Math-2
Lesson 8-2

Vertically Stretched Exponential Function

The "Parent" Exponential Function

$$y = b^x$$

exponent

base

$y = 2^x$ **(base 2 exponential function)**

$y = 3^x$ **(base 3 exponential function)**

$y = \left(\frac{1}{2}\right)^x$ **(base 1/2 exponential function)**

The base MUST BE positive and CANNOT equal 1.

$$b = (0, 1) \cup (1, \infty)$$

Fill in the output values of the table and graph the points.

$$f(x) = 2^x$$

Growth Factor is the base of the exponential

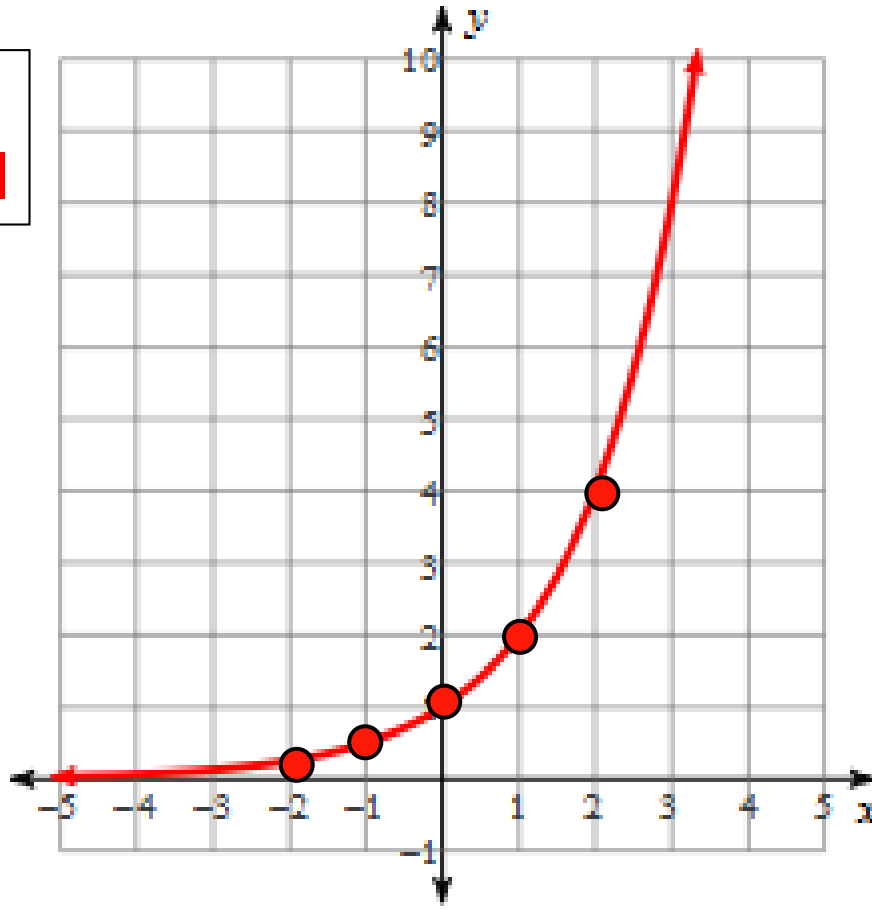
x	$2^{(\ )}$	y
-2	2^{-2}	0.25
-1	2^{-1}	0.5
0	2^{0}	1
1	2^{1}	2
2	2^{2}	4

$* 2$

$* 2$

$* 2$

$* 2$



$$\left(\frac{2}{1}\right)^{-2} = \left(\frac{1}{2}\right)^{2} = \frac{1}{4} = 0.25$$

"negative exponent property"

$$2^0 = 1$$

"zero exponent property"

Exponential Growth: the graph is increasing. Growth occurs when the base of the exponential is greater than 1.

$y = b^x$

'b' = 1 → no growth

'b' > 1 → growth

$f(x) = 1^x$

x	f(x)
-1	1
0	1
1	1

$g(x) = (1.1)^x$

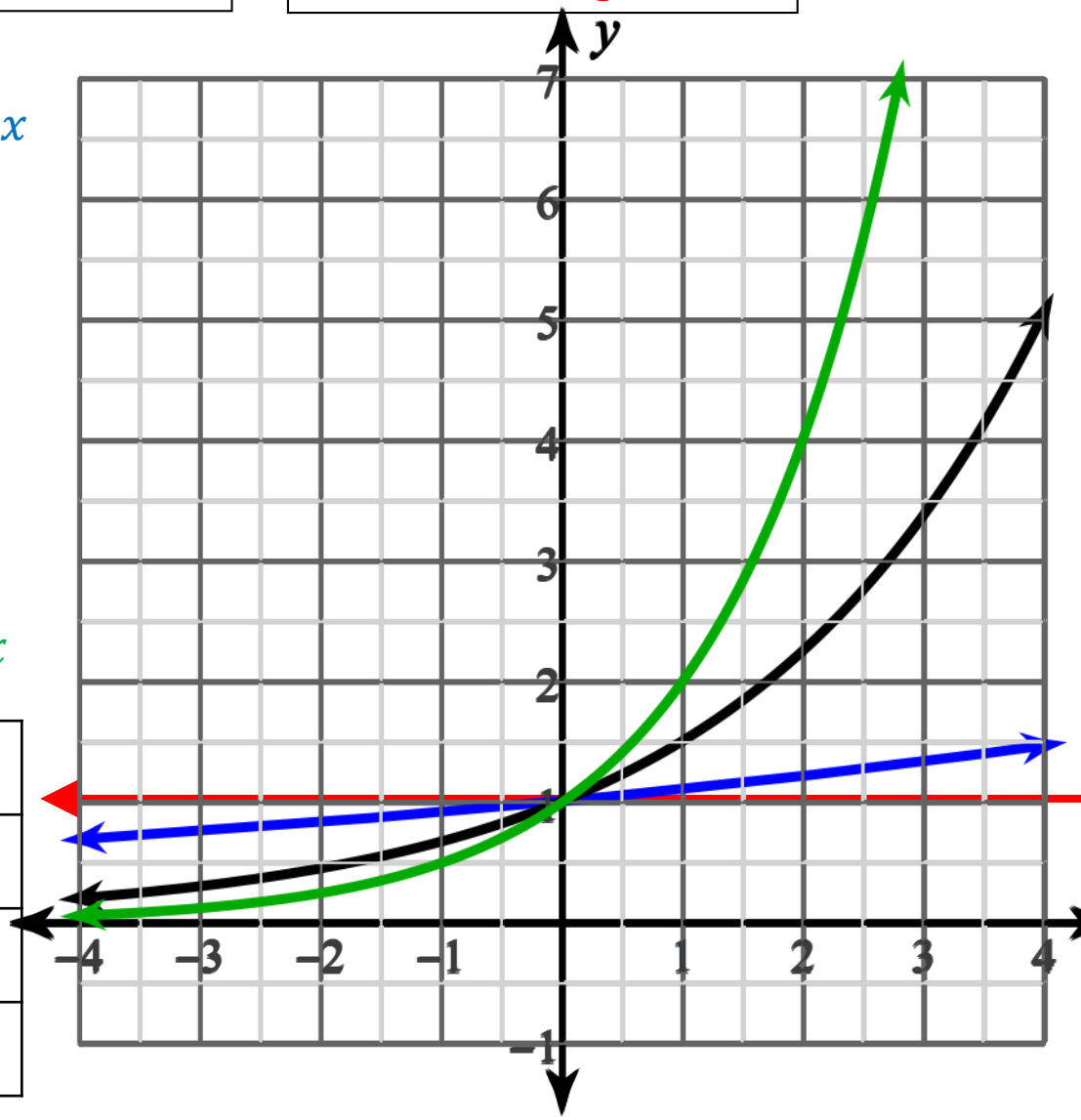
x	g(x)
-1	0.91
0	1
1	1.1

$h(x) = (1.5)^x$

x	h(x)
-1	0.67
0	1
1	1.5

$k(x) = (2)^x$

x	k(x)
-1	0.5
0	1
1	2



Exponential Decay: the graph is decreasing. decay occurs when the base of the exponential is between 0 and 1.

