## Math-2A <br> Lesson 7-4 <br> Analyzing the Graphs of Functions

## Which of the functions are symmetric across the $y$-axis?




$$
f(x)=x^{3}
$$



$$
f(x)=\sqrt[3]{x}
$$

Even Function: a function that is symmetric across the $y$-axis.

For which function does a reflection across the y-axis look exactly like a reflection across the x -axis?
 $f(x)=|x|$
$f(x)=x^{2}$
$f(x)=\sqrt{x}$




Odd Function: a function whose reflection across the $y$-axis looks exactly like its reflection across the x-axis.

## Average Rate of Change

 change" between $\mathrm{x}=2$ and 4 ?

Means "what is the slope of the graph between the two points $\left(2, y_{1}\right)$ and $\left(4, \mathrm{y}_{2}\right)$ ?


The Function is Increasing
$\rightarrow$ if you draw a tangent line at a point on the graph and it has a positive slope, the function is increasing at that point.
The slope of a tangent line at any point on the graph for the interval $x=(2, \infty)$ is positive.
What about when $x=2$ ?
The slope of a tangent line at $x=2$ is zero (not increasing at that point).
We say: "the function is increasing on the $(x)$ interval: $(2, \infty)$

$$
f(x) \uparrow \text { on } x=(2, \infty)
$$



## The function is decreasing

$\rightarrow$ if you draw a tangent line at a point on the graph, and it has a negative slope, the function is decreasing at that point.

The slope of a tangent line at any point on the graph for the interval $x=(-\infty, 2)$ is negative.

What about when $\mathrm{x}=2$ ?
The slope of a tangent line at $x=2$ is zero (not decreasing at that point).

We say the function is decreasing on the interval $x=(-\infty, 2)$

$$
f(x) \downarrow \text { on } x=(-\infty, 2)
$$



Where is the function positive?
means: "What x-values have corresponding $y$-values that are positive"?

Or, "The graph is above the x -axis for what $x$-values"?

To see what these $x$-values are, shade the portion of the $x$-axis where the graph is above the $x$-axis.

We say: $\quad f(x)>0$ for $x=(-\infty, 1) \cup(3, \infty)$


## Is the Function "even"?

$\rightarrow$ means, "is the graph symmetrical about the $y$-axis"? NO.

## Is the Function "odd"?

$\rightarrow$ means, "if the graph is reflected across the y-axis, would it look exactly the as if it were reflected across the $x$-axis"?

Extrema: a point on a graph whose tangent line has a zero slope.


We classify extrema by their $y$-values.

Absolute minimum (maximum): an extrema whose $y$-value is the smallest (largest) y-value for the entire function.
relative maximum (minimum): an extrema whose $y$-value is the greater than (less than) the $y$-value of points near it.


In""inequality notation" we say the range is: $y \geq-1$
In "interval notation" we say the range is: $y=[-1, \infty)$
$y=|x+3|-2 \quad 1$. Where is the function increasing?

$$
f(x) \uparrow \text { on } x=(-3, \infty)
$$

2. Where is the function decreasing?

$$
f(x) \downarrow \text { on } x=(-\infty,-3)
$$

3. Where is the function positive?

$$
f(x)>0 \text { for } x=(-\infty,-3] \cup[6, \infty)
$$

4. Is the function even, odd or neither?

Neither (not symmetrical about the $y$-axis or the origin)
5. What are the "extrema"? Absolute Minimum at ( $-3,-4$ )
6. What is the "domain" of the function? All real numbers
7. What is the "range" of the function? $\quad y=[-2, \infty)$

How would you describe the function?

a) Increasing
b) Decreasing
c) Constant
d) Even
e) Odd
f) Positive
g) Negative

How would you describe the function?

a) Increasing
b) Decreasing
c) Constant
d) Even
(e) Odd
f) Positive
g) Negative

How would you describe the function on $x=[-5,-2)$ ?

(a) Increasing
b) Decreasing
c) Constant
d) Even
e) Odd
f) Positive
(g) Negative

How would you describe the function?

a) Increasing
b) Decreasing
c) Constant
d) Even
(e) Odd
f) Positive
g) Negative

How would you describe the function on $x=[-1,1]$ ?

a) Increasing
b) Decreasing
c) Constant
d) Even
(e) Odd
f) Positive
g) Negative

