

Math-2A

Lesson 2-8

Add, subtract, multiply Polynomials

Polynomial: an expression or an equation formed by the sum of “same-base” powers.

$$y = 7x^3 - 3x^2 + 6x + 1$$

Terms of a polynomial: the individual powers that are being added.

Example Polynomial equation: $0 = 3x^4 - 2x^3 + x^2 - 5x + 1$

Example Polynomial expression: $7x^3 - 3x^2 + 6x + 1$

Degree of a Polynomial: the largest exponent of the polynomial.

Standard form Polynomial: is written so the powers become smaller from left to right. $4x^5 - 2x^3 + 3x - 5$

Simplifying Polynomials

$$3x^5 - 2x^4 + x^5 - 5x^4 + 1$$

$$3x^5 + x^5 - 2x^4 - 5x^4 + 1 \rightarrow 4x^5 - 7x^4 + 1$$

What property allowed us to “rearrange the order” of the terms?

Commutative Property (of addition)

Combine “like” terms.

(combine “like” terms is English for addition).

simplify

$$3x^5 - 2x^3 - 4x - 4x^5 - 6x \\ - x^5 - 2x^3 - 10x$$

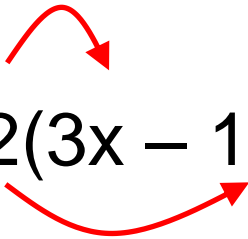
$$(2x^5 - 4x^3 - x + 4) - (-2x^5 - 5x)$$

$$4x^5 - 4x^3 + 4x + 4$$

If you add/subtract polynomials you get a sum that is also a polynomial.

Polynomials are “closed” for addition/subtraction!!!

Multiplying Polynomials

$$2(3x - 1)$$


Distributive property

$$6x - 2$$

simplify

$$3x^4(4x^2 - 2x) \Rightarrow 12x^6 - 6x^5$$


$$2x(5x^3 - 7x^2) \Rightarrow 10x^4 - 14x^3$$

If you multiply polynomials you get a product that is also a polynomial.

Polynomials are “closed” for multiplication!!!

$$(x + 3)(2x - 1) \quad \text{Distributive property (twice)}$$

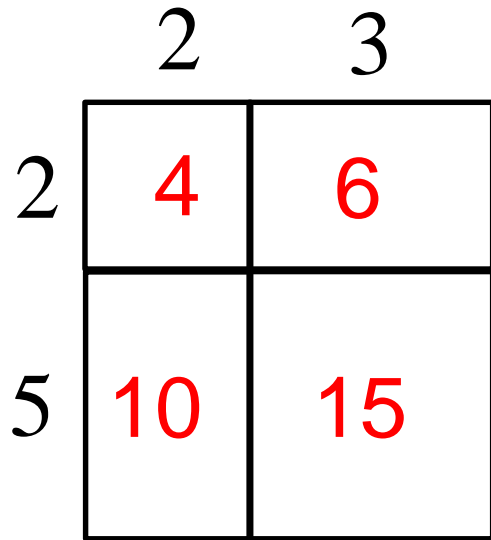
$$x(2x - 1) + 3(2x - 1)$$

$$2x^2 - x + 6x - 3 \quad \text{Combine "like" terms}$$

$$2x^2 + 5x - 3$$

The "Box Method" of multiplying Polynomials

Widths and lengths of the rectangles are shown.



Find the area of each rectangle

Find the total area of the 4 rectangles.

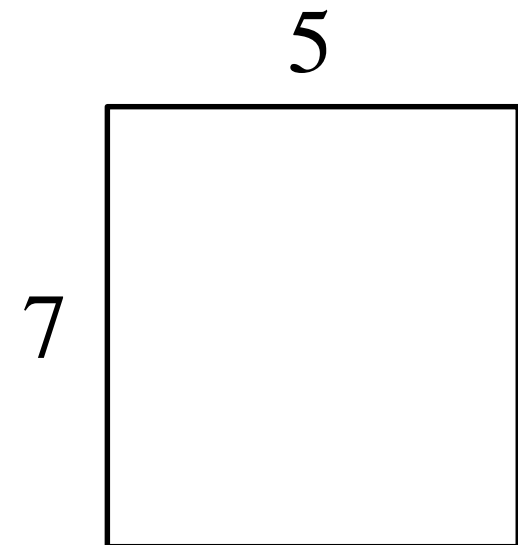
$$4 + 6 + 10 + 15 = 35$$

Combine the side lengths of the smaller rectangles to find the side lengths of the outer rectangle

Find the area of the outer rectangle.

