Math-2A
Lesson 8-1

Geometry Review, and
Midpoint and Distance Formulas

Explain, in your own words, what you think “Geometry” means.

Geometry (from the Ancient Greek:
geo- "earth",
-metron "measurement")
is a branch of mathematics concerned with questions of shape, size, relative position of figures, and the properties of space.

Point: the smallest “building block” of geometry.

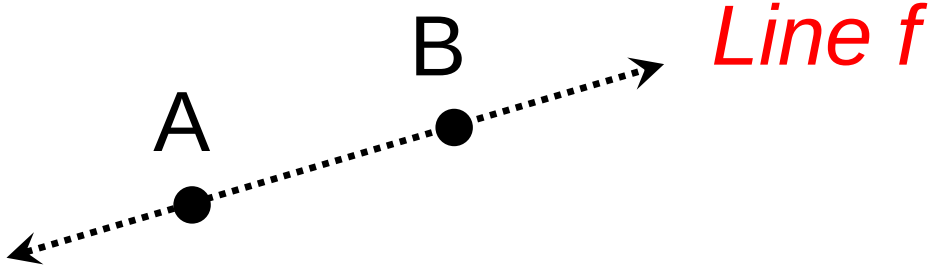
Points are labeled with capital letters.

Points have no size (they are infinitely small).

Line: is a collection of points that “line up” or are “straight”.

Colinear points: are points that are all on the same line.

Two Points: define the location and the direction of a line.



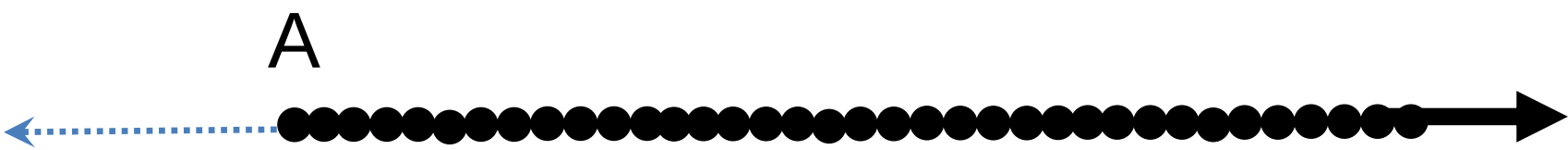
Lines are represented by two capital letters (for two points that are on the line) with a double arrow above the two letters $\overleftrightarrow{AB}$ or by a single lower case letter (Line *f*)

Line Segment: is made up of 2 endpoints and all the points between them that are colinear with the end points.

Line segments are represented by two capital letters (for the two end points of the line segment) with a dash above the two letters. $\overline{AB}$

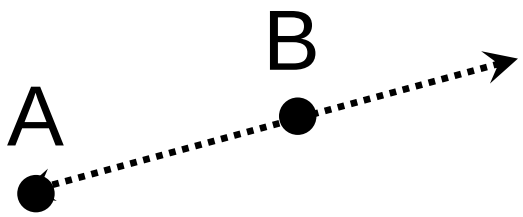
Length of a line segment is represented by two capital letters (for two end points) with no dash above the two letters. AB

Ray Is one half of a line. It has an endpoint and a direction.

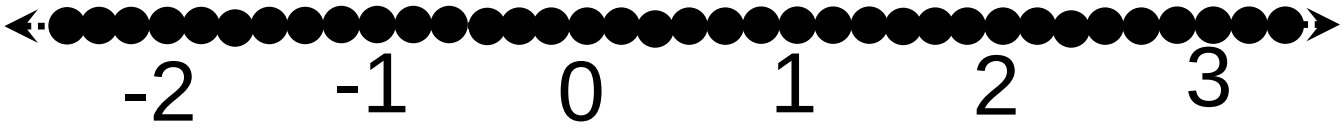


Rays are represented by two capital letters (for the end point and one other point of the ray) with a single direction arrow above the two letters. $\overrightarrow{AB}$

Two Points: define the location and the direction of a ray.



Number line. A line (a set of colinear points), where each point has been assigned a number. The numbers become larger from left to right.



Points have no size (they are infinitely small).

There are infinitely many **colinear** points between two points.