$y = b^x$

'b' = 1 → no growth

0 < 'b' < 1 → decay

$f(x) = 1^x$

x	f(x)
-1	1
0	1
1	1

$g(x) = (0.9)^x$

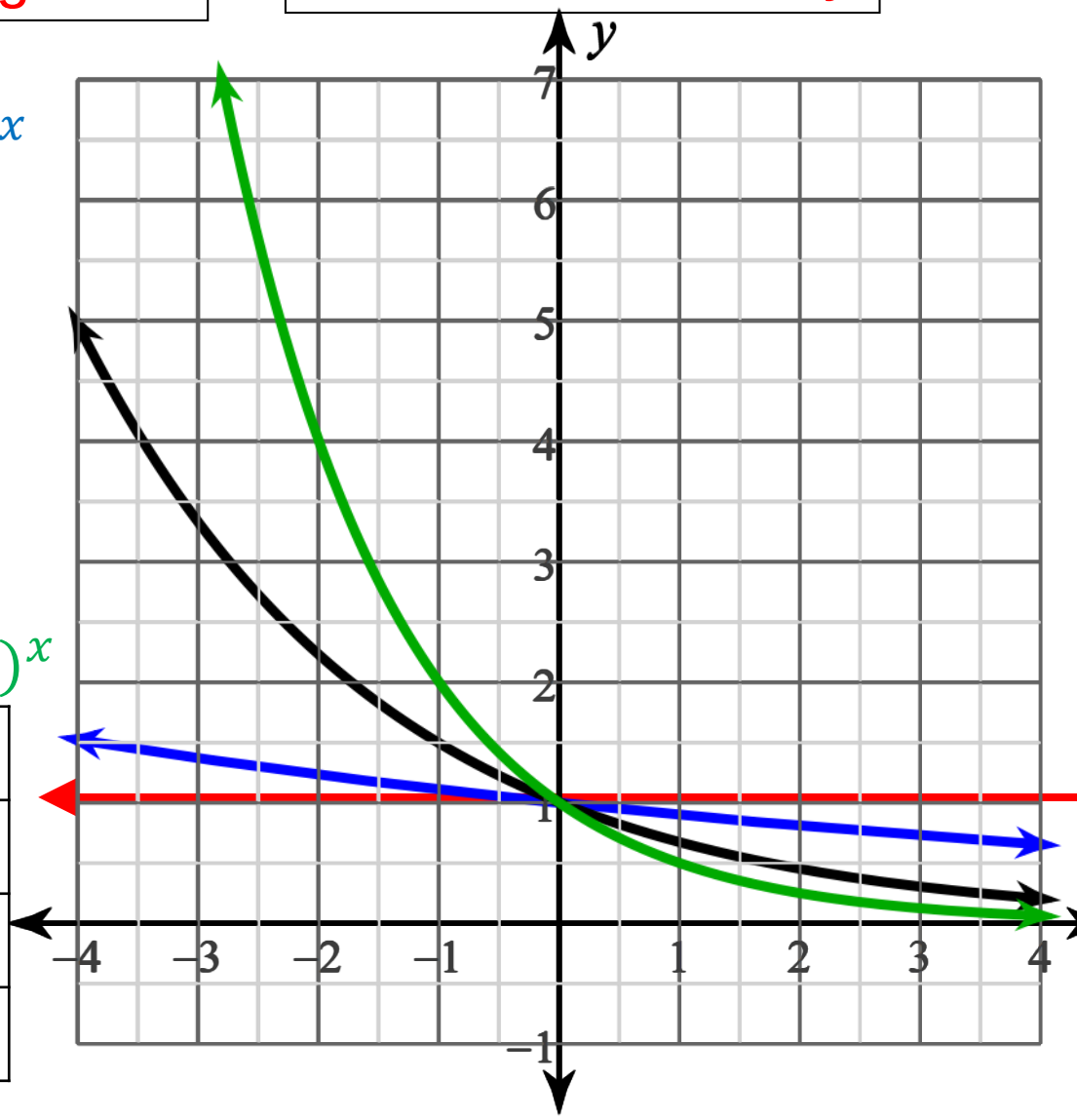
x	g(x)
-1	1.1
0	1
1	0.9

$h(x) = (0.67)^x$

x	h(x)
-1	1.5
0	1
1	0.67

$k(x) = (0.5)^x$

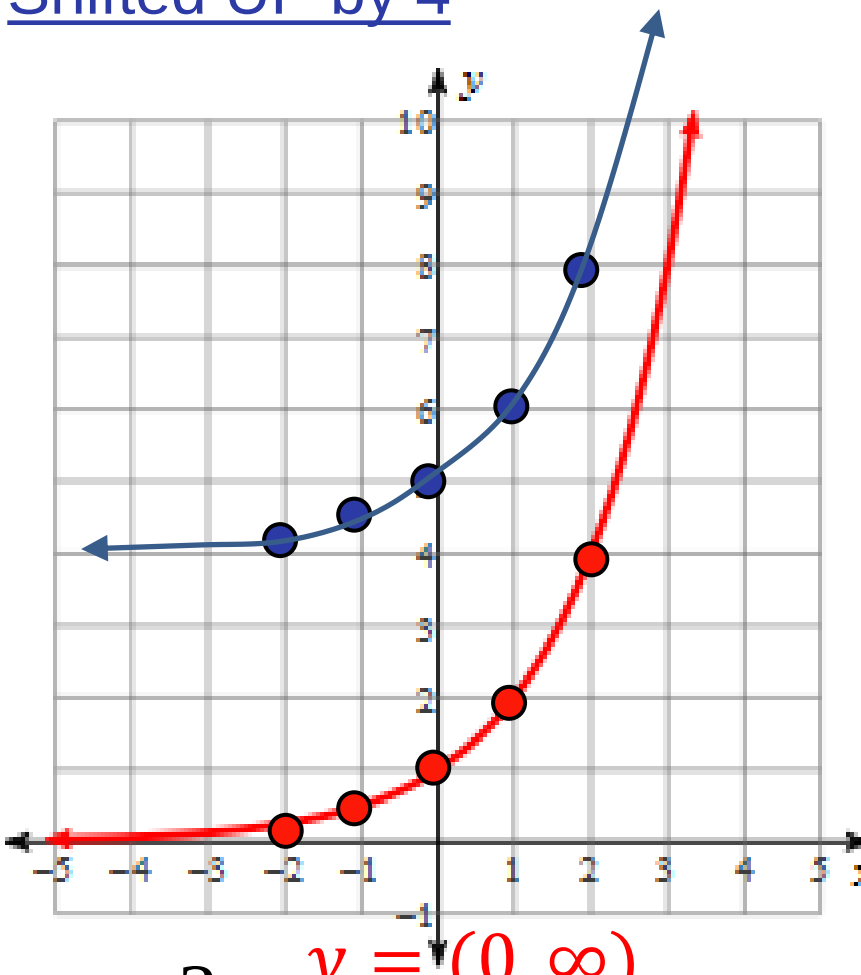
x	k(x)
-1	5
0	1
1	0.2



$f(x) = 2^x$ $k(x) = 2^x + 4$

Shifted UP by 4

x	$2^{(\ )}$	f(x)	$k(x)$
-2	2^{-2}	0.25	4.25
-1	2^{-1}	0.5	4.5
0	2^0	1	5
1	2^1	2	6
2	2^2	4	8



Horizontal asymptote: $y = 0$ $y = 4$

$range = ?$ $y = (0, \infty)$ $y = (4, \infty)$

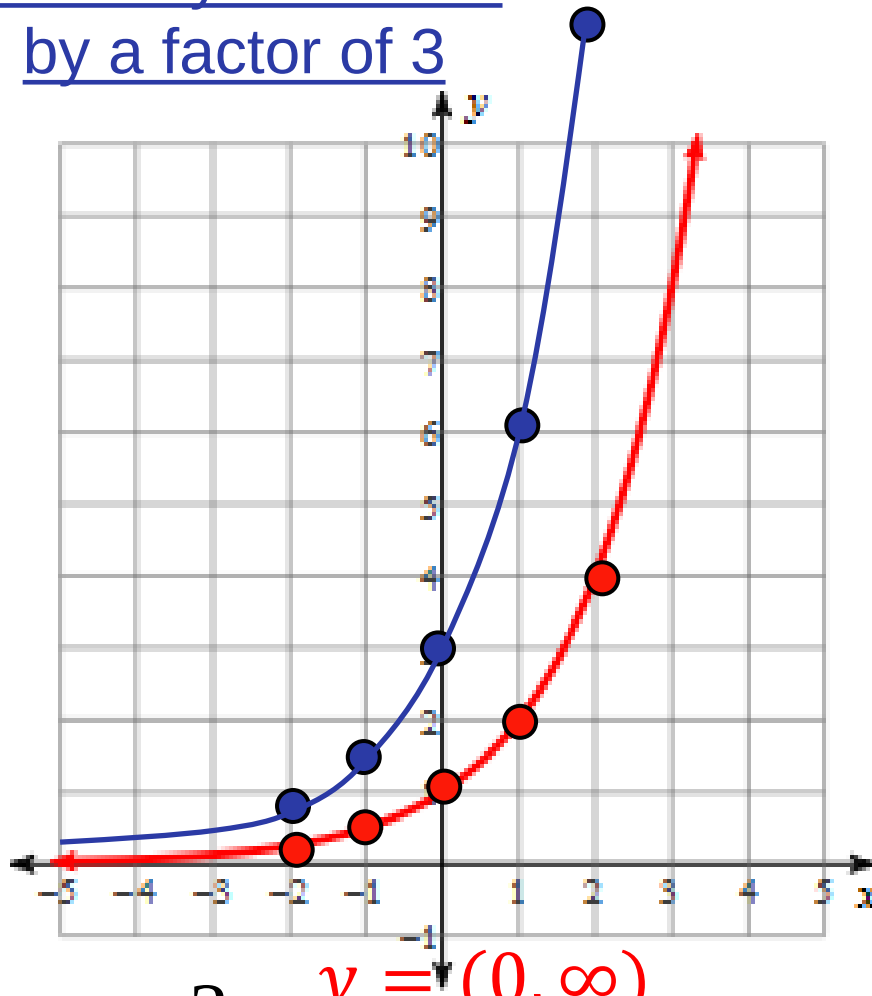
$Domain = ?$ $x = (-\infty, \infty)$ $x = (-\infty, \infty)$

y-intercept = ? $(0, 1)$ $(0, 5)$

$f(x) = 2^x$ $g(x) = 3(2)^x$

Vertically stretched by a factor of 3

x	$2^{(\ )}$	f(x)	$g(x)$
-2	2^{-2}	0.25	0.75
-1	2^{-1}	0.5	1.5
0	2^0	1	3
1	2^1	2	6
2	2^2	4	12