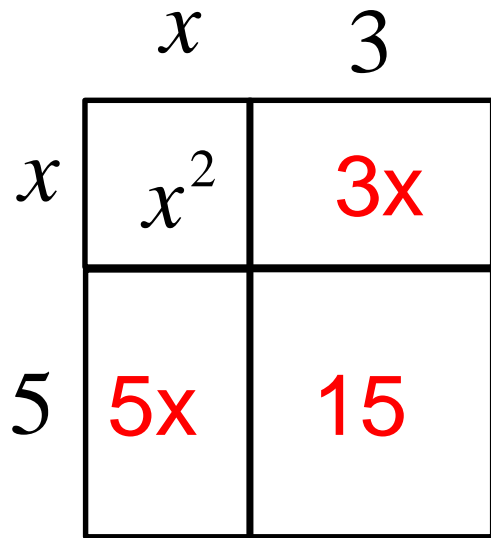
$$5 * 7 = 35$$

Explain the result to your neighbor.



The "Box Method" of multiplying Polynomials

Widths and lengths of the rectangles are shown.

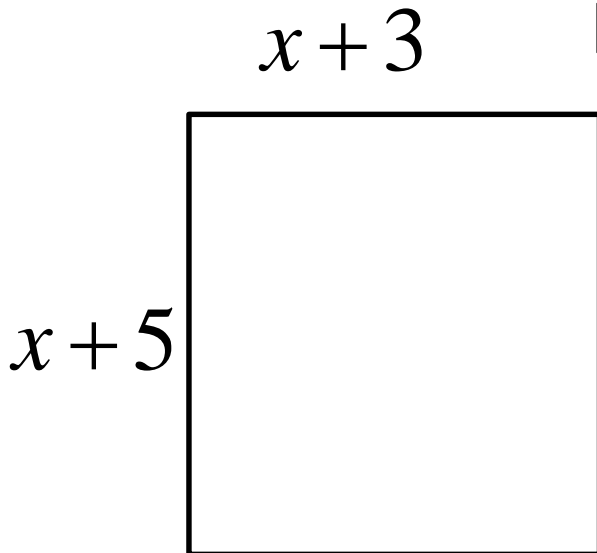


Find the area of each rectangle

Find the total area of the 4 rectangles.

$$x^2 + 3x + 5x + 15 \Rightarrow x^2 + 8x + 15$$

Combine the side lengths of the smaller rectangles to find the side lengths of the outer rectangle

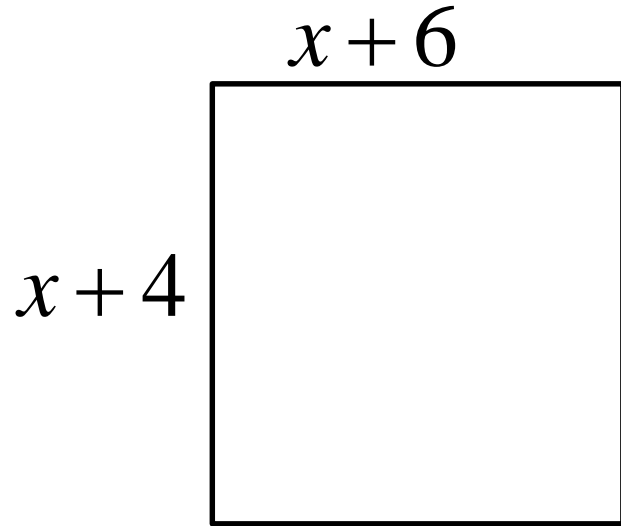


Find the area of the outer rectangle.

