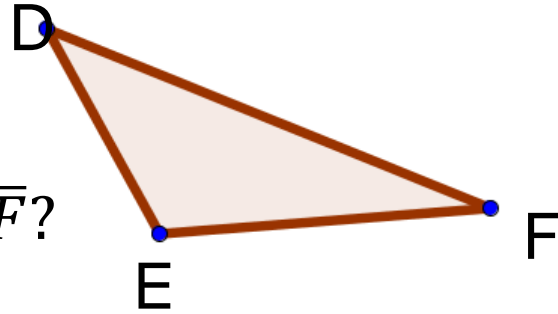
2) What is the included angle of sides \overline{DF} and \overline{EF} ?

$\angle F$

3) \overline{DF} is the included side of which two angles?

$\angle D$ and $\angle F$

4) What is the included side of $\angle D$ and $\angle E$ \overline{DE}



Congruence means same size and shape.

Angles

- two angles are congruent if they have the same measure (degrees)
- IF $\angle A \cong \angle B$ then $m\angle A = m\angle B$ (converse is true also)

Segments (sides)

- two line segments are congruent if they have the same length
- If $\overline{AB} \cong \overline{CD}$ then $AB = CD$ (converse is true also)

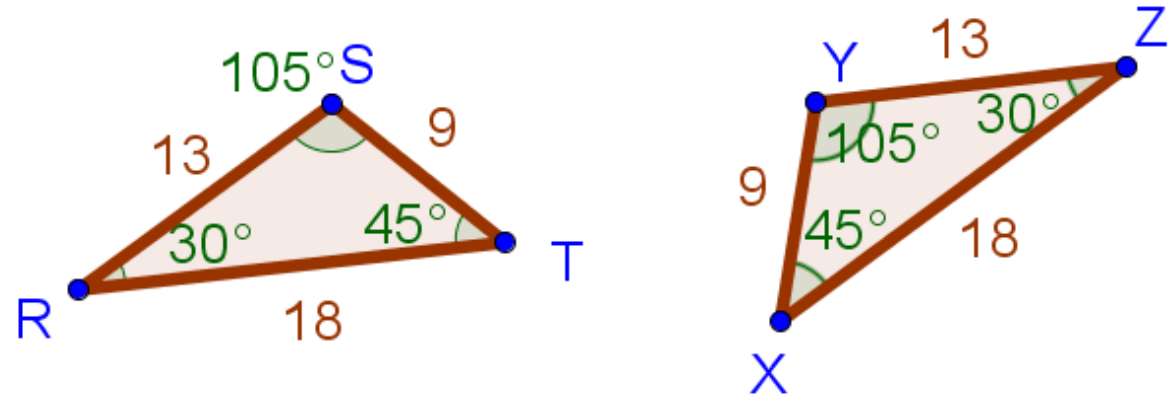
Triangles

two triangles are congruent

- If each angle in one triangle is congruent to its corresponding angle in the other triangle
- AND
- if each side in one triangle is congruent to its corresponding side in the other triangle.

Triangle Congruence

Why are $\triangle RST$ and $\triangle ZYX$ congruent? (That is, how do we prove it)

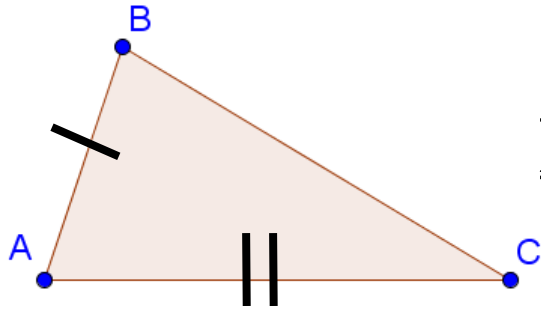


- All corresponding parts are congruent (CPCTC)
- This is just the definition of congruence

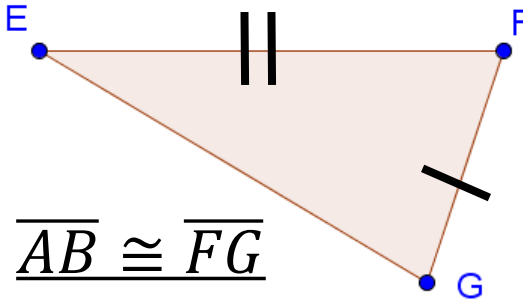
Do we need all 6 pairs of angles and sides to be congruent to prove the triangles are congruent?...