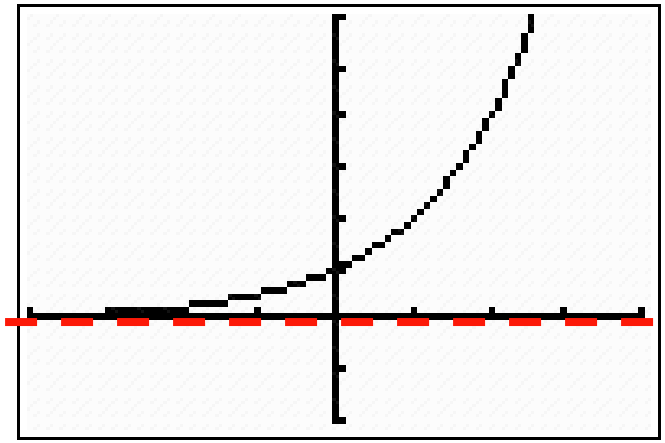
Horizontal asymptote: $y = 0$ $y = 0$

$range = ?$ $y = (0, \infty)$ $y = (0, \infty)$

$Domain = ?$ $x = (-\infty, \infty)$ $x = (-\infty, \infty)$

y-intercept = ? $(0, 1)$ $(0, 3)$

Transformations of the Exponential Function

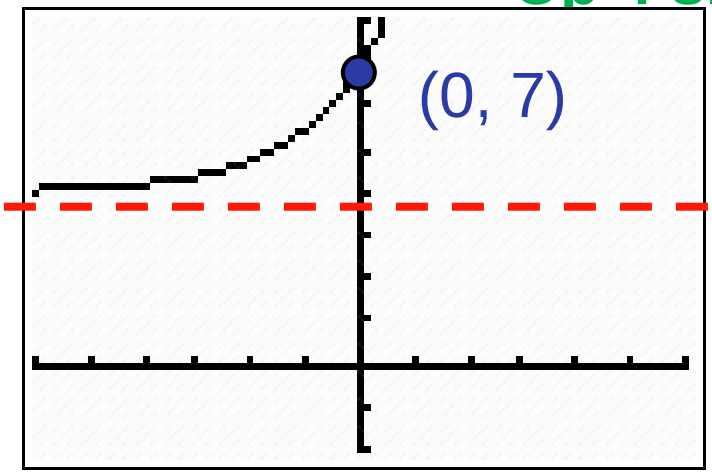


$f(x) = 2^x$ Base-2 Exponential Parent Function

VSF= 3

$h(x) = 3(2)^x + 4$

Up 4 shift



Transformation Form of the Exponential Function

$y = ab^x + k$ ← vertical shift and horizontal Asymptote

VSF:

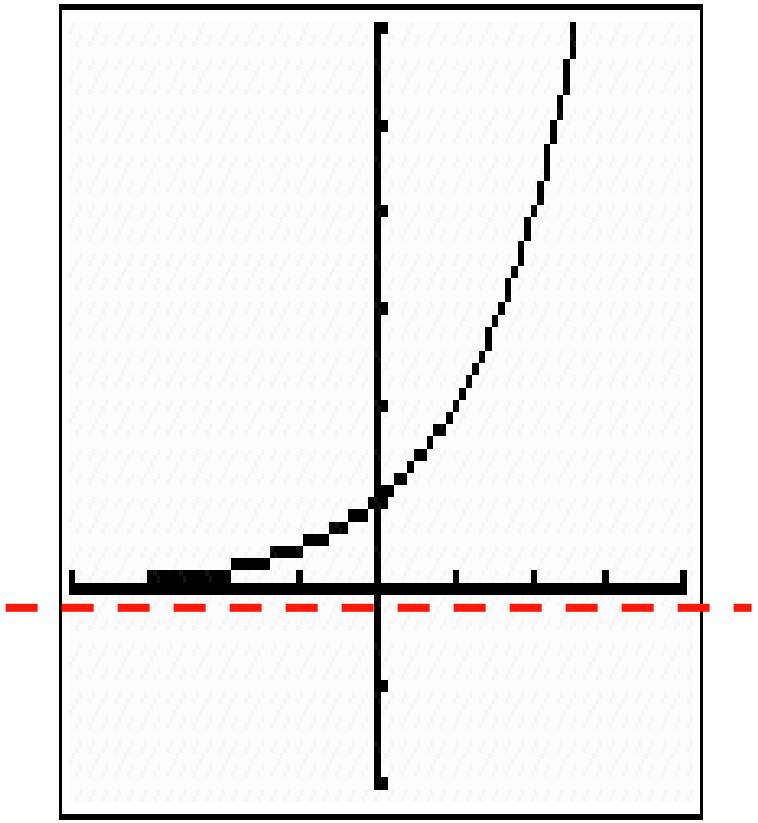
y-intercept: (0, a + k)

Growth Factor (the base of the exponential)

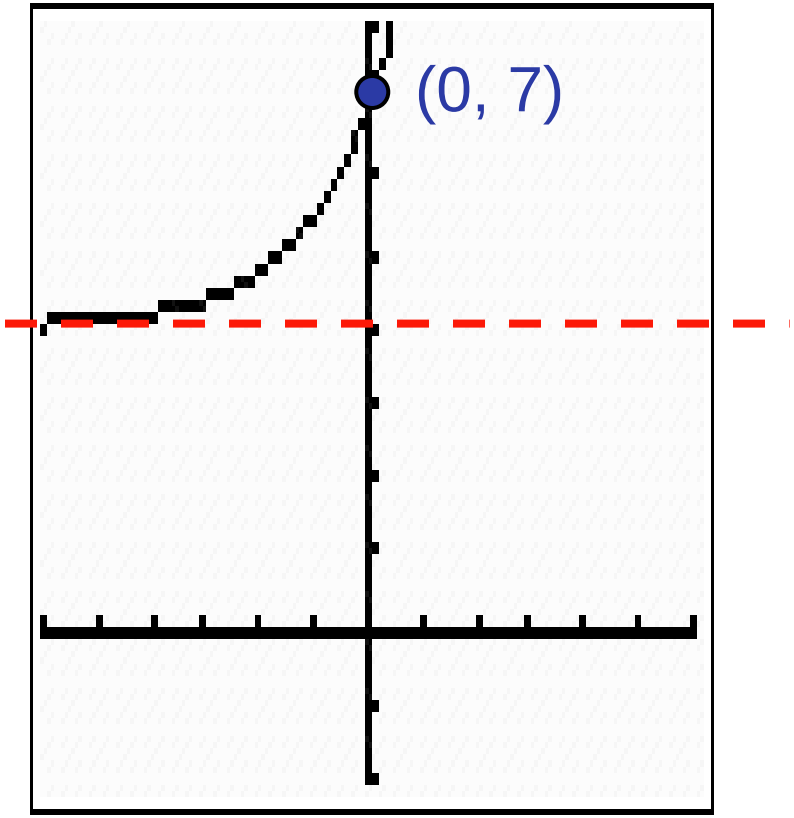
$h(0) = 3(2)^0 + 4$

$h(0) = 7$

$f(x) = 2^x$



$h(x) = 3(2)^x + 4$ **Up 4 shift**



$y = ab^x + k$

vertical shift and horizontal Asymptote

VSF

And how many units above the HA the y-intercept is.

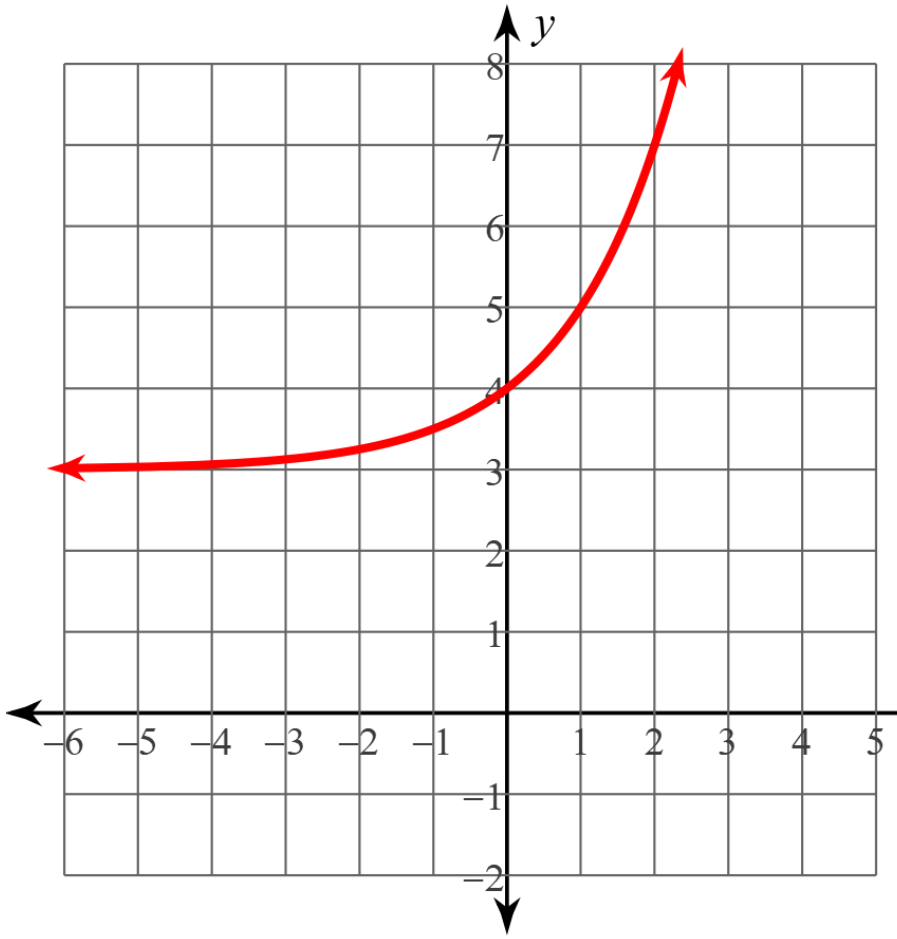
Growth Factor (the base of the exponential)

What is the equation of the graph?

