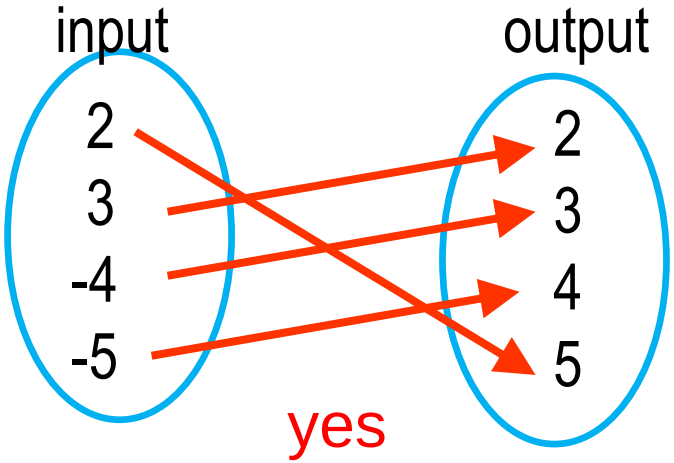
Math-2A

Lesson 13-1

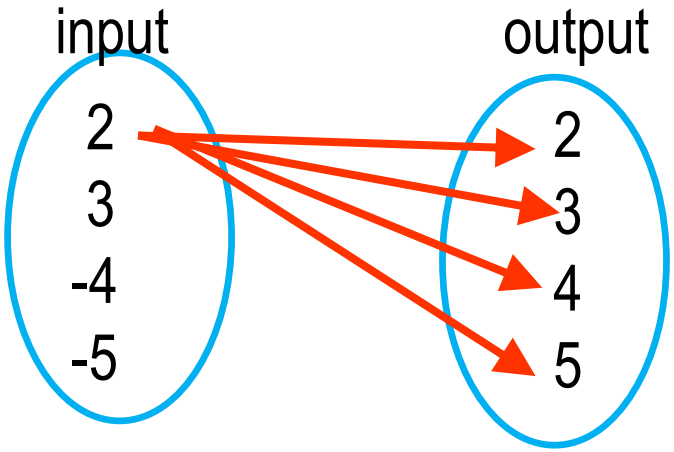
Relations and Functions

And the Linear Function

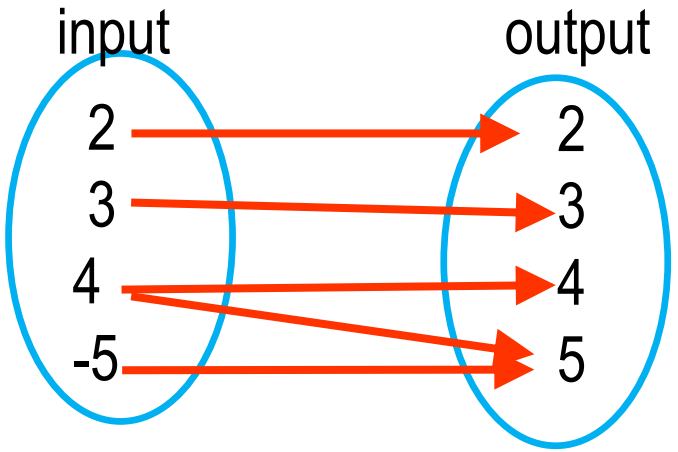
Is it a function?