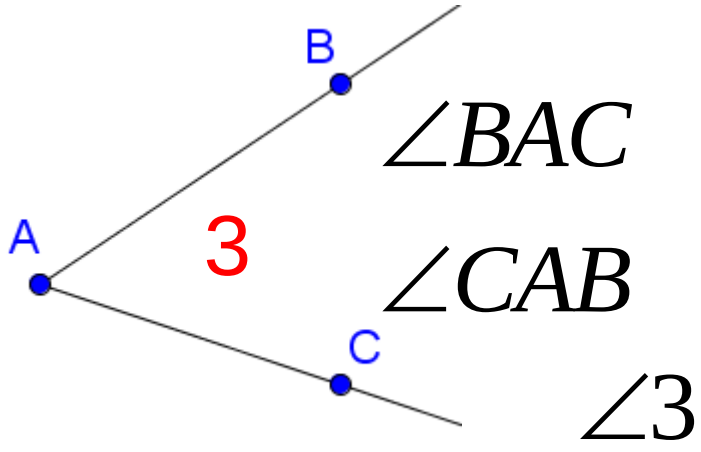
There are infinitely many numbers between two numbers on a number line.

X-Y Plane: infinitely many horizontal and vertical number lines where each point of intersection is a unique point with a location given by the numerical value of each number line where the two lines cross.

Angles can be represented symbolically three ways.

(1) angle symbol followed by one letter representing the vertex of the angle. (We can use this only if there is only one angle with that vertex). $\angle A$

(2) angle symbol followed by three letters representing a point on one side of the angle, the vertex (2nd letter), and a point on the other side of the angle.



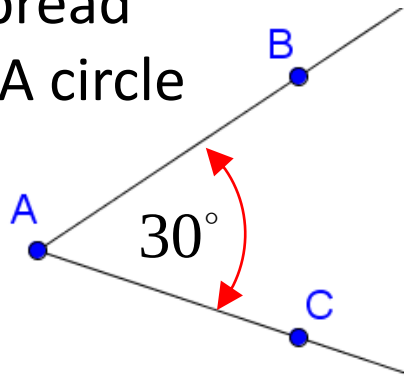
$\angle BAC$

$\angle CAB$

$\angle 3$

(3) Using an angle symbol followed by one number that is a label and NOT the measure of the angle.

Measure of an angle: how wide the sides are spread apart. For now, we measure angles in degrees. A circle has a measure of 360.˚



$$m\angle A = 30^\circ$$

Theta: "Θ" → a Greek letter used for the measure of an unknown angle.

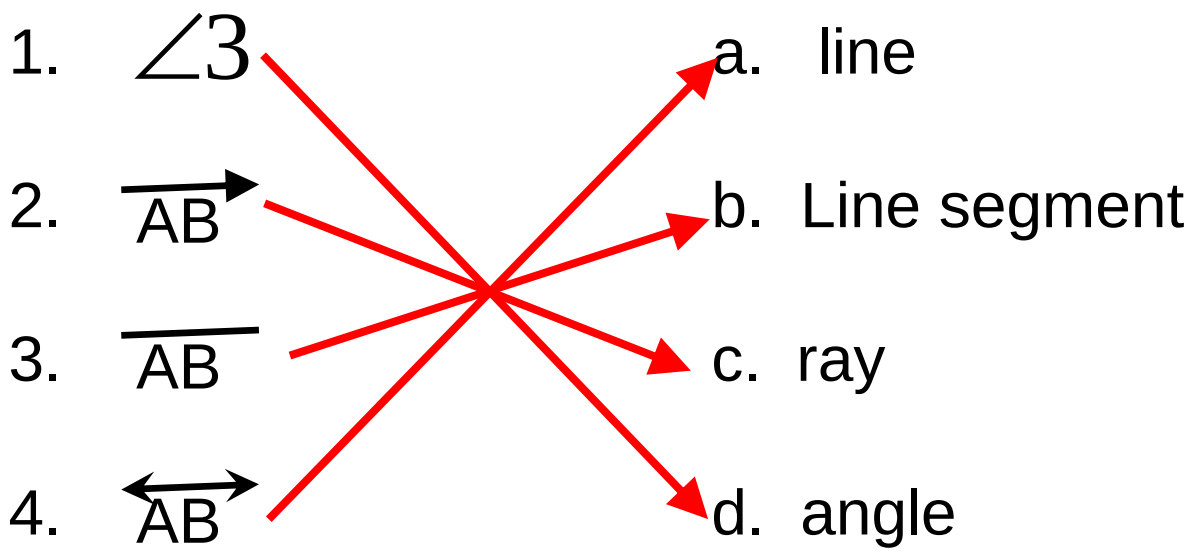
Angles are categorized based upon their measures.