1) Start with $g(x) = ab^x + k$

2) Find ‘k’

Horizontal asymptote:



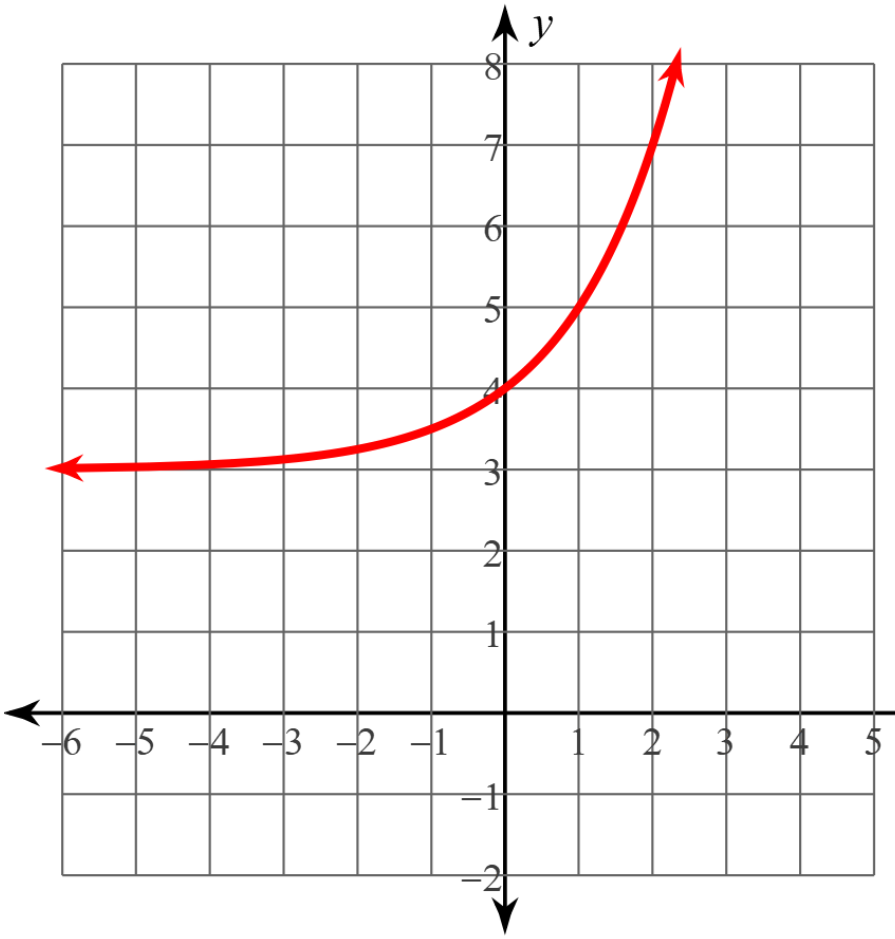
What is the equation of the graph?

1) Start with $g(x) = ab^x + k$

2) Find 'k'

Horizontal asymptote: y = 3

$k = 3$ $\quad y = ab^x + 3$



What is the equation of the graph?

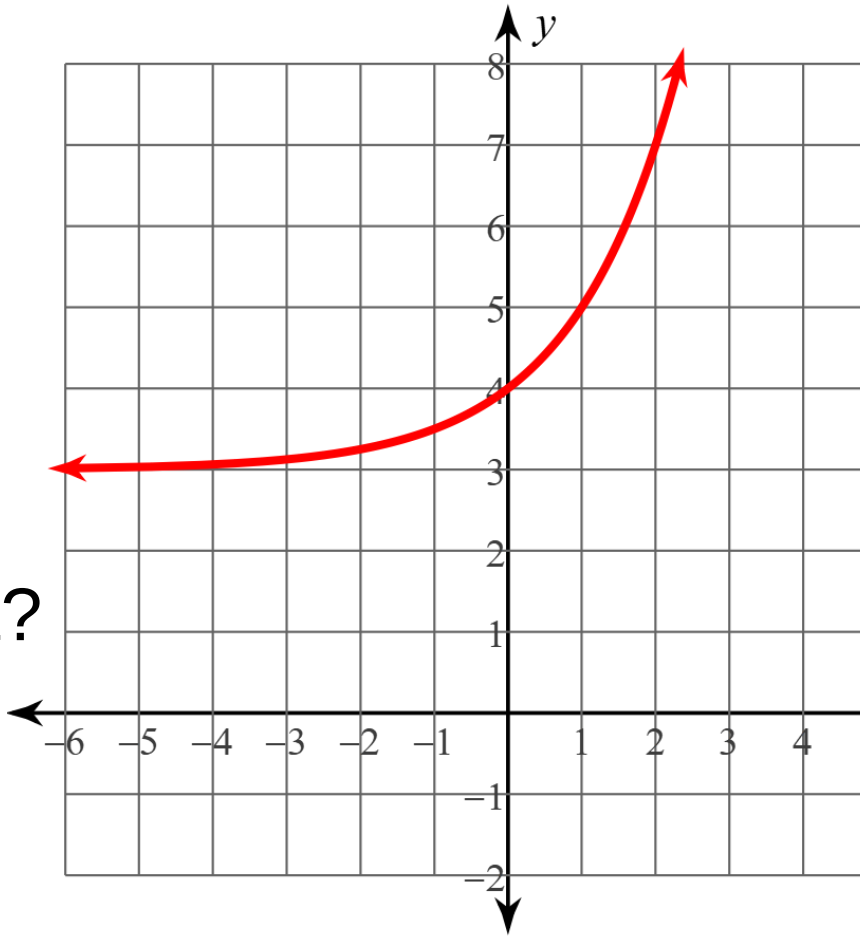
1) Start with $g(x) = ab^x + k$

2) Find 'k'

Horizontal asymptote: y = 3

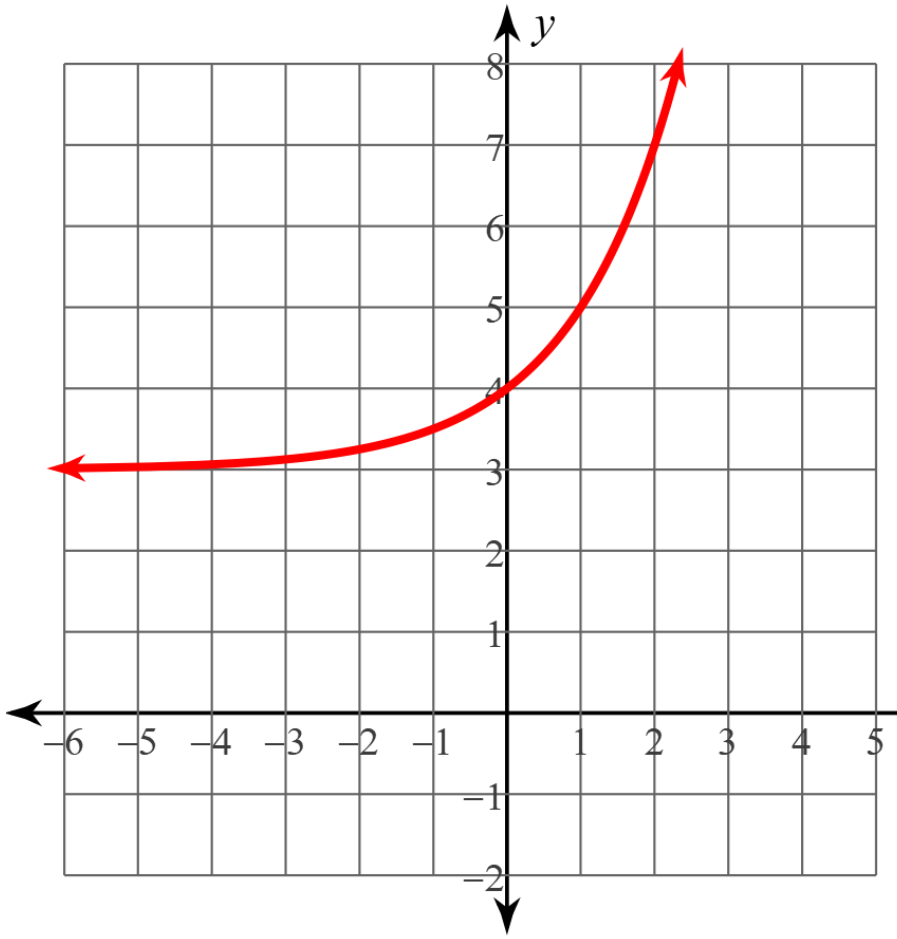
$k = 3$ $\quad y = ab^x + 3$

3) Find 'a' How many spaces above the HA is the y-intercept?



What is the equation of the graph?

1) Start with $g(x) = ab^x + k$



2) Find 'k'

Horizontal asymptote: y = 3

$k = 3$ $\quad y = ab^x + 3$

3) Find 'a' How many spaces above the HA is the y-intercept?

a = 1 → $y = b^x + 3$

4) Substitute a "nice" x-y pair from the graph into the equation.

(1, 5) → $5 = b^1 + 3$ → b = 2 $\quad y = 2^x + 3$

What is the equation of the graph?

