Math-2A

Lessons 2-7
Powers: Part 3
Zero and Negative Exponents

No Exponent? $$3x = 3^1 x^1$$

Usually, we don't write the exponent '1' (saves ink).

No Coefficient? $$x^3 = 1 * x^3 = 1^1 * x^3$$

Usually, we don't write the coefficient '1' (saves ink).

Negative? $$-x^2 = (-1) * x^2 = (-1)^1 * x^2$$

Usually, we don't write the coefficient '-1', we just put the "negative symbol" (saves ink).

Power: is repeated multiplication $x^4 = x * x * x * x$

multiplication: is repeated addition $3x = x + x + x$

How can you tell which definition to use?

Use the "context" of the problem

(adding two "like" terms)

$$3x + 4x \rightarrow (x + x + x) + (x + x + x + x) \rightarrow 7x$$

(adding two "like powers")

$$2x^2 + 3x^2 \rightarrow (x^2 + x^2) + (x^2 + x^2 + x^2) \rightarrow 5x^2$$

(multiplying two "same based powers")

$$x^2 * x^3 \rightarrow (x * x)(x * x * x) \rightarrow x^5$$

Multiply Powers Property When you multiply powers having the same base, you add the exponents.

$$(x^2)(x^3) = (x * x)(x * x * x) = x^5$$

'x' used as a factor five times

$$x^2 x^3 = x^{2+3} = x^5$$

Exponent of a Power Property When a power is inside parentheses and the parentheses has an exponent you multiply the exponents.

$$(x^2)^3 = \left(x^2\right)\left(x^2\right)\left(x^2\right) = (x * x)(x * x)(x * x) = x^6$$

'x^2' used as a factor 3 times → 'x' used as a factor 6 times

$$(x^2)^3 = x^{2*3} = x^6$$

Exponent of a Product Property Terms being multiplied inside of parentheses, with an exponent on the parentheses.

$$(xy)^2 = x*y*x*y \quad = x*x*y*y \quad = x^2y^2$$

Looks just like Exponent of a Power Property

$$(x^2y^3)^4 \quad = x^8y^{12}$$

DOES NOT WORK for "exponent of a sum"

$$(x+3)^2 \quad = (x+3)(x+3) \quad = x^2+6x+9$$

$$(x-3)^2 \neq x^2 - 3^2$$

$$(x-3)^2 = x^2+6x+9$$

Coefficients inside <u>Exponent of a Product</u>

$$(3x^3y^4)^2 = (3^1x^3y^4)^2 \quad = 3^2x^6y^8 \quad = 9x^6y^8$$

Constants (integer, etc.) usually have an exponent of ‘1’.

‘x’ is a number, we just don’t know what it is. You treat all numbers the same (whether they are variables or constants).

$$(3^3 * 2^4)^5 \quad = 3^{15} * 2^{20}$$

Can you multiply the 3 and 2 <u>before</u> removing the exponents?

NO! (PEMDAS)

Simplify

$$3x^2(4x^3)^2$$

$$\rightarrow 3*x^2*4^2*x^3 \quad \rightarrow 3*4^2*x^2*x^6 \quad \rightarrow 3*16*x^{2+6} \quad \rightarrow 48x^8$$

$$(2x^2)^5 \quad \rightarrow 2^5x^{10} \quad \rightarrow 32x^{10}$$

$$5(2x)^3(3x^4)^2 \quad \rightarrow 5*2^3*x^3*3^2*x^8$$

$$\rightarrow 5*8*9*x^{3+8} \quad \rightarrow 360x^{11}$$

$$(2y^5)^3 = ?$$

Watch the negatives! $(-x^3y^4)^2$

$$=((-1)^1x^3y^4)^2$$ Turn negative signs into multiplication by -1.

$$=(-1)^2x^6y^8 \quad = x^6y^8$$

$(-2x^2y^6)^3$ Negative coefficients have an exponent of ‘1’.

$$=((-2)^1x^2y^6)^3 \quad =(-2)^3x^6y^{18} \quad =-8x^6y^{18}$$

simplify

$$(-2x^2y^4z)^3 \to (-2)^3 * x^{2*3} * y^{4*3} * z^{1*3}$$

$$\to -8x^6y^{12}z^3$$

$$2(-3m^4x^3)^3 \to 2*(-3)^3 * m^{4*3} * x^{3*3}$$

$$\to 2(-27)*m^{12} * x^9$$

$$\to -54m^{12}x^9$$

Negative Exponent Property "Grab and drag"

When you "Grab and drag" the base and its exponent across the "boundary line" between numerator and denominator, you just change the sign of the exponent.

$$x^{-2} = \frac{1 * x^{-2}}{1} = \frac{1}{x^{2}}$$

Why did I insert the coefficient of '1' and denominator of '1' ?

If you "grab and drag" the only term in the numerator, the numerator does not disappear, and I needed to create a denominator.