(1) Acute angle: an angle whose measure is: $0^\circ < \Theta < 90^\circ$

(2) Right Angle: an angle whose measure is: $\Theta = 90^\circ$

(3) Obtuse angle: an angle whose measure is: $\Theta > 90^\circ$

Match the symbol with its description

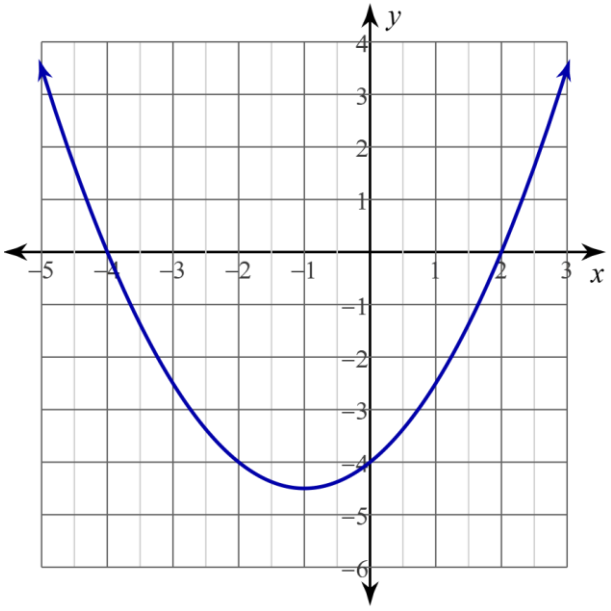


Are the following two angles the same angle or are they different angles? $\angle BAC,\ \angle CAB$

What is the vertex of the angle?

midpoint A point on a line segment that is half-way between the endpoints of the segment.

We can find the midpoint between any two numbers on a number line by ***averaging*** them. $\frac{a+b}{2}$



Find the midpoint between the two x-intercepts.

$$\frac{(-4)+(2)}{2} = \frac{-2}{2} = -1$$

For the parabola, what special point has an x-coordinate that is the midpoint between the two x-intercepts?

vertex

We can find the midpoint of a segment that is on the (x, y) plane using the following formula:

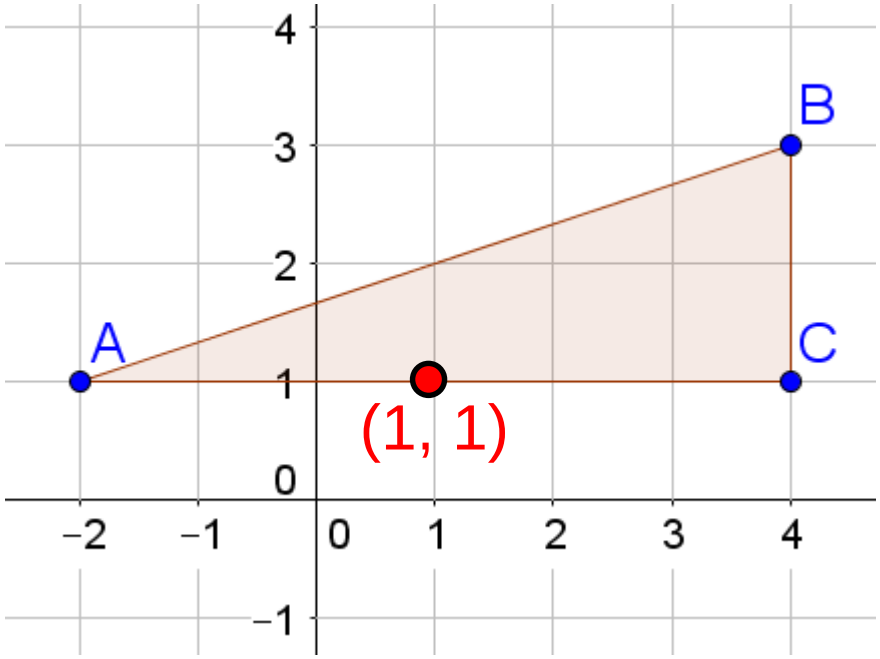
$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

The midpoint x-coordinate is the average of the x-coordinates of the two end points.