1) Start with $g(x) = ab^x + k$

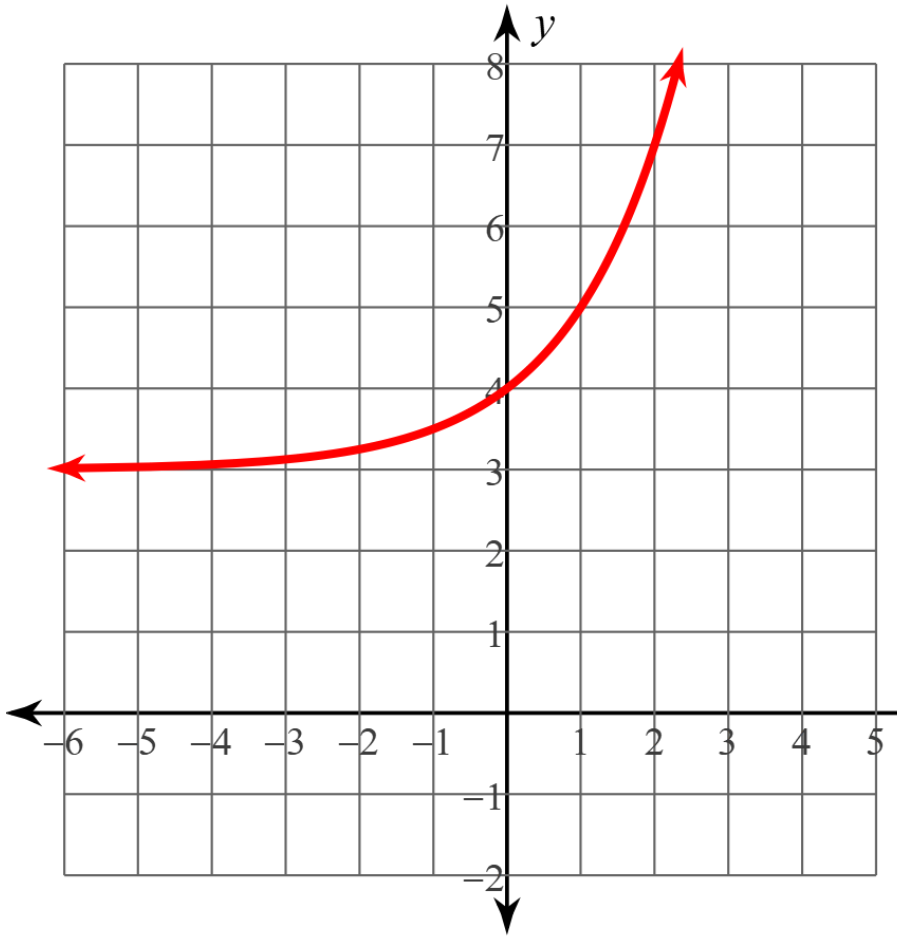
2) Find 'k'

Horizontal asymptote: y = 3

$k = 3$ $\quad y = ab^x + 3$

3) Find 'a' How many spaces above the HA is the y-intercept?

a = 1 → $y = b^x + 3$



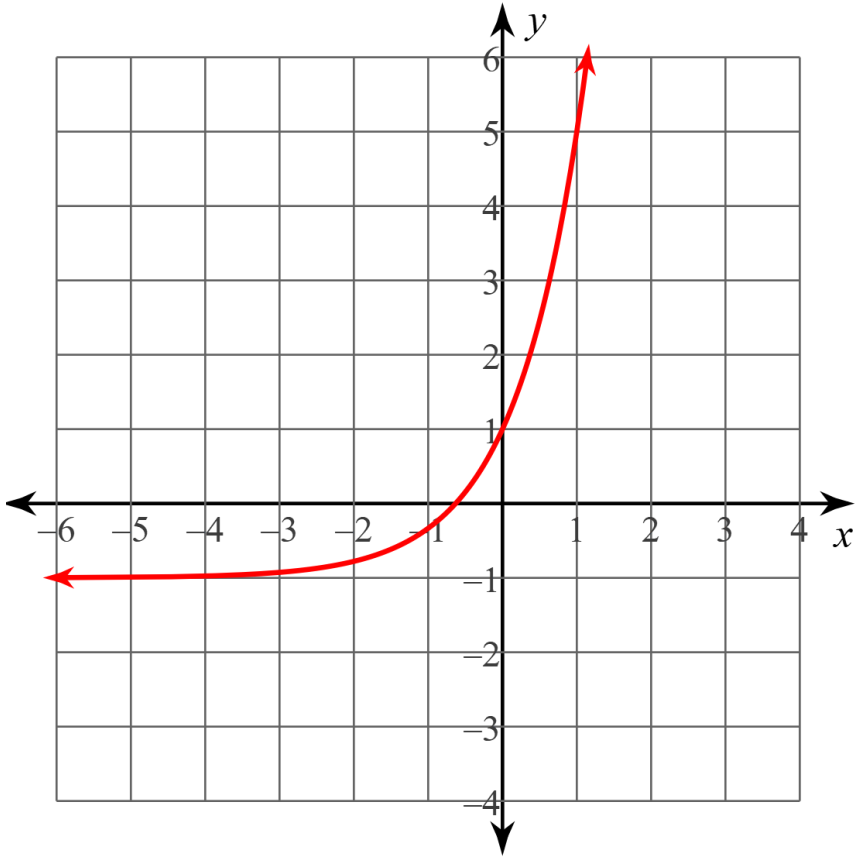
4) Substitute a "nice" x-y pair from the graph into the equation.

What is the equation of the graph?

1) Start with $g(x) = ab^x + k$

2) Find ‘k’

Horizontal asymptote:



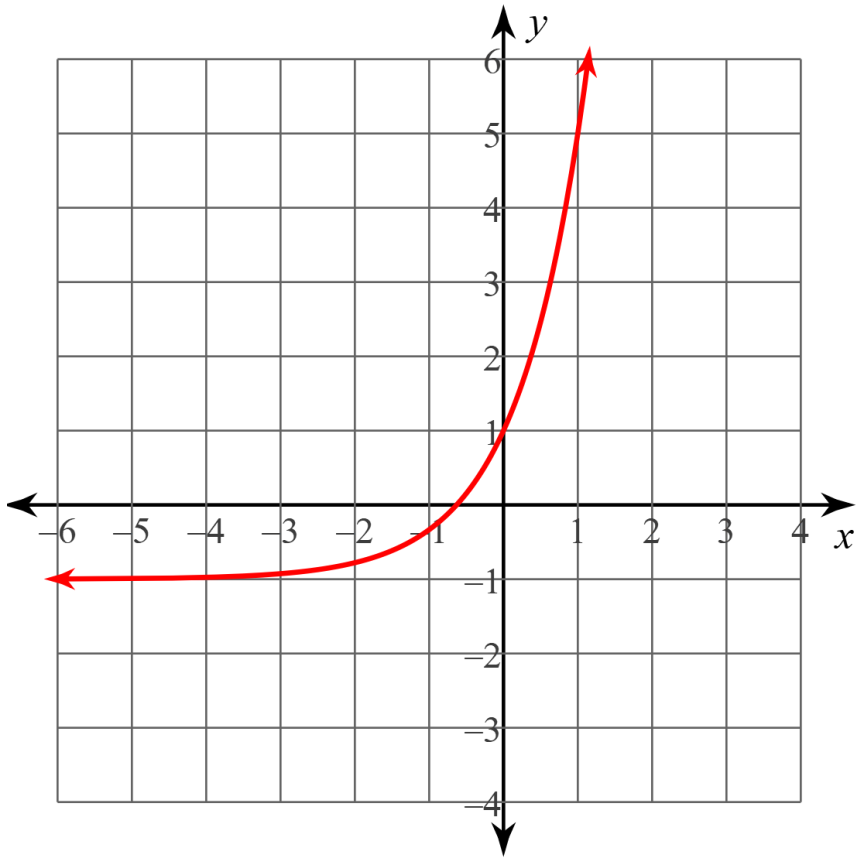
What is the equation of the graph?

1) Start with $g(x) = ab^x + k$

2) Find 'k'

Horizontal asymptote: y = -1

$k = -1$ $y = ab^x - 1$



What is the equation of the graph?

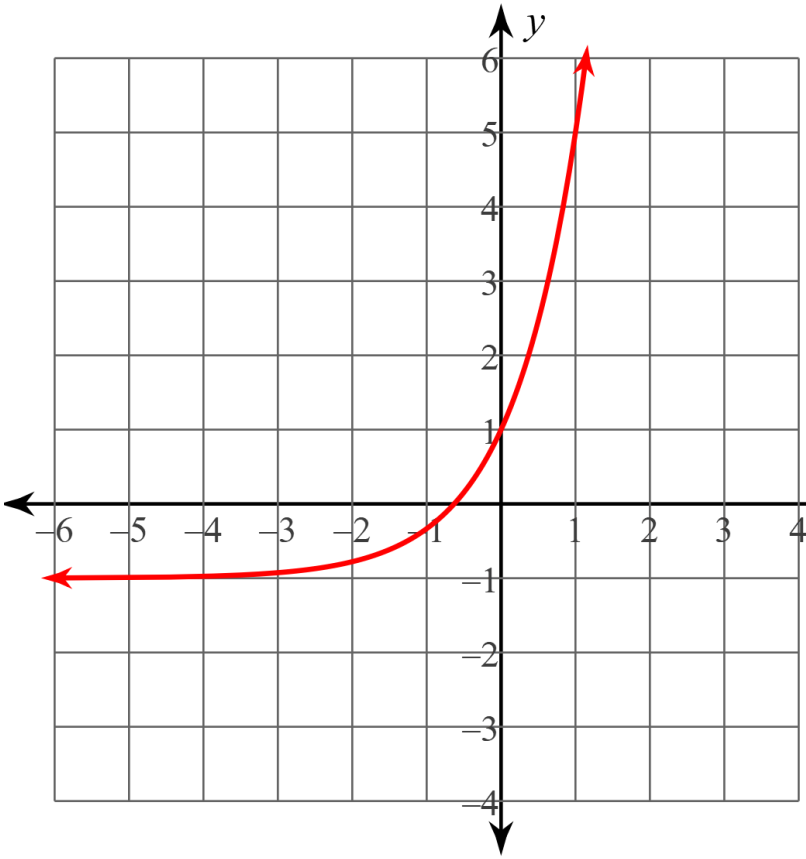
1) Start with $g(x) = ab^x + k$

2) Find 'k'

Horizontal asymptote: y = -1

$k = -1$ $y = ab^x - 1$

3) Find 'a' How many spaces above the HA is the y-intercept?