$$(x + 3)(x + 5) = x^2 + 8x + 15$$

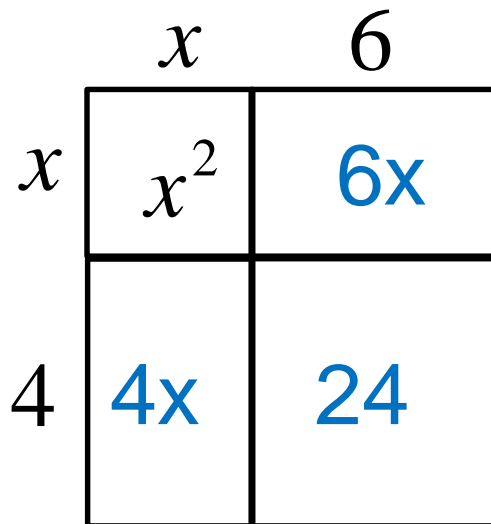
The "Box Method" of multiplying Polynomials

We start with the combined side lengths



We break the large rectangle into four smaller rectangles.

We break the combined side lengths to label the smaller rectangles



Find the area of each small rectangle.

Combine the areas to find the total area.

$$x^2 + 6x + 4x + 24 \Rightarrow x^2 + 10x + 24$$

$$(x + 6)(x + 4) = x^2 + 10x + 24$$

Use the "Box Method" to multiply Polynomial $(x+1)(x+7)$

| | | |
|-----|-------|-----|
| | x | 1 |
| x | x^2 | x |
| 7 | $7x$ | 7 |

$$x^2 + x + 7x + 7$$

$$\Rightarrow x^2 + 8x + 7$$

$$(x+1)(x+7) = x^2 + 8x + 7$$

Use the "Box Method" to multiply Polynomial $(x-3)(x-6)$

| | | |
|------|-------|-------|
| | x | -3 |
| x | x^2 | $-3x$ |
| -6 | $-6x$ | 18 |

$$x^2 - 3x - 6x + 18$$

$$\Rightarrow x^2 - 9x + 18$$

$$(x-3)(x-6) = x^2 - 9x + 18$$

simplify

$$(x-4)(x+3) \Rightarrow x^2 - x - 12$$

$$(2x+5)(x-4) \Rightarrow 2x^2 - 3x - 20$$

$$(x-3)(x+3) \Rightarrow x^2 - 9$$

$$(2x+5)(2x-5) \Rightarrow 4x^2 - 25$$

“Box Method”

“break apart” into individual terms (small rectangle lengths and widths)

$$(3x - 2)(2x^2 + x - 3)$$

| | | | |
|--------|--------|-----|--------|
| | $2x^2$ | x | (-3) |
| $3x$ | | | |
| (-2) | | | |

“multiply rows and columns”

$$(3x - 2)(2x^2 + x - 3)$$

| | | | |
|--------|--------|--------|--------|
| | $2x^2$ | x | (-3) |
| $3x$ | $6x^3$ | $3x^2$ | $-9x$ |
| (-2) | | | |