The midpoint y-coordinate is the average of the y-coordinates of the two endpoints.

Does the order of x_1 and x_2 matter?

Why not? Commutative property of addition.



Midpoint of $\overline{AC}$ = ?

Point A : $(x_1, y_1) = (-2,1)$

Point C : $(x_2, y_2) = (4,1)$

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\left(\frac{-2+4}{2}, \frac{1+1}{2}\right)$$

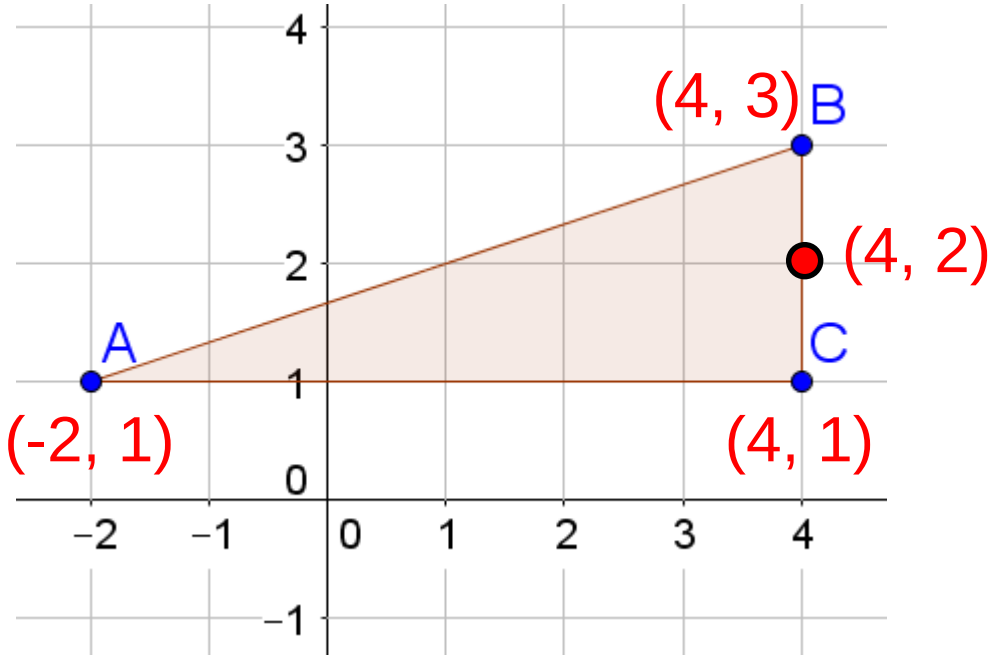
$$(1,\ 1)$$

$\text{Midpoint}_{\overline{AC}} : (1,\ 1)$

Is it necessary to use the midpoint formula to calculate the y-value of a midpoint on a horizontal line?

Why not?

Y-values are all the same on a horizontal line.



Is it necessary to use the midpoint formula to calculate the x-value of a midpoint on a vertical line?

Why not?

x-values are all the same on a vertical line.

Midpoint of $\overline{BC}$ = ?