What is the equation of the graph?

1) Start with $g(x) = ab^x + k$

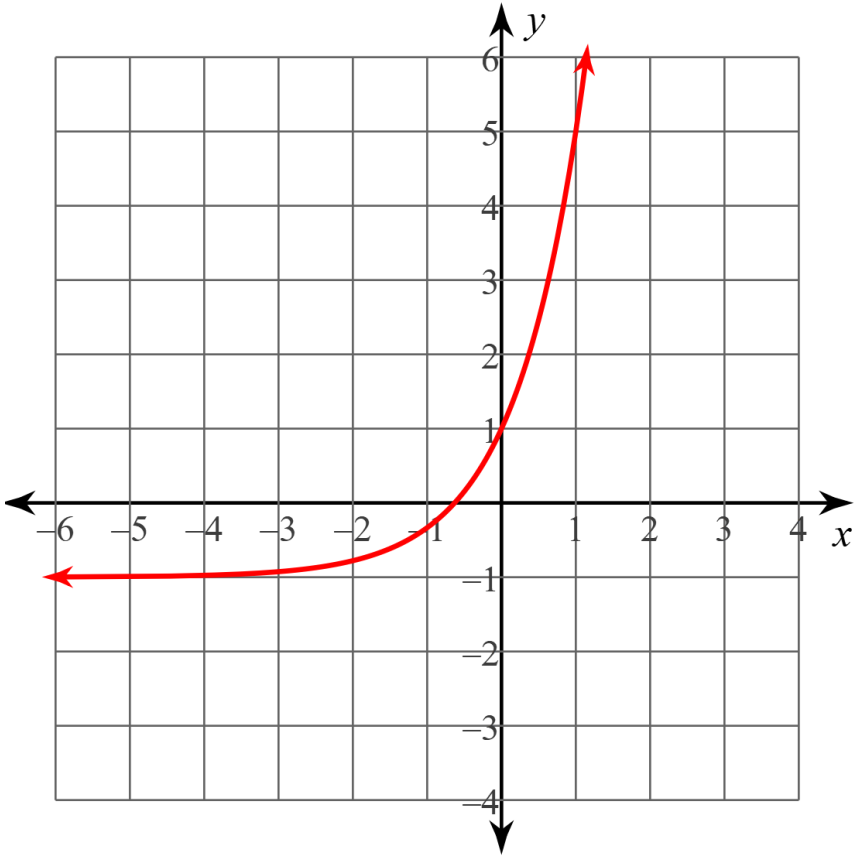
2) Find 'k'

Horizontal asymptote: y = -1

$k = -1$ $y = ab^x - 1$

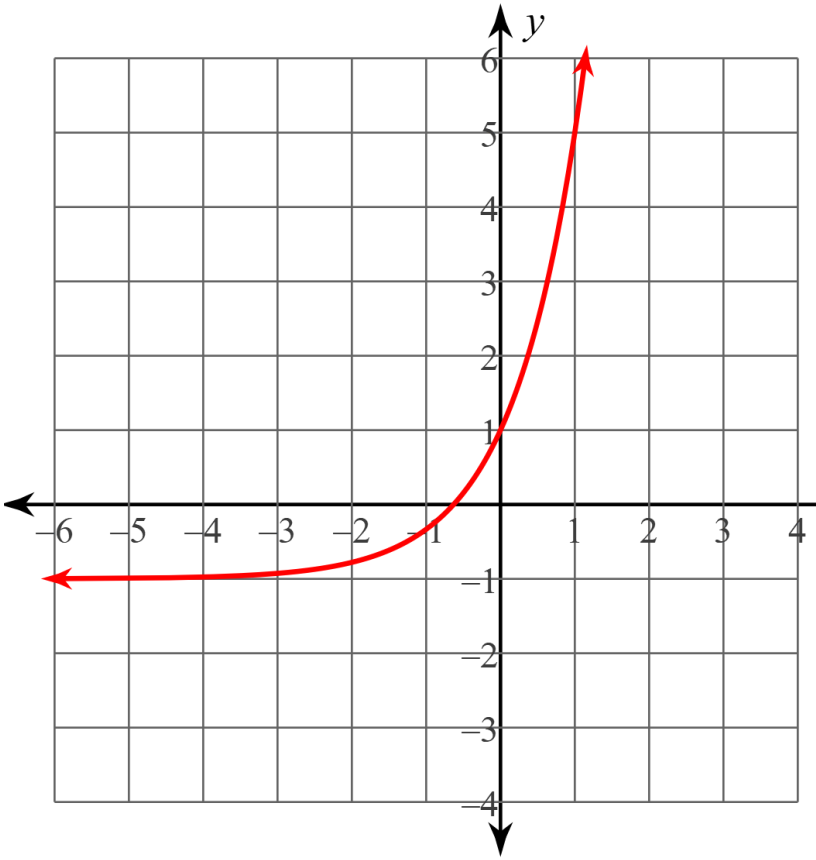
3) Find 'aHow many spaces above the HA is the y-intercept?

a = 2 → $y = 2b^x - 1$



What is the equation of the graph?

1) Start with $g(x) = ab^x + k$



2) Find 'k'

Horizontal asymptote: y = -1

$k = -1$ $y = ab^x - 1$

3) Find 'a' How many spaces above the HA is the y-intercept?

a = 2 → $y = 2b^x - 1$

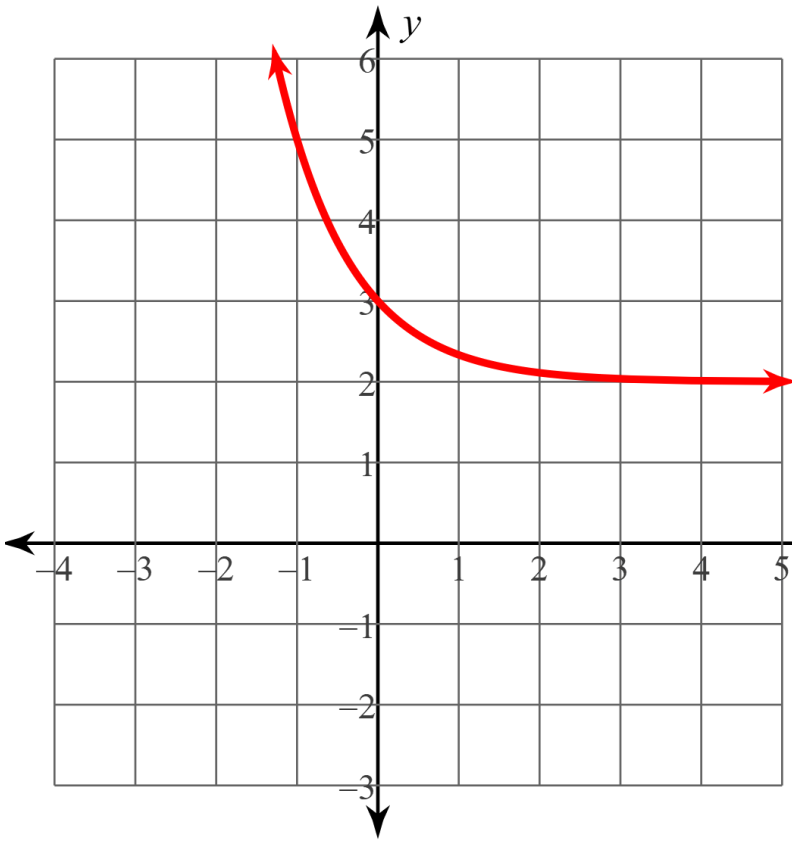
4) Substitute a "nice" x-y pair from the graph into the equation.

(1, 5) → $5 = 2b^1 - 1$ → b = 3 $y = 2(3^x) - 1$

What is the equation of the graph?