yes



No: input value ‘2’ has more than one output

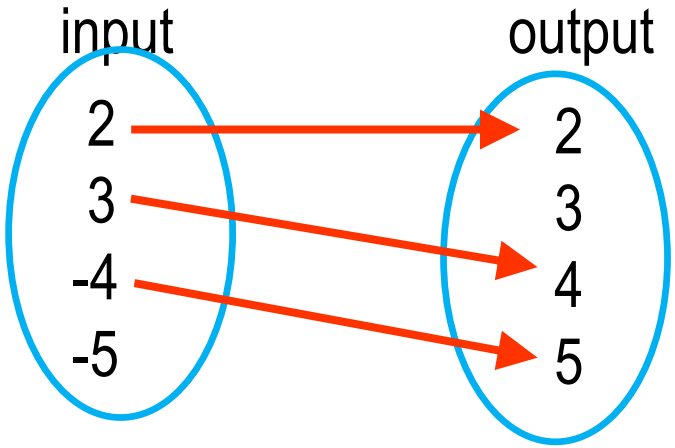


Relation but NOT a function since input value ‘4’ has 2 outputs.

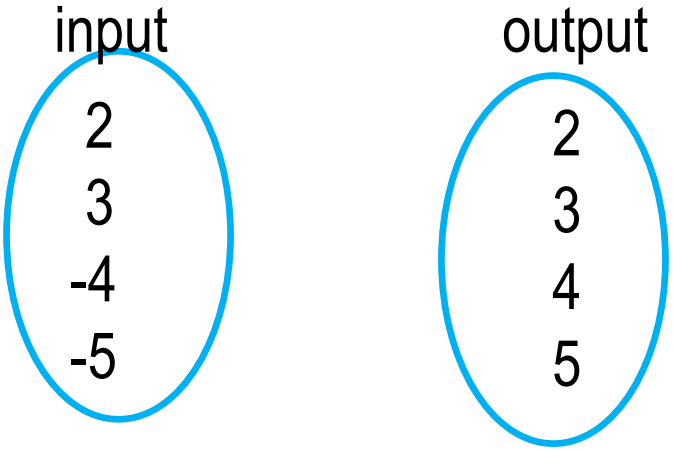
Relation: A pairing of input values to output values.

Function: A relation where each input has exactly one output.

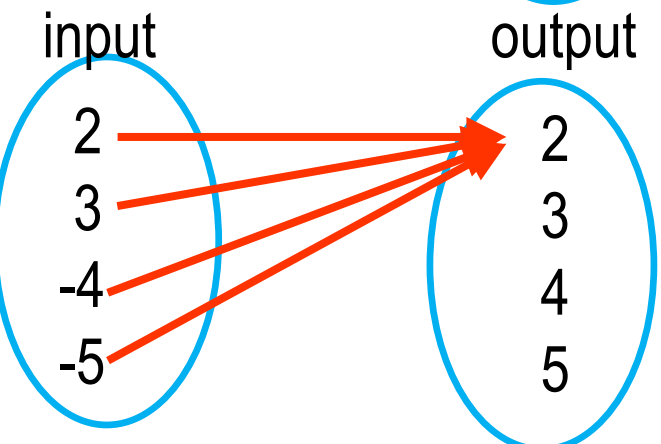
Is it a function?



yes



No (There aren't any pairings of inputs to outputs.)



Yes Each input has exactly one output (even though it's the same output)

There are at least 6 ways to show a relation between input and output values.

Ordered Pairs: (2, 4), (3, 2), (-4, 3)

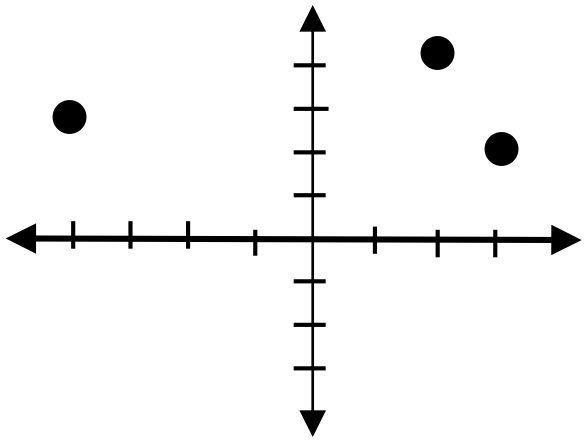
Data table:

