Math-2A

7-6: Review Exponents, Radicals, Quadratic Equations, and Polynomials

Properties of Exponents

What is a power?

Power: An expression formed by repeated multiplication of the base.

$$3x^4$$

Exponent

Coefficient

Base

The exponent applies to the number or variable immediately to its left, not to the coefficient !!!

Factor: a number that is being multiplied.

x^4 means "base x used as a factor 4 times"

Base

Exponent

$$x^4 = x * x * x * x$$

Power: is repeated multiplication $x^4 = x * x * x * x$

multiplication: is repeated addition $3x = x + x + x$

(<u>adding</u> two terms)

$$3x + 4x \quad = (x + x + x) + (x + x + x + x)$$

$$3x + 4x = 7x$$

$$2x^2 + 3x^2 \; = (x^2 + x^2) + (x^2 + x^2 + x^2)$$

$$2x^2 + 3x^2 = 5x^2$$

(<u>multiplying</u> two terms)

$$x^2 * x^3 = (x * x)(x * x * x)$$

$$x^2 * x^3 = x^5$$

Multiply Powers Property

$$(x^2)(x^3) = (x * x)(x * x * x)$$

This is 'x' used as a factor how many times?

$$(x^2)(x^3) = x^2 x^3 = x^{2+3} = x^5$$

'x' used as a factor <u>five</u> times

When you multiply powers having the same base, you <u>add the exponents</u>.

Exponent of a Power Property $(x^2)^3$

$$(x^2)^3 = (x * x)(x * x)(x * x)$$

This is ‘x’ used as a factor how many times?

$$(x^2)^3 = \quad = x^6$$

‘x’ used as a factor six times

$$(x^2)^3 = x^{2*3} = x^6$$

you multiply the exponents.

Exponent of a Product Of Powers Property

$$(xy)^2 = (xy)(xy) = x*y*x*y = x*x*y*y$$

$$= x^2 y^2$$

$$(xy)^m = x^m y^m$$

This makes it seem like you can "distribute" in the exponent. This only works with the power of a product!!

$$(x+3)^2 \neq x^2 + 3^2$$

$$(x+3)^2 = (x+3)(x+3)$$

$$= x^2 + 6x + 9$$

Exponent of a Product of Powers

$$(3x^3y^4)^2 = (3^1x^3y^4)^2 = 3^2x^6y^8$$

Very similar to exponent of a power.

$$3x^2(4x^3) = ? = 3*4*(x^2)(x^3) = 12x^5$$

You can re-arrange the order of multiplication.

simplify

$$(-2x^2y^4z)^3 \quad = -8x^6y^{12}z^3$$

$$2(-m^4x^3)^5 \quad -2m^{20}x^{15}$$

$$-3(-2x^2yz^3)^4 \quad -48x^8y^4z^{12}$$

Negative Exponent Property "Grab and drag"

$$x^{-2} = \frac{1 * x^{-2}}{1} = \frac{1}{x^{2}}$$

When you "Grab and drag" the base and its exponent across the "boundary line" between numerator and denominator, you just change the sign of the exponent.

$$x^{2} y^{-2} = \frac{x^{2}}{y^{2}}$$

$$\left(\frac{1}{x^{3}}\right)^{-2} = \frac{1}{x^{-6}} = \frac{1}{x^{-6}} = x^{6}$$

<u>Negative</u> Exponent Property

Possible errors

$$4x^{-2} = \frac{4 * x^{-2}}{1} = \frac{4}{x^2}$$

When you "Grab and drag" the <u>base and its exponent</u> across the "boundary line" between numerator and denominator, you just <u>change the sign</u> of the exponent.

DO NOT GRAB the coefficient!

$$\frac{4 * x^{-2}}{1} \neq \frac{1}{4x^2}$$

Zero Exponent Property

Any base raised to the zero power simplifies to one.

$$10^3 = 1000$$

$$10^2 = 100$$

$$10^1 = 10$$

$$10^0 = 1$$

$$2^0 = 1$$

$$(2x)^0 = 1$$

$$2x^0 = 2*1 = 2$$

Combination: (1) Negative Exponent, (2) Product of Powers, (3) Power of a Power, (4) Power of a Quotient

$$\left(\frac{3x^{2}}{2x^{-4}y}\right)^{2} = \left(\frac{3x^{2}x^{4}}{2y}\right)^{2} = \left(\frac{3x^{6}}{2y}\right)^{2} = \left(\frac{3^{1}x^{6}}{2^{1}y^{1}}\right)^{2}$$