1) Start with $g(x) = ab^x + k$

2) horizontal asymptote

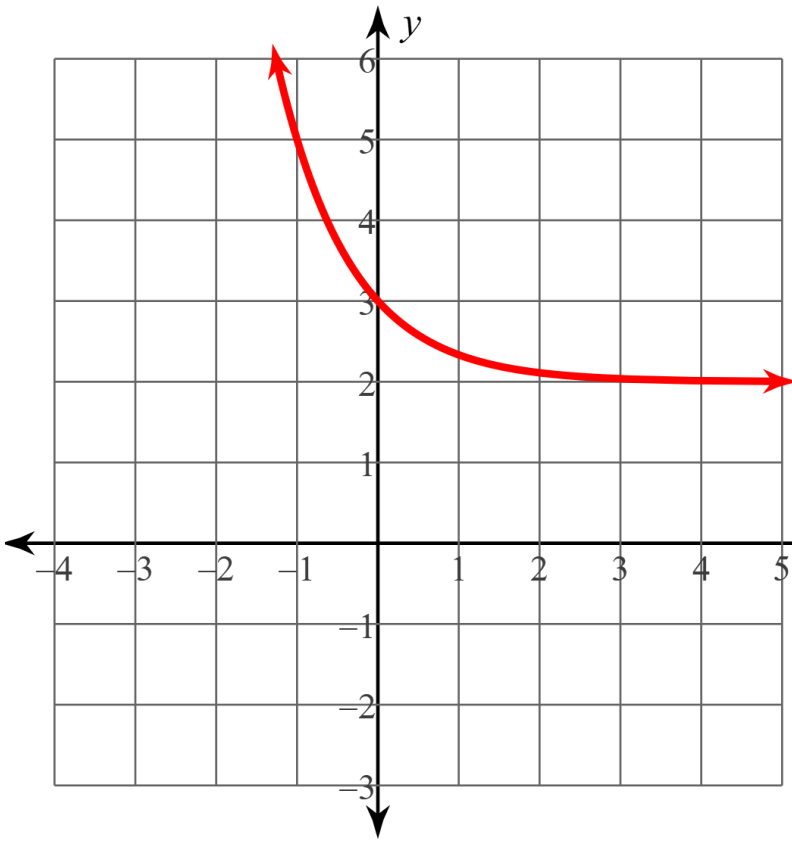


What is the equation of the graph?

1) Start with $g(x) = ab^x + k$

2) horizontal asymptote y = 2

$k = 2$ $y = ab^x + 2$



What is the equation of the graph?

1) Start with $g(x) = ab^x + k$

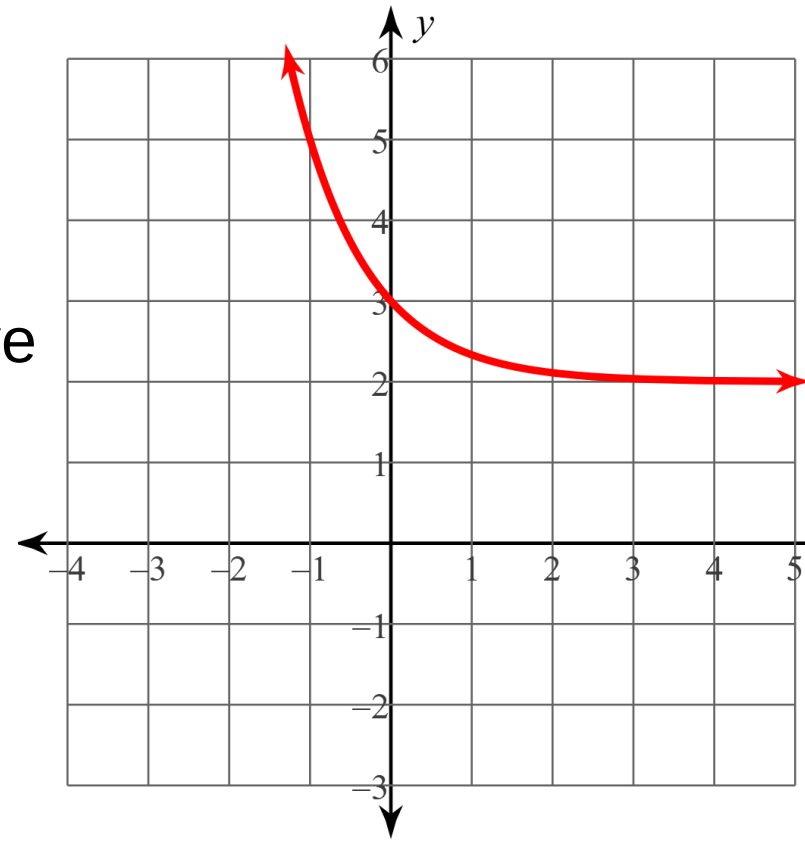
2) horizontal asymptote y = 2

$k = 2$ $y = ab^x + 2$

3) y-intercept How many spaces above the HA is the y-intercept?

$a = 1$ $y = 1 * b^x + 2$

$y = b^x + 2$



What is the equation of the graph?

1) Start with $\boxed{g(x) = ab^x + k}$

2) horizontal asymptote y = 2

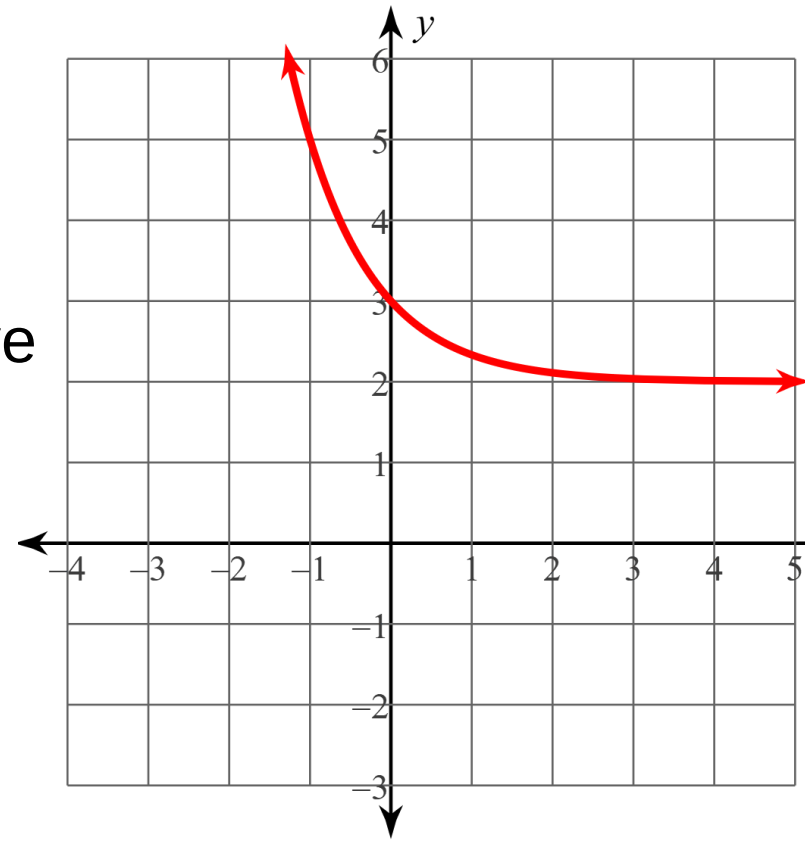
$k = 2$ $\boxed{y = ab^x + 2}$

3) y-intercept How many spaces above the HA is the y-intercept?

$a = 1$ $y = 1 * b^x + 2$

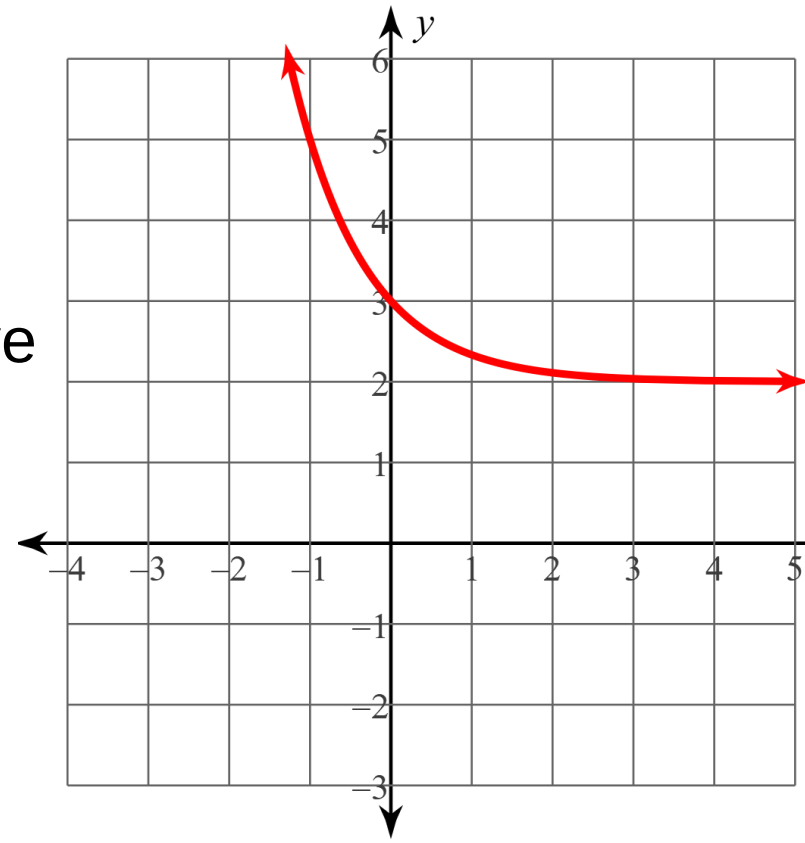
$y = b^x + 2$

4) "Nice" x-y pair $(-1, 5)$



What is the equation of the graph?

1) Start with $\boxed{g(x) = ab^x + k}$



2) horizontal asymptote y = 2

$k = 2$ $\boxed{y = ab^x + 2}$

3) y-intercept How many spaces above the HA is the y-intercept?