x	2	3	-4
y	4	2	3

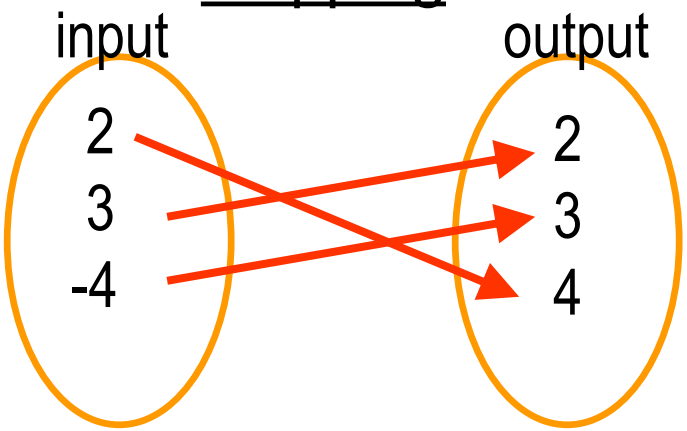
Equation: $y = 2x + 1$

Function notation: $f(2) = 4$

Graph:



Mapping



Are all of these representations the same?

Domain: the set made up of all of the input values that have corresponding output values.

Range: the set made up of all of the corresponding output values.

1. Identify the Domain

1. (2, 4), (3, 5), (-4, 2)

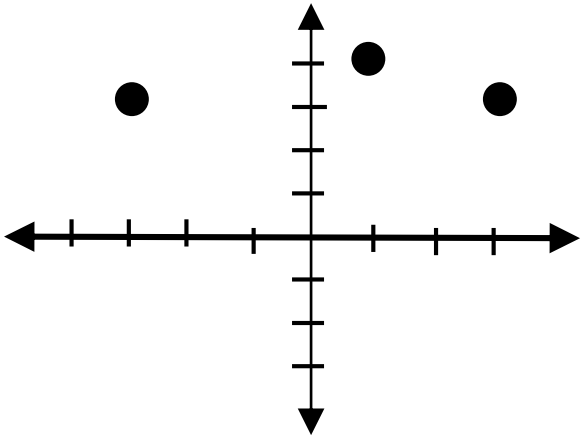
2. Identify the Range:

2.

x	6	9	-2
y	4	7	3

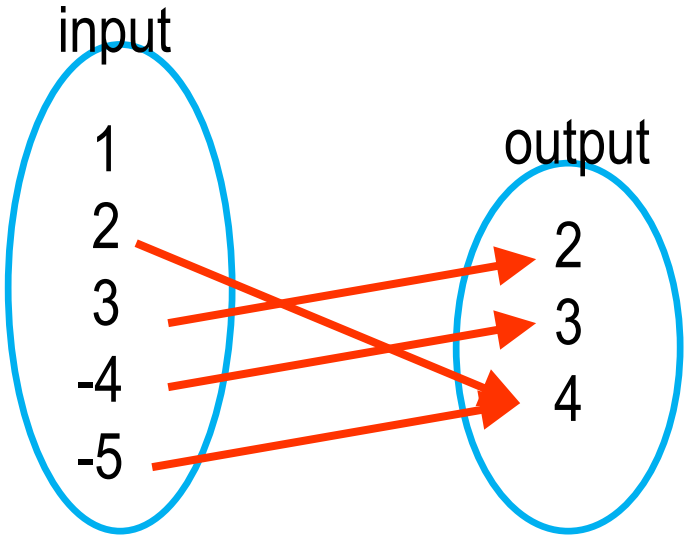
3. Identify the Domain:

3.



4. Identify the Domain:

4.