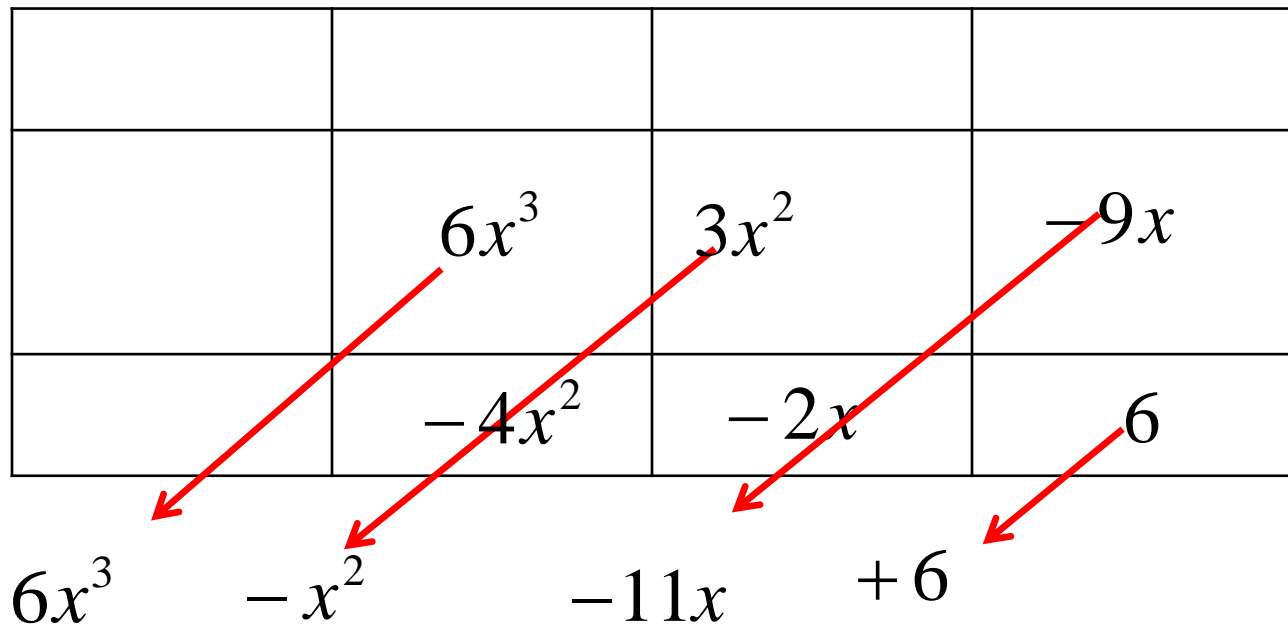
“multiply rows and columns”

$$(3x - 2)(2x^2 + x - 3) = ?$$

| | | | |
|--------|---------|--------|--------|
| | $2x^2$ | x | (-3) |
| $3x$ | $6x^3$ | $3x^2$ | $-9x$ |
| (-2) | $-4x^2$ | $-2x$ | 6 |

add (combine “like terms”)

$$(3x - 2)(2x^2 + x - 3)$$



Diagonals have “like terms”

Simplify

(Hint: use the box method) $(3x^2 + 2x - 1)^2$

$$(3x^2 + 2x - 1)(3x^2 + 2x - 1)$$

| | | | |
|--------|---------|--------|---------|
| | $3x^2$ | $2x$ | (-1) |
| $3x^2$ | $9x^4$ | $6x^3$ | $-3x^2$ |
| $2x$ | $6x^3$ | $4x^2$ | $-2x$ |
| (-1) | $-3x^2$ | $-2x$ | 1 |

$$(3x^2 + 2x - 1)(3x^2 + 2x - 1)$$

| | | | |
|--|---------|--------|---------|
| | | | |
| | $9x^4$ | $6x^3$ | $-3x^2$ |
| | $6x^3$ | $4x^2$ | $-2x$ |
| | $-3x^2$ | $-2x$ | 1 |

$$9x^4 + 12x^3 - 2x^2 - 4x + 1$$

add (combine "like terms")

Simplify

$$(x-2)(x+2) \Rightarrow x^2 - 4$$

$$(2x-4)(2x+4) \Rightarrow 4x^2 - 16$$

$$(x-y)(x+y) \Rightarrow x^2 - y^2$$

$$(x+3)(x+3) \Rightarrow x^2 + 6x + 9$$

$$(x+y)(x+y) \Rightarrow x^2 + 2xy + y^2$$

$$(x-5)^2 \Rightarrow x^2 - 10x + 25$$

$$(x^2 - 1)(x^2 - 1) \Rightarrow x^4 - 2x^2 + 1$$

$$(4x^2 + 2)(4x^2 - 2) \Rightarrow 16x^4 - 4$$

$$(x - 1)(x^3 - 3x + 4) \Rightarrow x^4 - x^3 - 3x^2 + 7x - 4$$