$a = 1$ $y = b^x + 2$

4) "Nice" x-y pair $(-1, 5)$

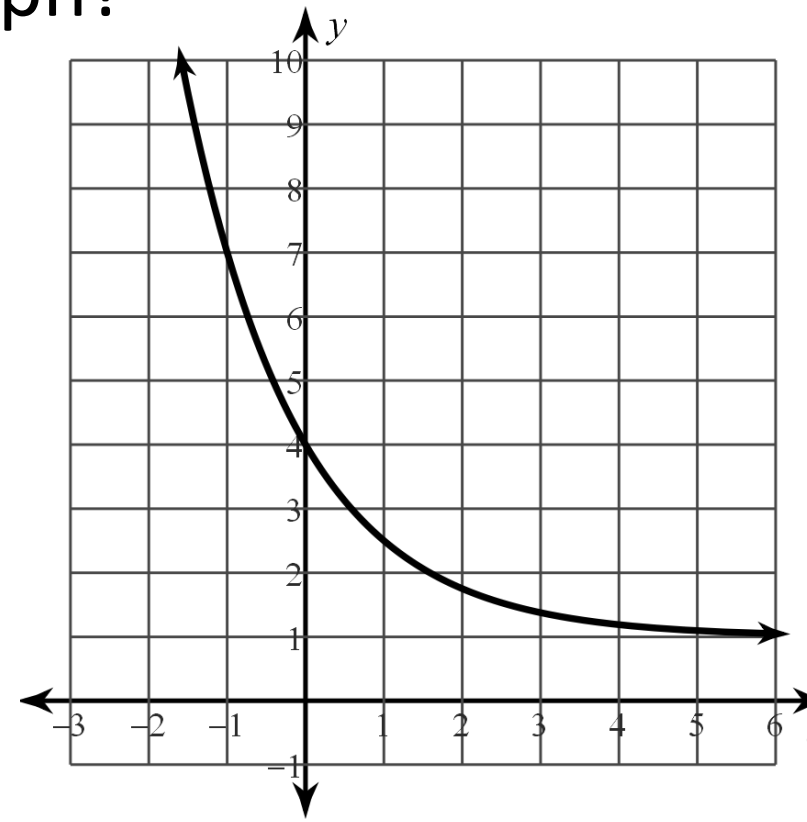
$5 = b^{-1} + 3$

$2 = b^{-1}$ $2 = \frac{1}{b}$ $b = \frac{1}{2}$ $y = \left(\frac{1}{2}\right)^x + 2$

What is the equation of the graph?

1) Start with $g(x) = ab^x + k$

2) horizontal asymptote

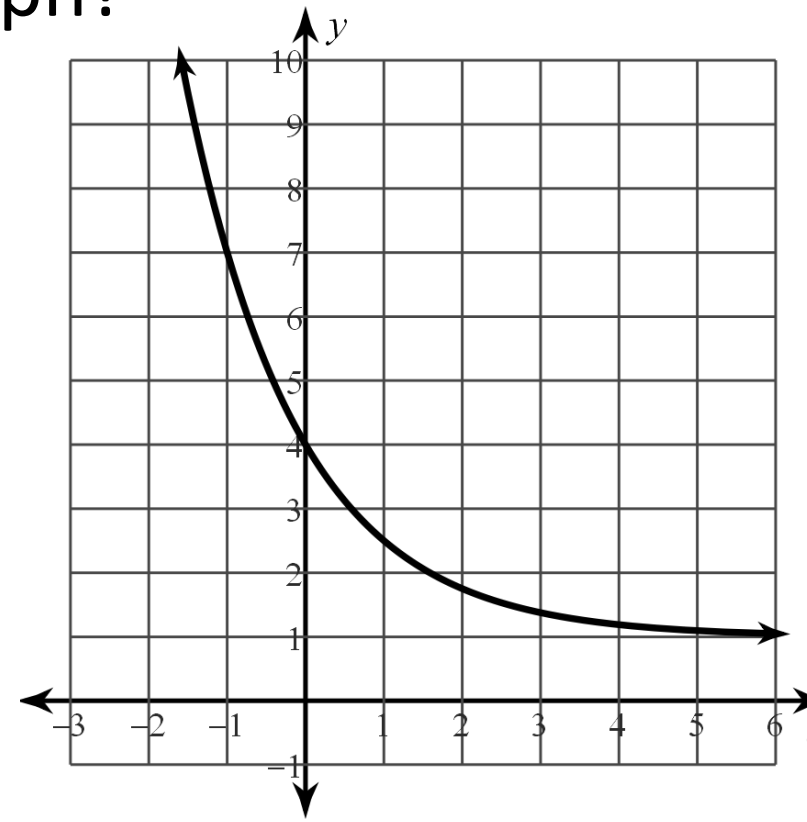


What is the equation of the graph?

1) Start with $g(x) = ab^x + k$

2) horizontal asymptote y = 1

$k = 1$ $y = ab^x + 1$



What is the equation of the graph?

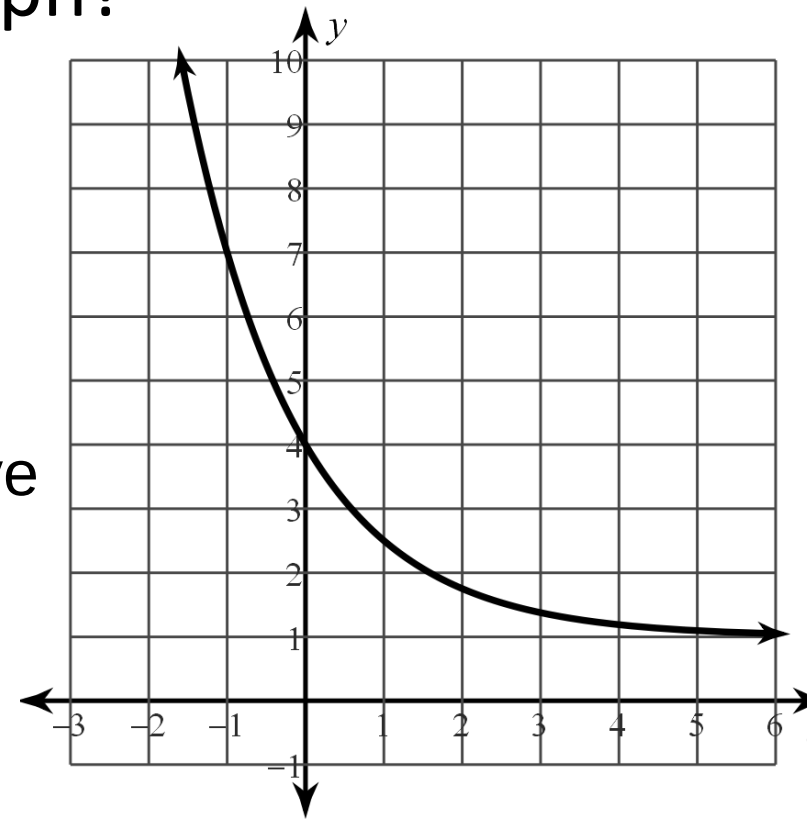
1) Start with $g(x) = ab^x + k$

2) horizontal asymptote y = 1

$k = 1$ $y = ab^x + 1$

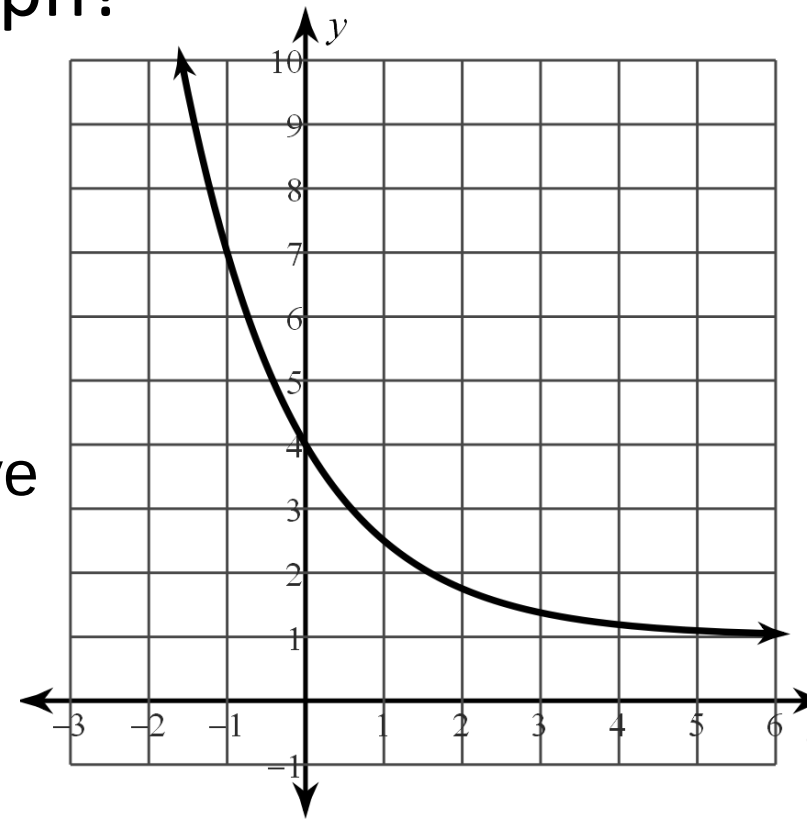
3) y-intercept How many spaces above the HA is the y-intercept?

$a = 3$ $y = 3b^x + 1$



What is the equation of the graph?

1) Start with $g(x) = ab^x + k$



2) horizontal asymptote y = 1

$k = 1$ $y = ab^x + 1$

3) y-intercept How many spaces above the HA is the y-intercept?

$a = 3$ $y = 3b^x + 1$

4) "Nice" x-y pair $(-1, 7)$

$7 = 3b^{-1} + 1$

$6 = 3b^{-1}$

$2 = b^{-1}$ $2 = \frac{1}{b}$ $b = \frac{1}{2}$ $y = 3\left(\frac{1}{2}\right)^x + 1$