1. Identify the Domain

1. (2, 4), (3, 5), (-4, 2)

2. Identify the Range:

2.

x	6	9	-2
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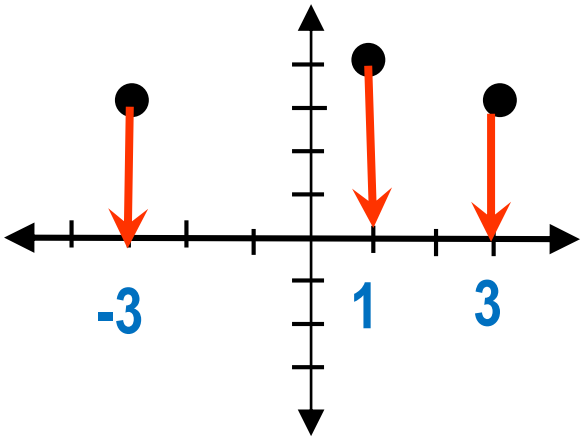
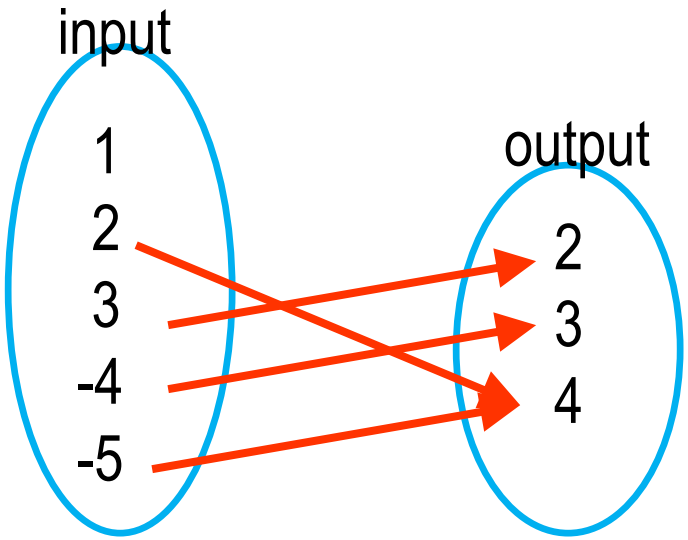


4. Identify the Domain:

4. 2, 3, -4, -5



We can convert between the six forms of a function.

To convert an equation into a table of x-y pairs, “plug in” (substitute) numbers into the equation in place of “x”.

$$y = 2x - 1$$

We replace ‘x’ (the place-holder) with a parentheses. Then we substitute into the parentheses the input value then simplify.

$$y = 2(\quad) - 1$$

x	0	1	2
y	-1	1	3

$$y = 2(0) - 1 \qquad y = -1$$

$$y = 2(1) - 1 \qquad y = 1$$

$$y = 2(2) - 1 \qquad y = 3$$

Equation → table

What is the y-intercept for each equation?

$y = 3x + 4$

x	0	1	2
y	4	7	10

$y = 4x - 2$

x	0	1	2
y	-2	2	6

$y = 5x + 3$

x	0	1	2
y	3	8	13

What do you notice when comparing the constant term in the equation to the numbers in the table?

