Math-2A Lesson 3-1 (Radicals)

$\sqrt{3}$ What number is equivalent to the square root of 3?

$x=\sqrt{3}$ Square both sides of the equation

$(x)^2=\left(\sqrt{3}\right)^2$ $x^2=3$

$x=\sqrt{3}$ is an equivalent statement to $x^2=3$

$\sqrt{3}$ ≈ 1.732

≈ 1.7321

≈ 1.73205

≈ 1.732051

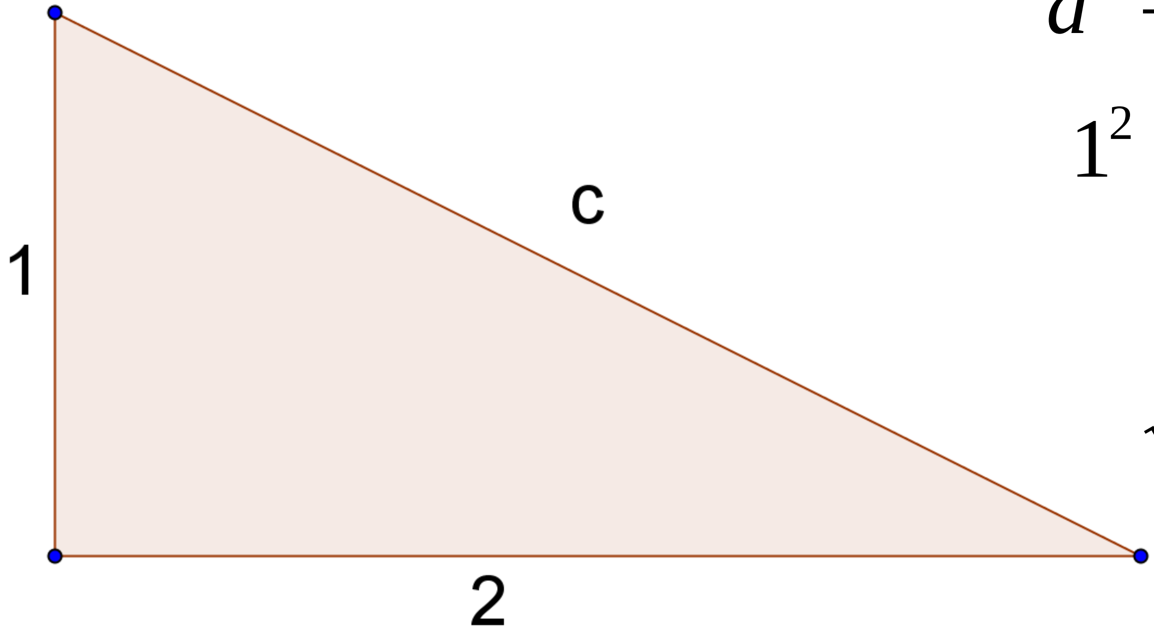
$\approx 1.7320508...$

There is no equivalent number

The decimal, is just an approximation.

The Irrational number system was created because of the side lengths of right triangles.

Pythagorean Theorem: If it's a right triangle, then side lengths can be related by:



$$a^2 + b^2 = c^2$$

$$1^2 + 2^2 = c^2$$

$$5 = c^2$$

$$\sqrt{5} = c$$

Vocabulary

Irrational numbers: cannot be written as a ratio of integers: ½, -⅔, etc.

The decimal version of an irrational number never terminates and never repeats. (5.13257306…).

If we see the radical symbol, the number is usually irrational (unless it is a “perfect square).

$$\sqrt{4} = 2\,(\text{rational }\#) \qquad \sqrt{3}$$

Radicals

Index number

Radical symbol

Radicand

$$\sqrt{3} \rightarrow \sqrt[2]{3}$$

$$x = \sqrt[2]{3}$$

$$x^2 = 3$$

The "square root of 3" means:
"what number squared equals 3?"

$$x = \sqrt[3]{4}$$

$$x^3 = 4$$

The "3rd root of 4" means:
"what number cubed equals 4?"

$$x = \sqrt[5]{2}$$

$$x^5 = 2$$

The "5th root of 2" means:
"what number used as a factor 5 times equals 2?"