Congruence Symbols

Segment congruence symbols (without giving measures) are just “tic” marks.



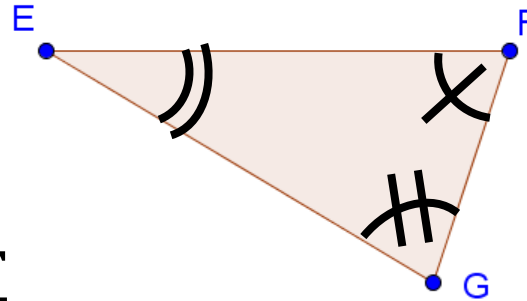
$$\overline{AC} \cong \overline{EF}$$



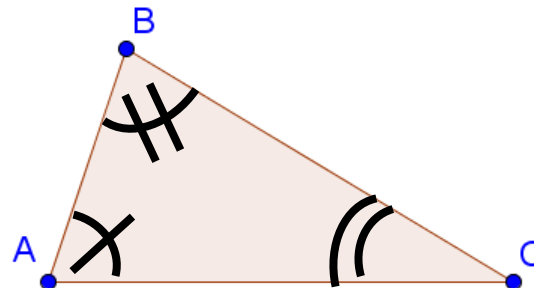
$$\overline{AB} \cong \overline{FG}$$

Angle congruence symbols can either be the “arc” symbol with either:

- (1) tic-marks
- (2) Repeated arcs



$$\angle A \cong \angle F$$

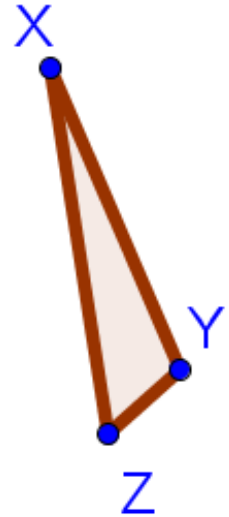
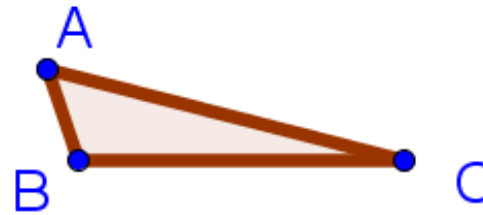


Congruence Statements

When stating **congruence**, the order is important

- The vertices of must be put in order so that the vertices in one triangle correspond to the vertexes in the other triangle. If Corresponding parts are congruent, then the triangles are congruent.
- For example: $\triangle ABC \cong \triangle ZYX$ because...

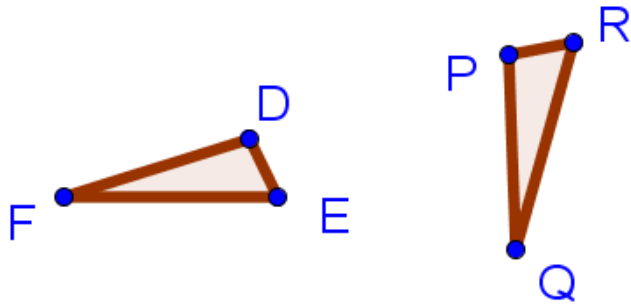
- $\angle A \cong \angle Z$ $\angle B \cong \angle Y$
- $\angle C \cong \angle X$ $\overline{AB} \cong \overline{ZY}$
- $\overline{BC} \cong \overline{YX}$ $\overline{CA} \cong \overline{XZ}$