$$\text{Point B}: (x_1, y_1) = (4,3)$$

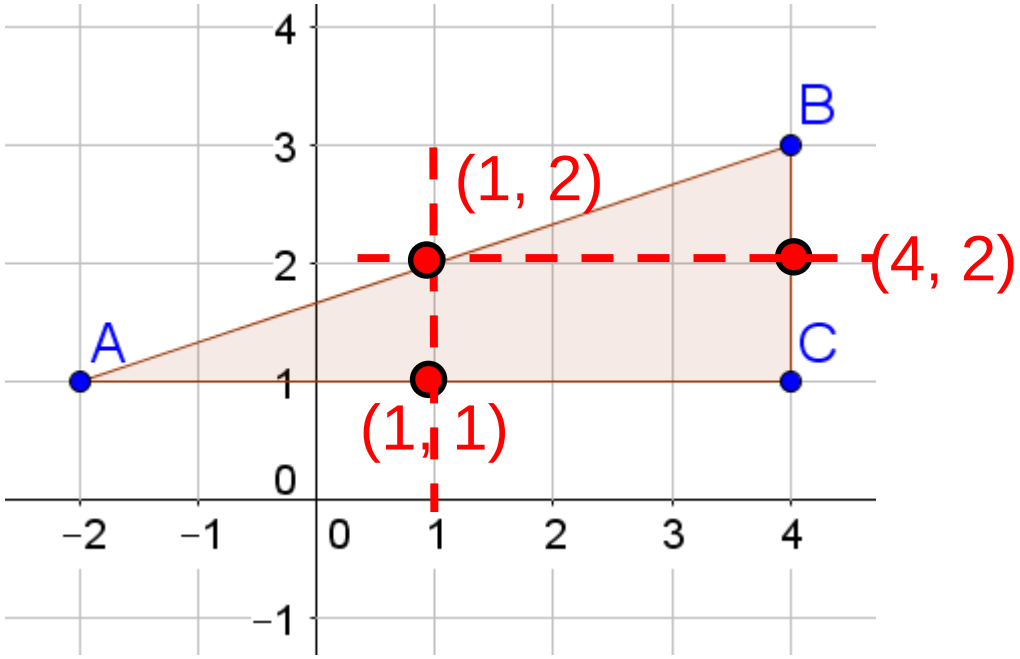
$$\text{Point C}: (x_2, y_2) = (4,1)$$

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$\left(\frac{4+4}{2}, \frac{3+1}{2} \right)$$

$$(4,\ 2)$$

$$\text{Midpoint}_{\overline{BC}}: (4,\ 2)$$



Midpoint of $\overline{AB}$ = ?

Point A : $(x_1, y_1) = (-2,1)$

Point B : $(x_2, y_2) = (4,3)$

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\left(\frac{-2+4}{2}, \frac{1+3}{2}\right)$$

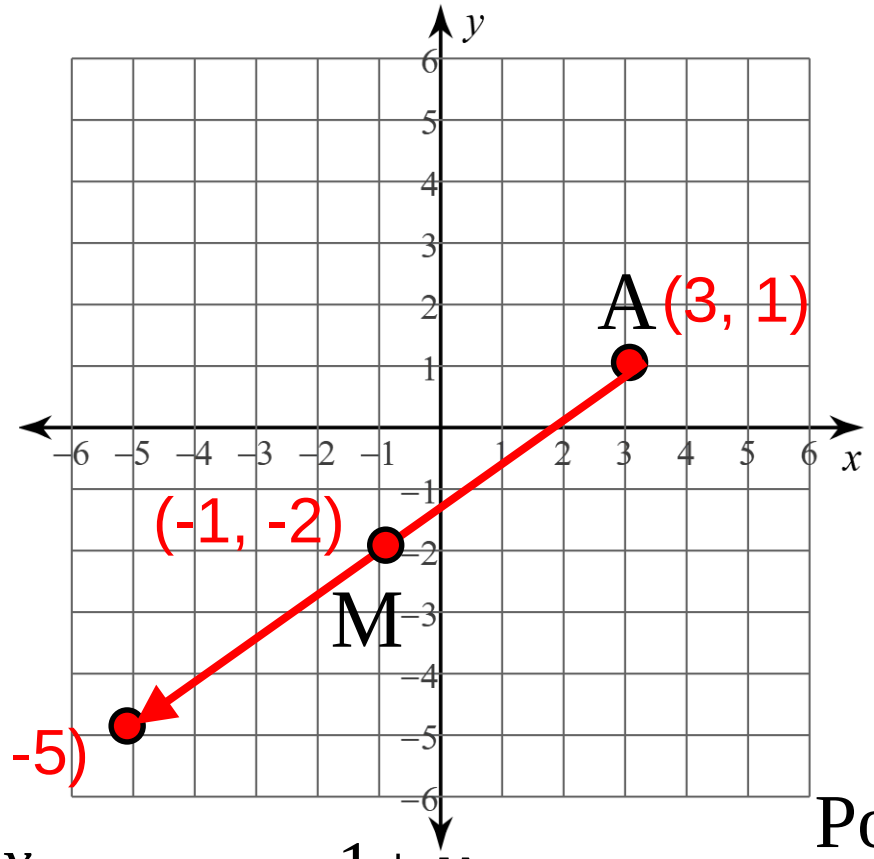
$$\text{Midpoint}_{\overline{AB}} : (1,\ 2)$$