Radicals

Index number

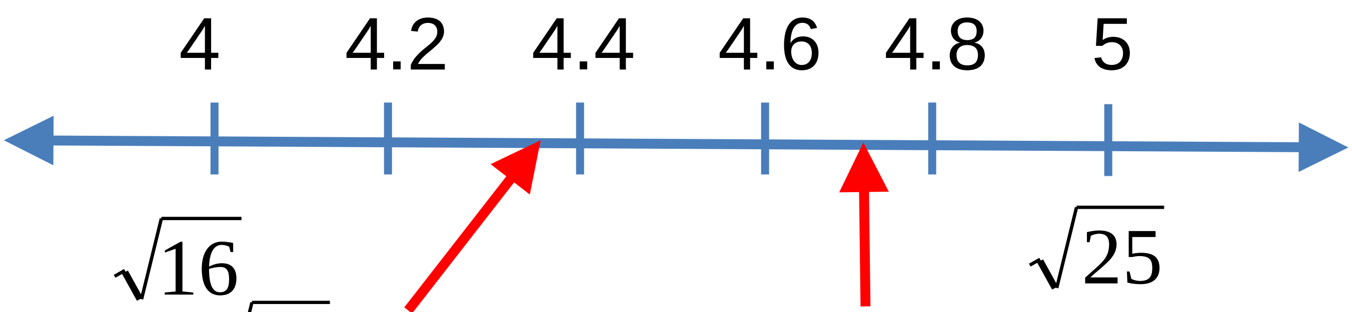
Radical symbol

$$\rightarrow \sqrt[2]{3}$$

Radicand

Fill in the blanks. Try to recognize the perfect squares.

$x = \sqrt[2]{16}$	$y = \sqrt[2]{19}$	$z = \sqrt[2]{22}$	$k = \sqrt[2]{25}$
$x^2 = 16$	$y^2 = 19$	$z^2 = 22$	$k^2 = 25$
$x =$ ____	$y =$ ____	$z =$ ____	$k =$ ____



$\sqrt{19} \approx$ _____... $\sqrt{22} \approx$ _____...

Adding and subtracting radicals

Can these two terms be combined using addition? $3x + 2x$

Write 3x as repeated addition $x + x + x$

Write 2x as repeated addition $x + x$

$$3x + 2x \rightarrow x + x + x + x + x \rightarrow 5x$$

When multiplication is written as repeated addition, "like terms" look exactly alike.

$$3\sqrt{x} + 2\sqrt{x} \rightarrow \sqrt{x} + \sqrt{x} + \sqrt{x} + \sqrt{x} + \sqrt{x} \rightarrow 5\sqrt{x}$$

$$3\sqrt{6} + 2\sqrt{6} \rightarrow \sqrt{6} + \sqrt{6} + \sqrt{6} + \sqrt{6} + \sqrt{6} \rightarrow 5\sqrt{6}$$

Define "like powers" "Same base, same exponent".

$$3x^4 + 2x^4 \to 5x^4$$

Define "like radicals" "Same radicand, same index number".

$$3\sqrt{6} + 2\sqrt{6} \to 5\sqrt{6}$$

Which of the following are "like radicals" that can be added?

$$\sqrt{2} + \sqrt{3}$$

$$\sqrt[4]{5} + \sqrt[4]{5}$$

$$2\sqrt{3} + 3\sqrt{2}$$

$$3\sqrt[5]{2} + 4\sqrt[5]{2}$$

$$\sqrt[4]{2} + \sqrt[3]{2}$$

$$6\sqrt[3]{4} + 6\sqrt[4]{4}$$

$$\sqrt{3}+\sqrt{2} \rightarrow \sqrt{3+2}=\sqrt{5}$$

Are they equivalent?