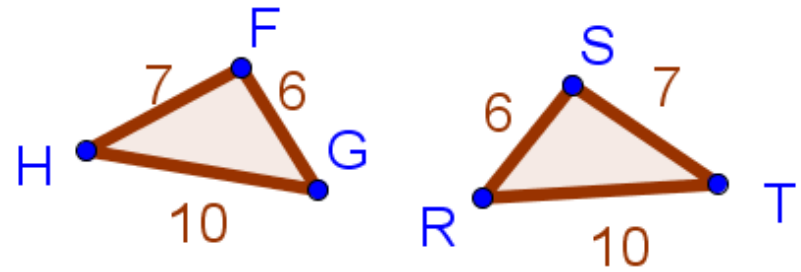
- “corresponding parts of congruent triangles are congruent” or “CPCTC”

Each pair of triangles is congruent:

Write a congruence statement for each pair of triangle.



$$\triangle DEF \cong \underline{\triangle PRQ}$$

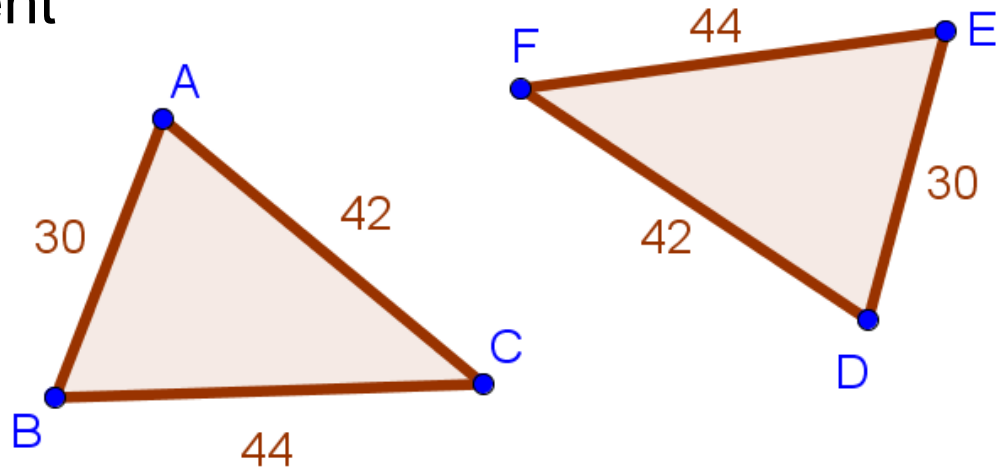


$$\triangle RST \cong \triangle GFH$$

We can prove Triangle Congruence using congruence of only three pairs of corresponding parts.

Side-Side-Side (SSS) Congruency Axiom: if all three pairs of corresponding sides of a triangle are congruent, then the triangles are congruent

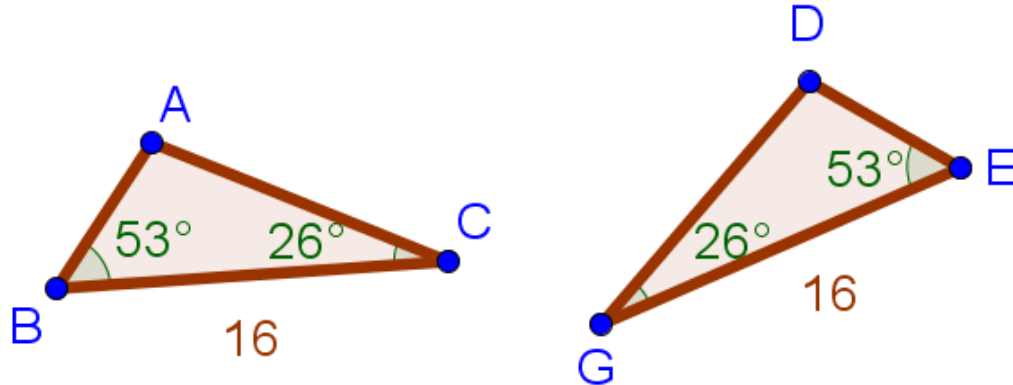
- $\overline{AB} \cong \overline{DE}$
- $\overline{BC} \cong \overline{EF}$
- $\overline{CA} \cong \overline{FD}$



- Therefore,
- $\underline{\Delta ABC \cong \Delta DEF}$ by **SSS**

Angle-Side-Angle (ASA) Congruency Axiom: if two angles and their included side are congruent, then the two triangles are congruent.