Is it necessary to use the midpoint formula to calculate the x-value and y-value of a midpoint on a line that is neither horizontal or vertcal?

Why?

Every x-y pair is unique on this type of line.

Midpoint of $\overline{AB}$ is (-1,-2) Point A is (3,1). What is Point B?



In which quadrant of the x-y plane is point B?

$$\text{Midpoint}_{\overline{AB}} : (-1, -2) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\frac{x_1 + x_2}{2} = -1 \qquad \frac{y_1 + y_2}{2} = -2$$

$$\text{Point A} : (x_1, y_1) = (3, 1)$$

$$\frac{3 + x_2}{2} = -1 \qquad \frac{1 + y_2}{2} = -2$$

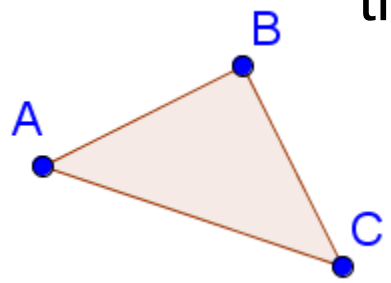
$$3 + x_2 = -2 \qquad 1 + y_2 = -4$$

$$x_2 = -5 \qquad y_2 = -5$$

Using the “building blocks” of Geometry that we have learned so far (points, segments, lines, and rays), how would you define…?

A triangle?

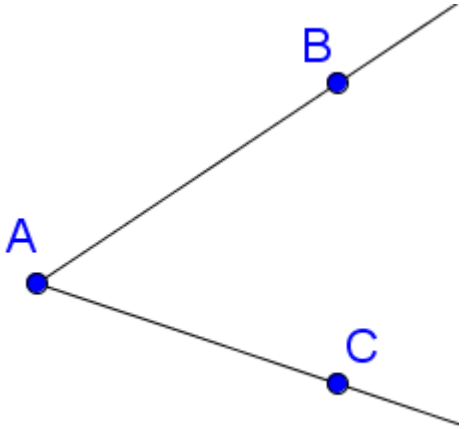
Three non-collinear segments that intersect at their endpoints.



Is a triangle made up of the points that make up the sides OR is it the points that are in the interior?

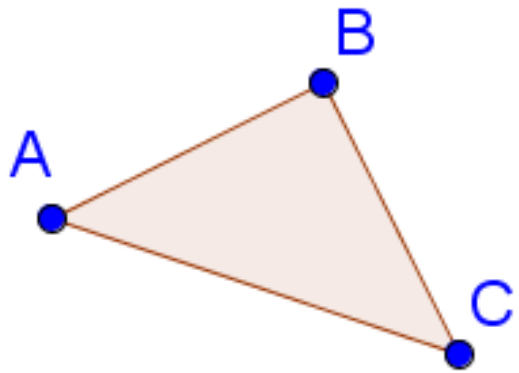
An angle?

Two rays that intersect at their endpoints.



Is an angle made up of the points that are on the sides OR is it the points that are on the interior?

Triangles are represented by the triangle symbol followed by the three letters representing the end points of the segments (each of which is called a vertex).



