Math-2A
Lesson 7-3

Solving Systems of Equations by Elimination

Solve an Equation: Find the value of ‘x’ that makes the equation “true.”

$$x - 2 = 8$$

$$+2 \quad +2$$

What Property allows adding the same number to the left and right sides of an equation?

The property of equality!!

$$x - 2 = 8$$

$$2 = 2$$

The ***Addition Property of Equality*** means we are adding two equations to each other.

Elimination Method Add or subtract multiples of one equation to the other equation to eliminate one of the variables.

What Property allows adding equations?

$$3x - 2 = -2x + 8$$
$$+2x \qquad\qquad +2x$$

$$3x - 2 = -2x + 8$$
$$2x \quad = \quad 2x$$

The property of equality!!

Adding two equations means "adding equivalent values to the left and right sides of an equation".

Slide #3: Easiest Problem

$$x - 3y = 5$$
$$-x + 5y = 3$$

$$2y = 8$$

$$y = 4$$

$$x - 3(4) = 5$$

$$x = 17$$

Solution: (17, 4)

Even though the equation that we add does not look the same left and right of the '=' sign,

the "=" sign guarantees left side is equivalent to the right side of the equation.

Replace 'y' with 4 in either of the original equations, then solve for 'x'.

Slide #4: Requires some work

$2x - 2y = 6$
$-x + 6y = 7$

If we just add these two equations, no variables will be eliminated.

Add or subtract *multiples of one equation* to/from the other to eliminate one of the variables

$$2x - 2y = 6$$
$$(2)\,[-x + 6y] = [7]\,(2)$$

$$2x - 2y = 6$$
$$-2x + 12y = 14$$

$$10y = 20$$
$$\boxed{y = 2}$$

$x = ?$

Substitute y = 2 into either of the original equations, then solve for 'x'.

$$2x - 2(2) = 6$$
$$2x - 4 = 6$$
$$2x = 10$$
$$\boxed{x = 5}$$

Solution: (5, 2)

Slide #5: The Hardest problem

(8) [9x – 5y] = [18] (8)

(5) [-10x + 8y] = [2] (5)

$$72x - 40y = 144$$

$$-50x + 40y = 10$$

$$22x = 154$$

$$\boxed{x = 7}$$

Requires the most work: you must multiply both equations equation different numbers to obtain same coefficient but opposite sign on one of the variables. (In this case let's eliminate the y-variable first.

Substitute x = 7 into either of the original equations, then solve for 'y'.

$$9(7) - 5y = 18$$

$$63 - 5y = 18$$

$$-5y = -45$$

$$\boxed{y = 9}$$

Solution: (7, 9)

In summary,

there are 3 Levels of Difficulty for Elimination Problems

$$x - 3y = 5$$
$$-x + 5y = 3$$

Easy: (1st example on slide #3)) → same coefficient but opposite sign on one of the variables. If you just add the equations, one of the variables is eliminated.

$$2x - 2y = 6$$
$$(2)\left[-x + 6y\right] = \left[7\right](2)$$

Requires some work: (2^{nd} example on slide #4) you must multiply one equation by a number to obtain same coefficient but opposite sign on one of the variables.

$$(8)\left[9x - 5y\right] = \left[18\right](8)$$
$$(5)\left[-10x + 8y\right] = \left[2\right](5)$$

Requires the most work: (3^{rd} example on slide #5) you must multiply both equations equation different numbers to obtain same coefficient but opposite sign on one of the variables.

Solve using elimination.

$$\begin{aligned} 2x - y &= 2 \\ 4x + 2y &= 8 \end{aligned}$$

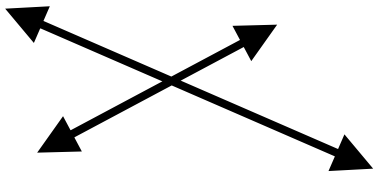
$$\begin{aligned} -6x - 3y &= 12 \\ 12x + 4y &= -8 \end{aligned}$$

$$\begin{aligned} -6x - 10y &= 2 \\ -12x - 20y &= 4 \end{aligned}$$

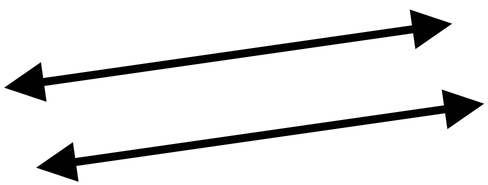
$$\begin{aligned} -3x + 8y &= -6 \\ 2x + 6y &= 4 \end{aligned}$$

Categories of Solutions:

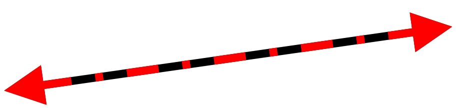
Ways 2 lines can be graphed:



Cross → one solution



Parallel →no solutions



Same line → infinitely many solutions

How do you know how many solutions there are using the elimination method (1, 0, or infinitely many) ?

When you perform the elimination step and both variables disappears and you get a number equal to another number:

a. and it's true:
(3 = 3 or 0 = 0) → Infinitely many solutions (same line)

b. and it's false:
(-2 = 3 or 10 = 0) → No solution (parallel lines)