$$= \frac{3^{1*2}x^{6*2}}{2^{1*2}y^{1*2}} = \frac{3^{2}x^{12}}{2^{2}y^{2}} = \frac{9x^{12}}{4y^{2}}$$

$$\left(\frac{2x^2}{3y^{-2}z^3}\right)^2 \qquad = \frac{4x^4y^4}{49}$$

$$\left(\frac{5x^4}{3y^{-2}}\right)^{-1} \qquad = \frac{3}{5x^4y^2}$$

Adding and subtracting radicals

Can these two terms be combined using addition? $3x + 2x$

Write 3x as repeated addition $x + x + x$

Write 2x as repeated addition $x + x$

$$3x + 2x \rightarrow x + x + x + x + x \rightarrow 5x$$

When multiplication is written as repeated addition, “like terms” look exactly alike.

$$3\sqrt{x} + 2\sqrt{x} \rightarrow \sqrt{x} + \sqrt{x} + \sqrt{x} + \sqrt{x} + \sqrt{x} \rightarrow 5\sqrt{x}$$

$$3\sqrt{6} + 2\sqrt{6} \rightarrow \sqrt{6} + \sqrt{6} + \sqrt{6} + \sqrt{6} + \sqrt{6} \rightarrow 5\sqrt{6}$$

$$\boxed{\sqrt{a} * \sqrt{b} = \sqrt{ab}}$$

Simplify the following:

$$3\sqrt{8} * 5\sqrt{2}$$

$$3 * \sqrt{8} * 5 * \sqrt{2}$$

$$3 * 5 * \sqrt{8} * \sqrt{2}$$

$$15 * \sqrt{8} * \sqrt{2}$$

$$15 * \sqrt{16}$$

$$15 * 4 = 60$$

$$2\sqrt{3} * 3\sqrt{5} \quad \rightarrow 6\sqrt{15}$$

$$7\sqrt{6} * 2\sqrt{5} \quad \rightarrow 14\sqrt{30}$$

$$\sqrt{5} + 3\sqrt{5} \quad \rightarrow 4\sqrt{5}$$

$$7\sqrt{6} + 2\sqrt{6} \quad \rightarrow 9\sqrt{6}$$

<u>Simplify radicals</u>: use the Product of Radicals Property to factor ("break apart") the radical into a "perfect square" times a number.

$$\sqrt{a} * \sqrt{b} = \sqrt{ab}$$

$$\sqrt{18} \rightarrow \sqrt{9} * \sqrt{2} \rightarrow 3 * \sqrt{2} \rightarrow 3\sqrt{2}$$

Simplify $\sqrt{24} \rightarrow \sqrt{4} * \sqrt{6} \rightarrow 2\sqrt{6}$

$$3\sqrt{32x^2} \rightarrow 3 * \sqrt{16} * \sqrt{x^2} + \sqrt{2} \rightarrow 3 * 4 * x * \sqrt{2} \rightarrow 12x\sqrt{2}$$

$$\sqrt[3]{x^4} \rightarrow \sqrt[3]{x^3} * \sqrt[3]{x} \rightarrow x\sqrt[3]{x}$$

$$\sqrt[4]{3x^5y} \rightarrow \sqrt[4]{x^4} * \sqrt[4]{3xy} \rightarrow x\sqrt[4]{3xy}$$

Can we add "unlike" radicals?

$$\sqrt{a} * \sqrt{b} = \sqrt{ab}$$

Simplify $7\sqrt{6} + 2\sqrt{24} \rightarrow 7\sqrt{6} + (2 * \sqrt{4} * \sqrt{6})$

$$\rightarrow 7\sqrt{6} + (2 * 2 * \sqrt{6})$$

$$\rightarrow 7\sqrt{6} + 4\sqrt{6}$$

$$\rightarrow 11\sqrt{6}$$

$$-3\sqrt{32} + 2\sqrt{8} \rightarrow (-3 * \sqrt{16} * \sqrt{2}) + (2 * \sqrt{4} * \sqrt{2})$$

$$\rightarrow (-3 * 4 * \sqrt{2}) + (2 * 2 * \sqrt{2})$$

$$\rightarrow -12\sqrt{2} + 4\sqrt{2}$$

$$\rightarrow -8\sqrt{2}$$

Another way to Simplify Radicals Factor, factor, factor!!!

$$\sqrt{54} \rightarrow \sqrt[2]{54} \rightarrow \sqrt[2]{2*27} \rightarrow \sqrt[2]{2*3*9} \rightarrow \sqrt[2]{2*3*3*3}$$

What is the factor that is used (Index number) ‘2’ times under the radical?