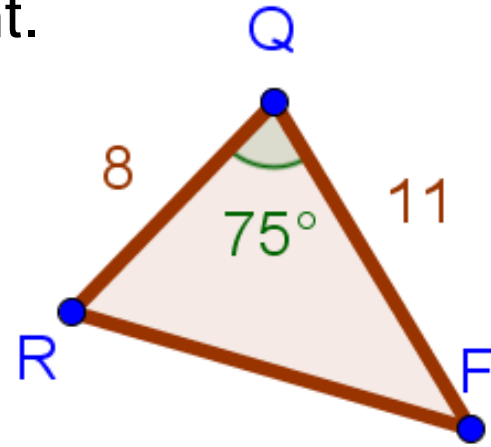
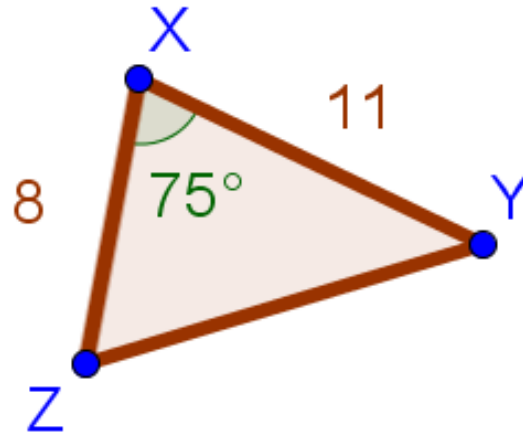
- $\underline{\angle ABC \cong \angle DEG}$
- $\underline{\overline{BC} \cong \overline{EG}}$
- $\underline{\angle BCA \cong \angle EGD}$



- Therefore,
- $\underline{\triangle ABC \cong \triangle DEG}$ by **ASA**

Side-Angle-Side (SAS) Congruency Axiom: if two pairs of corresponding sides and the pair of included angles are congruent, then the triangles are congruent.

- $\overline{XZ} \cong \overline{QR}$
- $\angle ZXY \cong \angle RQF$
- $\overline{XY} \cong \overline{QF}$



Therefore,

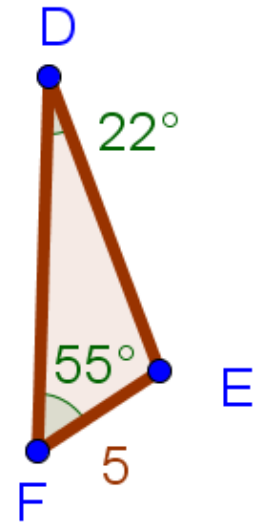
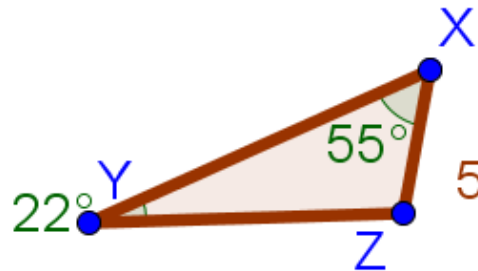
- $\triangle XYZ \cong \triangle QFR$ by **SAS**

Angle-Angle-Side (AAS) Congruency Axiom: If two pairs of corresponding angles are congruent and one pair of corresponding sides are congruent (which are NOT the included side), then the two triangles are congruent.

- $\angle ZXY \cong \angle EFD$
- $\angle XYZ \cong \angle FDE$
- $\overline{XZ} \cong \overline{FE}$