$y = 3x + 4$

$y = 3(0) + 4$

x	0	1	2
y	4	7	10

$y = 4x - 2$

$y = 4(0) - 2$

x	0	1	2
y	−2	2	6

$y = 5x + 3$

$y = 5(0) + 3$

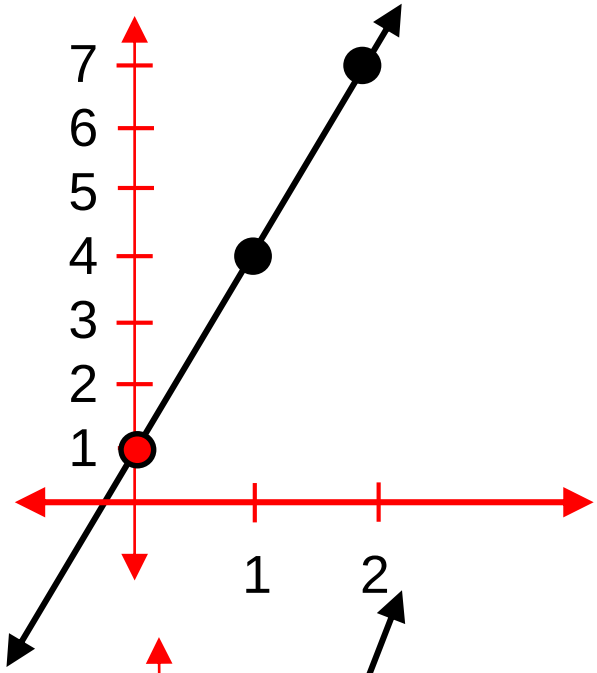
x	0	1	2
y	3	8	13

The constant term of the equation is always mapped from the input value zero.

Fill in the table then graph x-y pairs from the table.

$y = 3x + 1$

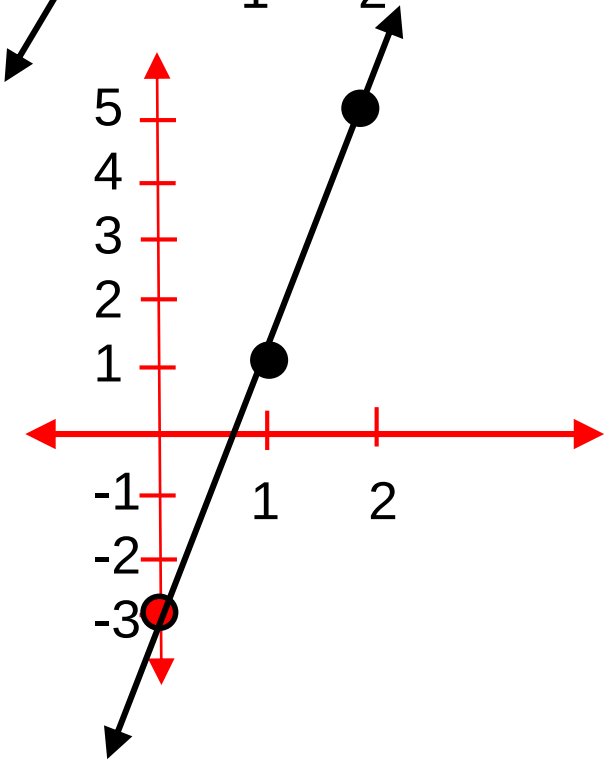
x	0	1	2
y	1	4	7



y-intercept: the x-y pair where a graph crosses the y-axis.

$y = 4x - 3$

x	0	1	2
y	−3	1	5



Solution of a two-variable equation: all x-y pairs that make the equation true.

What do you notice when comparing the coefficient of the input variable to the numbers in the table?

$y = 3x + 4$

x	0	1	2
y	4	7	10

+3 +3

$y = 4x - 2$

x	0	1	2
y	−2	2	6

+4 +4

$y = 5x + 3$

x	0	1	2
y	3	8	13

+5 +5

If the input value changes by one, the coefficient of the input variable is the change in 'y' between adjacent terms in the table.

Why isn't the change in 'y' between adjacent terms equal to the coefficient of 'x'?

Fill in the tables.

$y = 2x + 1$

x	0	2	4
y	1	5	9

+2 +2 +4 +4

We changed the input value to 'x' by '2' for each adjacent value in the table instead of '1'.

$y = 3x - 5$

x	0	2	4
y	−5	1	7

+2 +2 +6 +6

$y = 4x + 2$

x	0	2	4
y	2	10	18

+2 +2 +8 +8

How can you use the change in 'x' and the change in 'y' in the tables to calculate the coefficient of 'x'?

Delta a Greek letter (that looks like a triangle) used in engineering and math to denote “change.”