ΔABC

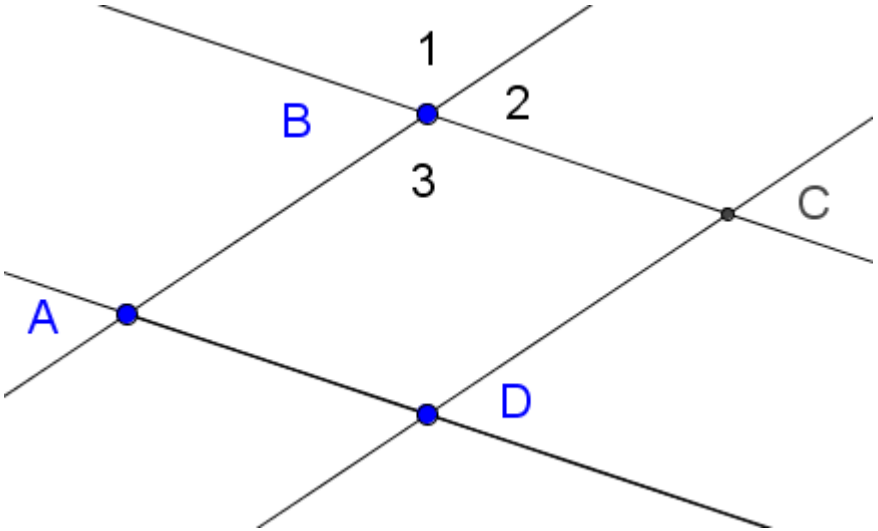
ΔBCA

ΔCAB

etc.

If you are referring to one triangle only, the order of the letters doesn't matter.

Your turn:



(1) Can $\angle 3$ be named $\angle B$? If not, why can't it be?

No because we don't know which angle it is if we just say $\angle B$.

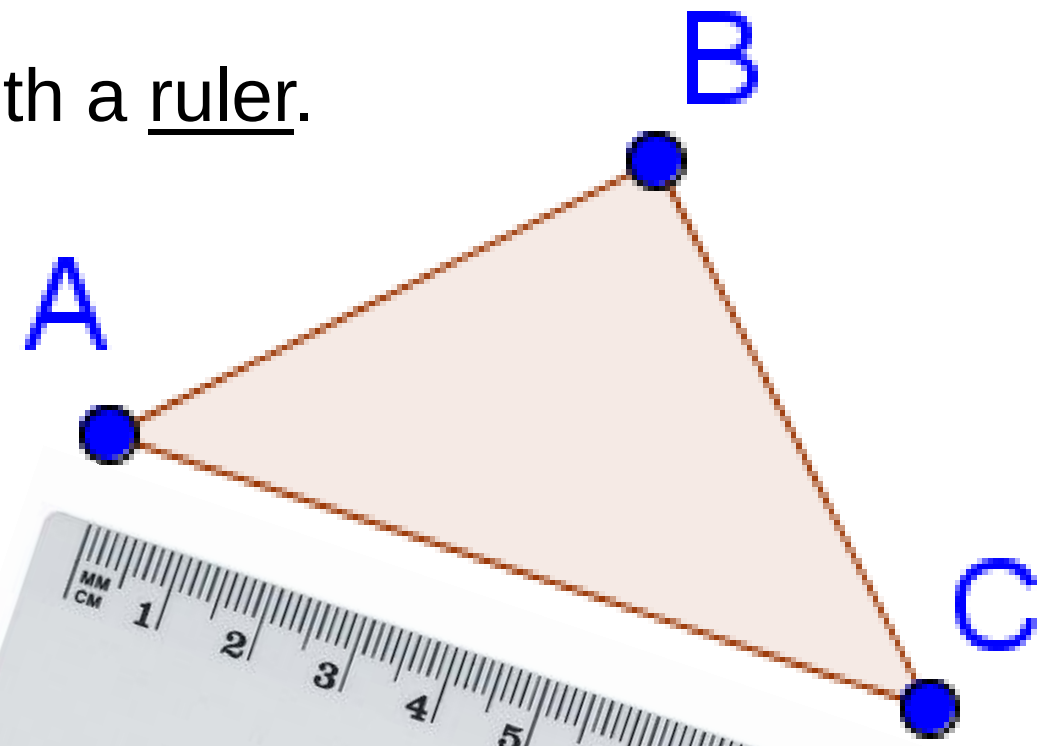
(2) Represent $\angle 3$ two other ways.

$\angle ABC$

$\angle CBA$

How are the lengths of segments measured?

With a ruler.