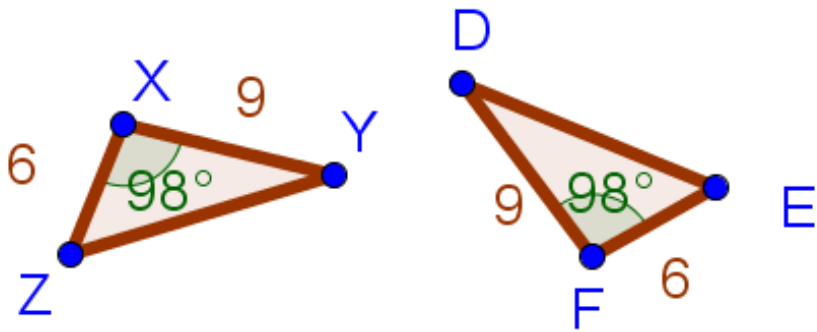
• Therefore,

- $\triangle XYZ \cong \triangle FDE$ by **AAS**

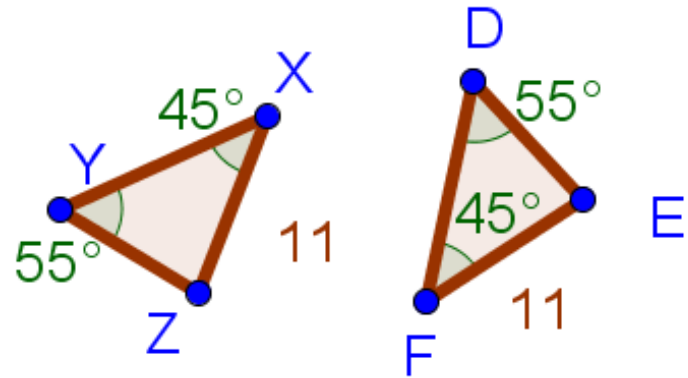


- Determine which congruence condition proves the congruence for each the following pairs of triangles.
- Write a congruence statement for each of the following pairs of triangles.

1)

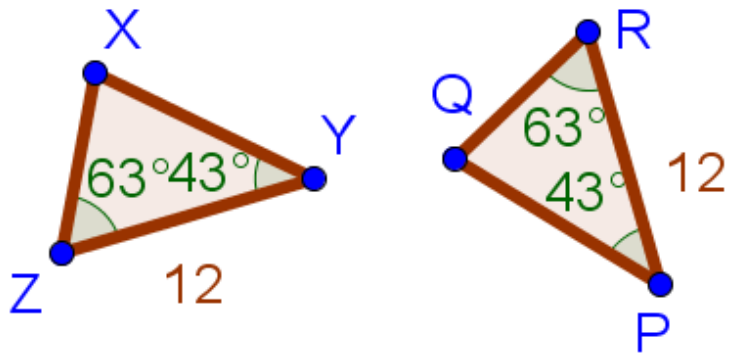


2)

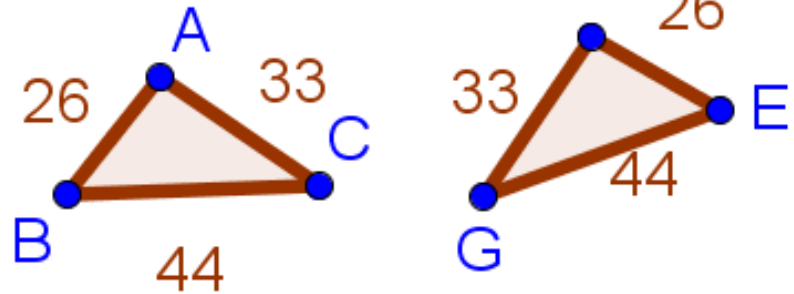


- Determine which congruence condition proves the congruence for each the following pairs of triangles.
- Write a congruence statement for each of the following pairs of triangles.

3)