$$\sqrt{3} \approx 1.7321... \qquad \sqrt{2} \approx 1.4142...$$

$$\sqrt{3}+\sqrt{2} \approx 3.1462... \qquad \sqrt{5} \approx 2.2630...$$

$$\sqrt{3}+\sqrt{2} \neq \sqrt{5}$$

$$\sqrt{a}+\sqrt{b} \neq \sqrt{a+b}$$

This is NOT a property of radicals.
NEVER DO THIS!!!!

$$\sqrt{4}+\sqrt{9} \rightarrow \sqrt{13}$$

$$\sqrt{4}+\sqrt{9} \rightarrow 2+3 \rightarrow 5 \neq \sqrt{13}$$

Simplify the following:

$$3\sqrt{2}+5\sqrt{2} \rightarrow 8\sqrt{2}$$

$$5\sqrt{3}-4\sqrt{3} \rightarrow \sqrt{3}$$

$$\sqrt{5}+3\sqrt{5} \rightarrow 4\sqrt{5}$$

$$7\sqrt{6x}+2\sqrt{6x} \rightarrow 9\sqrt{6x}$$

$$3\sqrt{x}+2\sqrt{x} \rightarrow 5\sqrt{x}$$

$$5\sqrt{2x}-\sqrt{5x}+3\sqrt{5x} \rightarrow 5\sqrt{2x}+2\sqrt{5x}$$

$7\sqrt{6}+2\sqrt{24}$ not “like terms” in their present form

$\sqrt{3} * \sqrt{2}$

$\sqrt{3*2} \rightarrow \sqrt{6}$

Will this work?

$\sqrt{3} \approx 1.7321...$ $\sqrt{2} \approx 1.4142...$

$\sqrt{3} * \sqrt{2} \approx 2.4495$

$\sqrt{6} \approx 2.4495...$

Product of Radicals Property

$\sqrt{a} * \sqrt{b} \rightarrow \sqrt{a*b}$

$$\sqrt{5} * \sqrt{2} = \sqrt{10}$$

$\sqrt{4} * \sqrt{9} \rightarrow \sqrt{4*9}$

$2*3 \rightarrow \sqrt{36}$

$2*3 \rightarrow 6$

$6 = 6$

Are these equivalent?

$\sqrt{a} * \sqrt{b} = \sqrt{ab}$

Although I only gave two examples, it actually DOES WORK for zero and all positive numbers.

$$\sqrt{a} * \sqrt{b} = \sqrt{ab}$$

Simplify the following:

$$3\sqrt{8} * 5\sqrt{2}$$

$$3 * \sqrt{8} * 5 * \sqrt{2}$$

$$3 * 5 * \sqrt{8} * \sqrt{2}$$

$$15 * \sqrt{8} * \sqrt{2}$$

$$15 * \sqrt{16}$$

$$15 * 4 = 60$$

$$2\sqrt{3} * 3\sqrt{5} \quad \rightarrow 6\sqrt{15}$$

$$7\sqrt{6} * 2\sqrt{5} \quad \rightarrow 14\sqrt{30}$$

$$\sqrt{5} + 3\sqrt{5} \quad \rightarrow 4\sqrt{5}$$

$$7\sqrt{6} + 2\sqrt{6} \quad \rightarrow 9\sqrt{6}$$

Simplify radicals: use the Product of Radicals to "factor" the radical into a "perfect square" times a number.

$$\sqrt{a} * \sqrt{b} = \sqrt{ab}$$

$$\sqrt{18} \quad \rightarrow \sqrt{9} * \sqrt{2} \quad \rightarrow 3 * \sqrt{2} \quad \rightarrow 3\sqrt{2}$$

Simplify

$$\sqrt{24} \ \rightarrow \sqrt{4} * \sqrt{6} \ \rightarrow 2\sqrt{6}$$

$$3\sqrt{32x^2} \quad \rightarrow 3 * \sqrt{16} * \sqrt{x^2} + \sqrt{2} \quad \rightarrow 3 * 4 * x * \sqrt{2} \quad \rightarrow 12x\sqrt{2}$$

$$-2\sqrt{56x^3y} \quad \rightarrow -2 * \sqrt{x^2} * \sqrt{8 * 7xy}$$

$$\rightarrow -2 * x * \sqrt{4} * \sqrt{2 * 7xy}$$

$$\rightarrow -4x\sqrt{14xy}$$

Perfect Square: the result of multiplying a number times itself.