<u>Negative</u> Exponent Property "Grab and drag"

When you "Grab and drag" the <u>base and its exponent</u> across the "boundary line" between numerator and denominator, you just <u>change the sign</u> of the exponent.

$$x^{-2} = \frac{1}{x^2}$$

$$\frac{2}{m^{-3}} = \frac{2}{1 * m^{-3}} = \frac{2m^3}{1} = 2m^3$$

$$\frac{3y}{2y^{-4}} = \frac{3y}{2 * y^{-4}} = \frac{3y * y^4}{2} = \frac{3y^5}{2}$$

Do we "Grab and drag" the <u>coefficient</u> of a base with a negative exponent?

NO! The negative exponent ONLY applies to the base.

<u>Negative</u> Exponent Property "Grab and drag"

$$\frac{3y^4}{2y^2} = \frac{3y^4}{2 * y^2} = \frac{3y^4 * y^{-2}}{2} = \frac{3y^{4-2}}{2} = \frac{3y^2}{2}$$

Can we use the negative exponent property on <u>positive exponents?</u>

YES! When it crosses the numerator—denominator boundary, it just changes sign ((+ → -)

This is how we <u>simplify</u> <u>same-based powers</u> that are both in the numerator AND denominator.

$$\frac{4x^2 y^{-2}}{y^3} = \frac{4x^2}{y^3 * y^2} = \frac{4x^2}{y^{3+2}} = \frac{4x^2}{y^5}$$

$$\left(\frac{x^7}{x^3}\right)^{-2} = \left(\frac{x^7}{1 * x^3}\right)^{-2} = \left(\frac{x^7 * x^{-3}}{1}\right)^{-2} = \left(\frac{x^{7-3}}{1}\right)^{-2} = \left(\frac{x^4}{1}\right)^{-2}$$

$$= \frac{x^{4(-2)}}{1^{-2}} = \frac{x^{-8}}{1^{-2}} = \frac{1^2}{x^8} = \frac{1}{x^8}$$

$$\left(\frac{x^7}{x^3}\right)^{-2} = \left(\frac{x^7}{1*x^3}\right)^{-2} = \left(\frac{x^7 * x^{-3}}{1}\right)^{-2} = \left(\frac{x^{7-3}}{1}\right)^{-2} = \left(\frac{x^4}{1}\right)^{-2}$$

$$= \frac{x^{4(-2)}}{1^{-2}} = \frac{x^{-8}}{1^{-2}} = \frac{1^2}{x^8} = \frac{1}{x^8}$$

“There are many paths to “math Nirvana”.

$$\left(\frac{x^7}{x^3}\right)^{-2} = \left(\frac{x^3}{x^7}\right)^{2} = \left(\frac{1*x^3}{x^7}\right)^{2} = \left(\frac{1}{x^7 x^{-3}}\right)^{2} = \left(\frac{1}{x^4}\right)^{2} = \frac{1}{x^8}$$

Negative Exponent really means: “convert a number (or a power) to its <u>reciprocal</u> (to change the sign of the exponent).

$$\left(\frac{2x^4y^2}{3x^3y^{-3}}\right)^{-2} \to \left(\frac{x^{-3}y^3 * 2x^4y^2}{3}\right)^{-2} \to \left(\frac{2 * x^{-3}x^4 * y^3y^2}{3}\right)^{-2}$$

$$\to \left(\frac{2x^{4-3}y^5}{3}\right)^{-2}$$

$$\left(\frac{2xy^5}{3}\right)^{-2} \to \left(\frac{3}{2xy^5}\right)^2 \to \frac{9}{4x^2y^{10}}$$

Negative Exponent Property

Possible errors

$$4x^{-2} = \frac{4 * x^{-2}}{1} = \frac{4}{x^2}$$

When you “Grab and drag” the base and its exponent across the “boundary line” between numerator and denominator, you just change the sign of the exponent.

DO NOT GRAB the coefficient!

$$\frac{4 * x^{-2}}{1} \neq \frac{1}{4x^2}$$

Negative Exponent Property   Possible errors

$$-4 = \frac{-4 * 1}{1}$$

$$\boxed{\frac{1}{-4^{-1}}}$$

$$-4 \neq \frac{1}{4}$$

$$\boxed{\frac{1}{4}}$$

Which of these is correct?

The Negative Exp. Prop. Changes the sign of the exponent ***NOT*** *the sign of the coefficient!*

It's better to think of -4 as (-1)*4 so you don't get tempted to "drag" negative coefficients.

$$-4 = \frac{-1 * 4}{1}$$

Zero Exponent Property

Any base raised to the zero power simplifies to one.

$$10^3 = 1000 \qquad 2^3 = 8 \qquad 2^0 = 1$$

$$10^2 = 100 \qquad 2^2 = 4 \qquad (2x)^0 = 1$$

$$10^1 = 10 \qquad 2^1 = 2 \qquad 2x^0 = 2*1 = 2$$

$$10^0 = 1 \qquad 2^0 = 1$$

Multiply Powers Property

$$x^3x^0 = x^{3+0} = x^3$$

Zero Exponent Property

$$x^3x^0 = x^3*1 = x^3$$

If you got a different result from two properties, then one of the properties is a “lie” since properties guarantee equivalence.