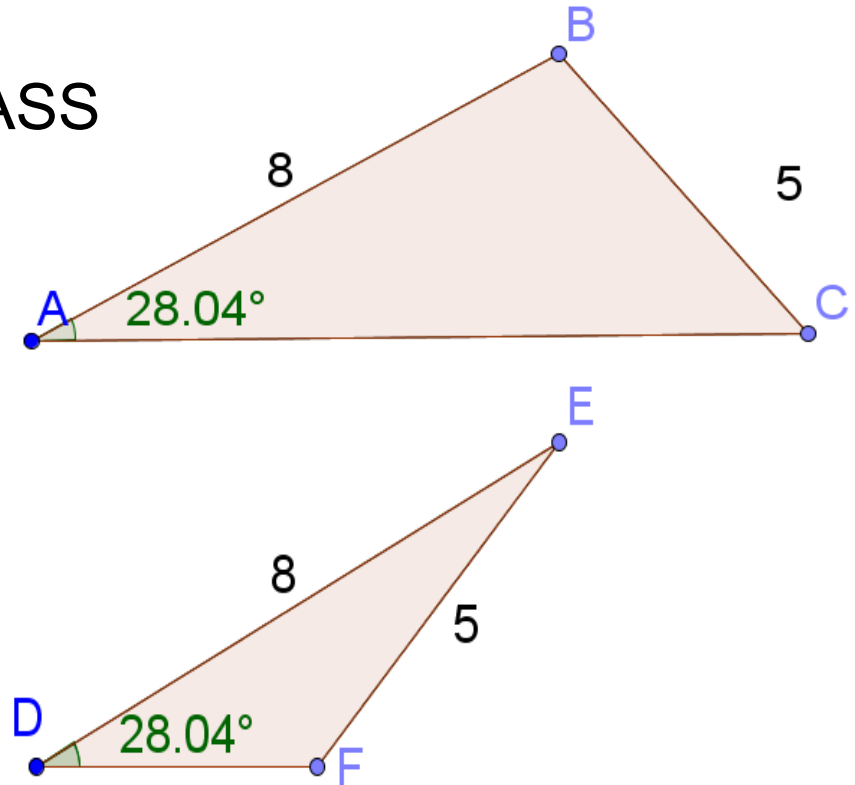
4)



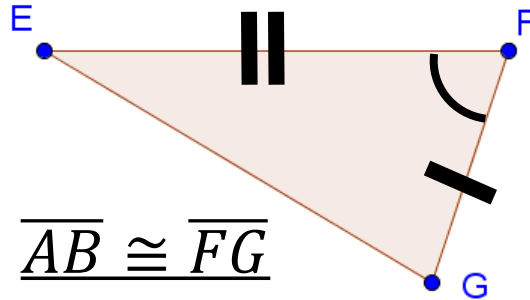
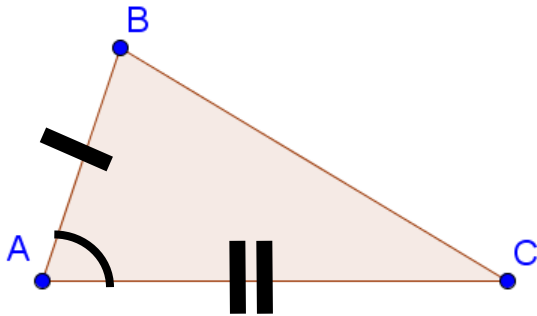
Angle-Side-Side (ASS) Condition (is NOT a congruency)

Let's look at an example of ASS

- $\angle A \cong \angle D$
- $\overline{AB} \cong \overline{DE}$
- $\overline{BC} \cong \overline{EF}$



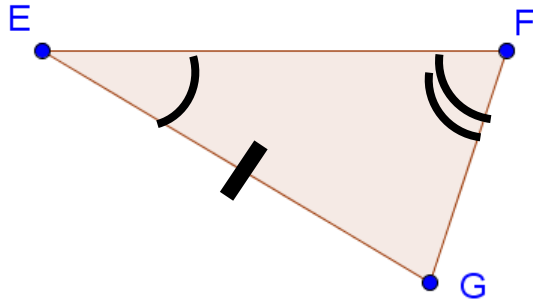
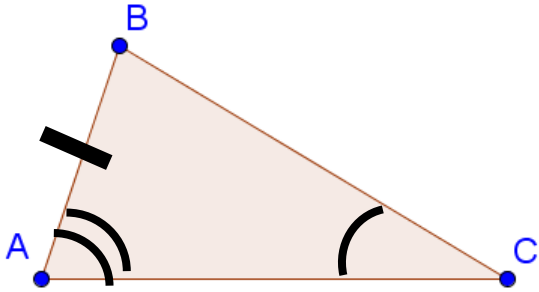
Write a congruence statement that identifies the additional information needed to prove these two triangles are congruent by **SAS**.