$3*3=9$ "9" is a perfect square because it is equivalent to 3*3.

$3^2=9$ Find the first 10 perfect squares.

$1^2 =$ 1

$2^2 =$ 4

$3^2 =$ 9

$4^2 =$ 16

$5^2 =$ 25

$6^2 =$ 36

$7^2 =$ 49

$8^2 =$ 64

$9^2 =$ 81

$10^2 =$ 100

Simplify radicals: use the Product of Radicals to "break apart" the radical into a "perfect square" times a number.

$$\sqrt{a} * \sqrt{b} = \sqrt{ab}$$

Simplify

$$7\sqrt{6} + 2\sqrt{24} \rightarrow 7\sqrt{6} + (2 * \sqrt{4} * \sqrt{6})$$

$$\rightarrow 7\sqrt{6} + (2 * 2 * \sqrt{6})$$

$$\rightarrow 7\sqrt{6} + 4\sqrt{6}$$

$$\rightarrow 11\sqrt{6}$$

$$-3\sqrt{32} + 2\sqrt{8} \rightarrow (-3 * \sqrt{16} * \sqrt{2}) + (2 * \sqrt{4} * \sqrt{2})$$

$$\rightarrow (-3 * 4 * \sqrt{2}) + (2 * 2 * \sqrt{2})$$

$$\rightarrow -12\sqrt{2} + 4\sqrt{2}$$

$$\rightarrow -8\sqrt{2}$$

<u>Simplify radicals</u>: use the Product of Radicals to “factor” the radical into a “powers of exponent ‘m’ ” times a number.

$$\sqrt[m]{a} * \sqrt[m]{b} = \sqrt[m]{ab} \qquad \sqrt[3]{x^4} \rightarrow \sqrt[3]{x^3} * \sqrt[3]{x} \rightarrow x\sqrt[3]{x}$$

<u>Simplify</u>

$$\sqrt[4]{3x^5y} \rightarrow \sqrt[4]{x^4} * \sqrt[4]{3x} \rightarrow x\sqrt[4]{3x}$$

$$3\sqrt[3]{16x^2y^5} \rightarrow 3 * \sqrt[3]{8} * \sqrt[3]{y^3} * \sqrt[3]{2x^2y^2}$$

$$\rightarrow 3 * \sqrt[3]{2^3} * \sqrt[3]{y^3} * \sqrt[3]{2x^2y^2}$$

$$\rightarrow 3 * 2 * y * \sqrt[3]{2x^2y^2}$$

$$\rightarrow 6y\sqrt[3]{2x^2y^2}$$

Another way to Simplify Radicals

<u>Factor, factor, factor!!!</u>

$$\sqrt{54} \rightarrow \sqrt[2]{54} \rightarrow \sqrt[2]{2*27} \rightarrow \sqrt[2]{2*3*9} \rightarrow \sqrt[2]{2*3*3*3}$$

What is the factor that is used '2' times under the radical?

"Factor out" the factor (that is used 2 times).

$$\rightarrow 3\sqrt[2]{2*3} \quad \rightarrow 3\sqrt{6}$$

Using Properties of Exponents to reduce the writing:

$$\sqrt[4]{32x^6} \rightarrow \sqrt[4]{32*x^4*x^2} \rightarrow x\sqrt[4]{32*x^2}$$

$$\rightarrow x\sqrt[4]{2^4*2^1*x^2} \quad \boxed{\rightarrow 2x\sqrt[4]{2x^2}}$$