Bring the out factor that is used 2 times.

$$\rightarrow 3\sqrt[2]{2*3} \quad \rightarrow 3\sqrt{6}$$

Using Properties of Exponents to reduce the writing:

$$\sqrt[4]{32x^6} \rightarrow \sqrt[4]{32*x^4*x^2}$$

$$\rightarrow x\sqrt[4]{32*x^2}$$

$$\rightarrow x\sqrt[4]{2^4*2^1*x^2}$$

$$\rightarrow 2x\sqrt[4]{2x^2}$$

Rationalizing the denominator: using mathematical properties to change an irrational number (or imaginary number) in the denominator into a rational number.

We take advantage of the idea:

$$\sqrt{2}*\sqrt{2} = \sqrt{2*2} = \sqrt{4} = 2$$

$$\sqrt{3}*\sqrt{3} = \sqrt{3*3} = \sqrt{9} = 3$$

$$\frac{1}{\sqrt{2}}*\frac{\sqrt{2}}{\sqrt{2}} \rightarrow \frac{\sqrt{2}}{2}$$

Identity Property of Multiplication

multiplying by '1' doesn't change the number.

$$\frac{2}{\sqrt{6}} * \frac{\sqrt{6}}{\sqrt{6}} \rightarrow \frac{2\sqrt{6}}{6} \rightarrow \frac{\cancel{2} * \sqrt{6}}{\cancel{2} * 3} \rightarrow \frac{\sqrt{6}}{3}$$

$$\frac{25}{\sqrt{15}} * \frac{\sqrt{15}}{\sqrt{15}} \rightarrow \frac{25\sqrt{15}}{15} \rightarrow \frac{\cancel{5} * 5 ** \sqrt{15}}{\cancel{5} * 3} \rightarrow \frac{5\sqrt{15}}{3}$$

$$\frac{14}{3\sqrt{21}} * \frac{\sqrt{21}}{\sqrt{21}} \rightarrow \frac{14\sqrt{21}}{3 * 21} \rightarrow \frac{2 * \cancel{7} ** \sqrt{21}}{3 * \cancel{7} * 3} \rightarrow \frac{2\sqrt{21}}{9}$$

Simplifying Polynomials

$$3x^5 - 2x^4 + x^5 - 5x^4 + 1$$

$$\boxed{3x^5 + x^5} - 2x^4 - 5x^4 + 1 \quad \rightarrow \; 4x^5 - 7x^4 + 1$$

What property allowed us to “rearrange the order” of the terms?

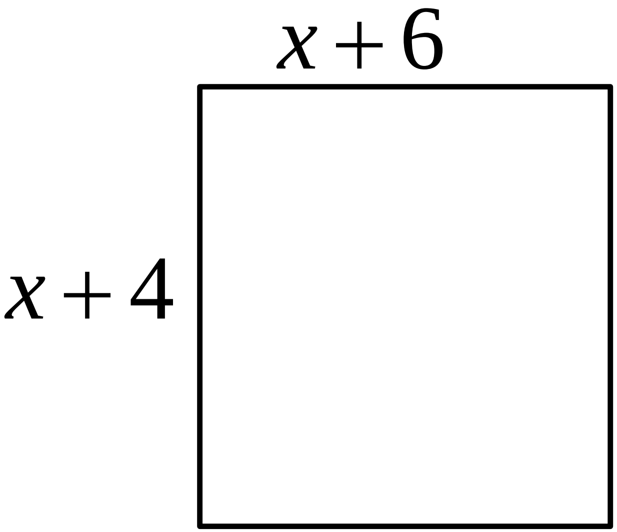
Commutative Property (of addition)

Combine “like” terms.

(combine “like” terms is English for addition).

The “Box Method” of multiplying Polynomials

We start with the combined side lengths



We break the large rectangle into four smaller rectangles.

We break the combined side lengths to label the smaller rectangles

	x	6
x	x^2	6x
4	4x	24

Find the area of each small rectangle.

Combine the areas to find the total area.

$$x^2+6x+4x+24 \Rightarrow x^2+10x+24$$

$$(x+6)(x+4)=x^2+10x+24$$

Use the “Box Method” to multiply Polynomial $(x+1)(x+7)$

	x	1
x	x^2	x
7	7x	7

$$x^2+x+7x+7$$

$$\Rightarrow x^2+8x+7$$

$$(x+1)(x+7)=x^2+8x+7$$

Use the “Box Method” to multiply Polynomial $(x-3)(x-6)$

	x	-3
x	x^2	-3x
-6	-6x	18

$$x^2-3x-6x+18$$

$$\Rightarrow x^2-9x+18$$

$$(x-3)(x-6)=x^2-9x+18$$