$$\begin{aligned}\underline{\overline{AB}} &\cong \underline{\overline{FG}} \\ \underline{\overline{AC}} &\cong \underline{\overline{EF}} \\ \underline{\angle A} &\cong \underline{\angle F}\end{aligned}$$

Write a triangle congruence statement: $\Delta ABC \cong \Delta FGE$

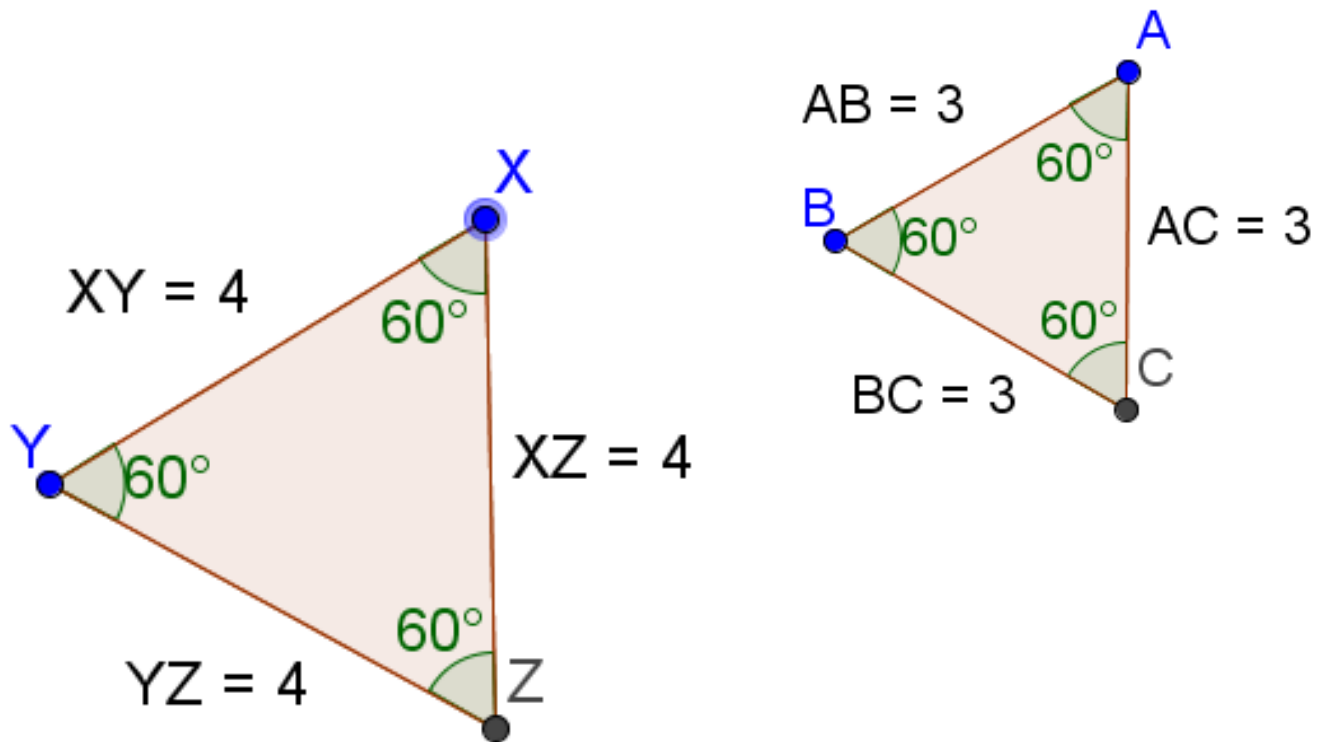
Are the triangles congruent? If so, write a congruence statement for the two triangles, and identify why are they congruent.



$\Delta ABC \cong \text{none}$

Angle-Angle-Angle (AAA) Condition

AAA controls shape only, not size



Angle-Side-Side (ASS) Condition

Let's look at an example of ASS

- $\angle A \cong \angle D$
- $\overline{AB} \cong \overline{DE}$
- $\overline{BC} \cong \overline{EF}$