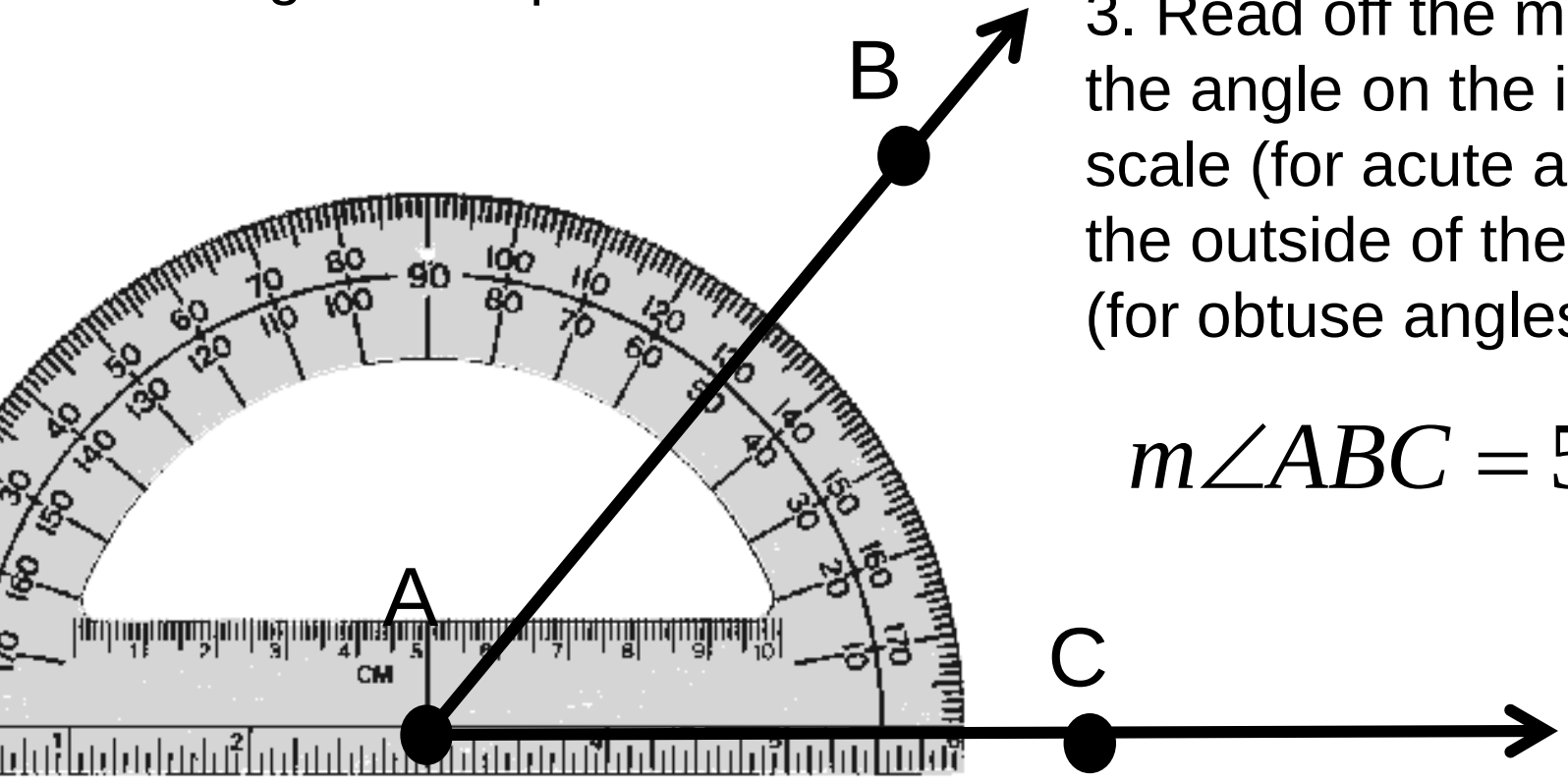
AC = ?

AC = 9 cm

How are angles measured? With a protractor

1. Put the hole of the protractor at the vertex of the angle.

2. Line up one side of the angle so that it goes through “0” on the edge of the protractor.



3. Read off the measure of the angle on the inside scale (for acute angles) or the outside of the scale (for obtuse angles).

$$m\angle ABC = 50^\circ$$