Δx Means the change in ‘x’

Δy Means the change in ‘y’

$y = 3x - 5$

$\Delta x = 2 \quad \Delta x = 2$

x	0	2	4
y	-5	1	7

$\Delta y = 6 \quad \Delta y = 6$

The coefficient of ‘x’ in the equation equals the change in ‘y’ of the table values divided by the change in ‘x’ of the table values.

$$3 = \frac{6}{2} = \frac{\Delta y}{\Delta x}$$

Fill in the table then graph the ordered pairs

$y = 2x - 2$

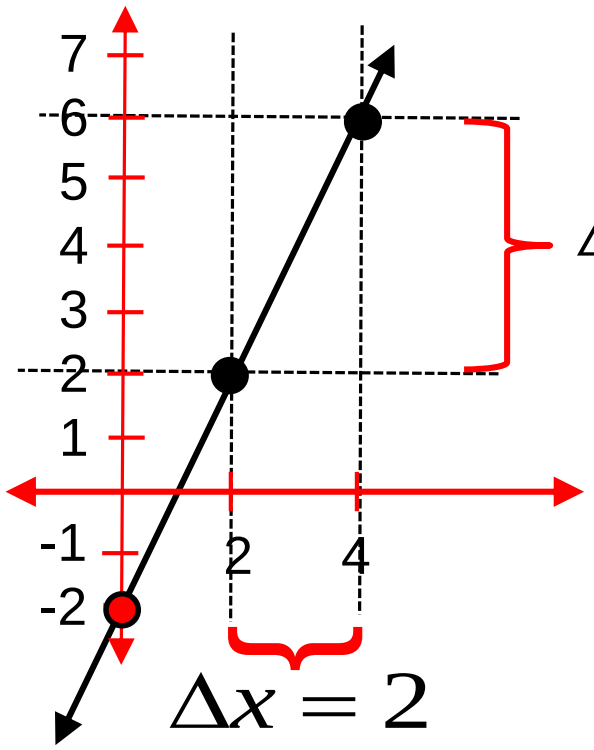
$\Delta x = 2 \quad \Delta x = 2$

x	0	2	4
y	-2	2	6

$\Delta y = 4 \quad \Delta y = 4$

Graphing the solution to the equation will result in infinitely points

→ they all form a line.



Slope (of a line) is its steepness given by $m = \frac{\Delta y}{\Delta x}$

Slope is the coefficient of 'x' when the equation is written in the form: y = mx + b

$$m = \frac{4}{2} = 2$$

Slope-intercept form of a linear equation:

the equation of a line written in the form:

$y = f(x)$

that gives the

slope of the line

and

the y-value where the graph crosses the y-axis.

$$y = mx + b$$

$$y = 3x + 2$$

Slope = 3 y-intercept: (0, 2)

Your turn: Is the data linear? If so, what is the equation that "fits" the data? $y = mx + b$

What does this number represent on the graph?

x	f(x)
-4	-7
-3	-5
-2	-3
-1	-1
0	1
1	3
2	5
3	7
4	9

The output value 'y' when input value x = 0.

$$y = m(0) + b \qquad y = b$$

The y-intercept always has ax x-value of zero.

$$(0, b) \qquad b = 1$$

Substitute b = 1 into the general equation.

$$y = mx + 1$$

What is the slope?

$$m = \frac{\Delta y}{\Delta x} \qquad m = \frac{2}{1}$$

$\Delta x = 1 \quad \Delta y = 2$

Substitute m = 2 into the general equation.

$$y = 2x + 1$$

Another way to do it:

$$y = mx + b \qquad b = 1 \qquad y = mx + 1$$

x	f(x)
-4	-7
-3	-5
-2	-3
-1	-1
0	1
1	3
2	5
3	7
4	9

Every x-y pair is a solution of the equation → makes the equation true.

Substitute any x-y pair in for 'x' and 'y' in the equation.

$$3 = m(1) + 1$$

Solve for 'm'. $m = 2$

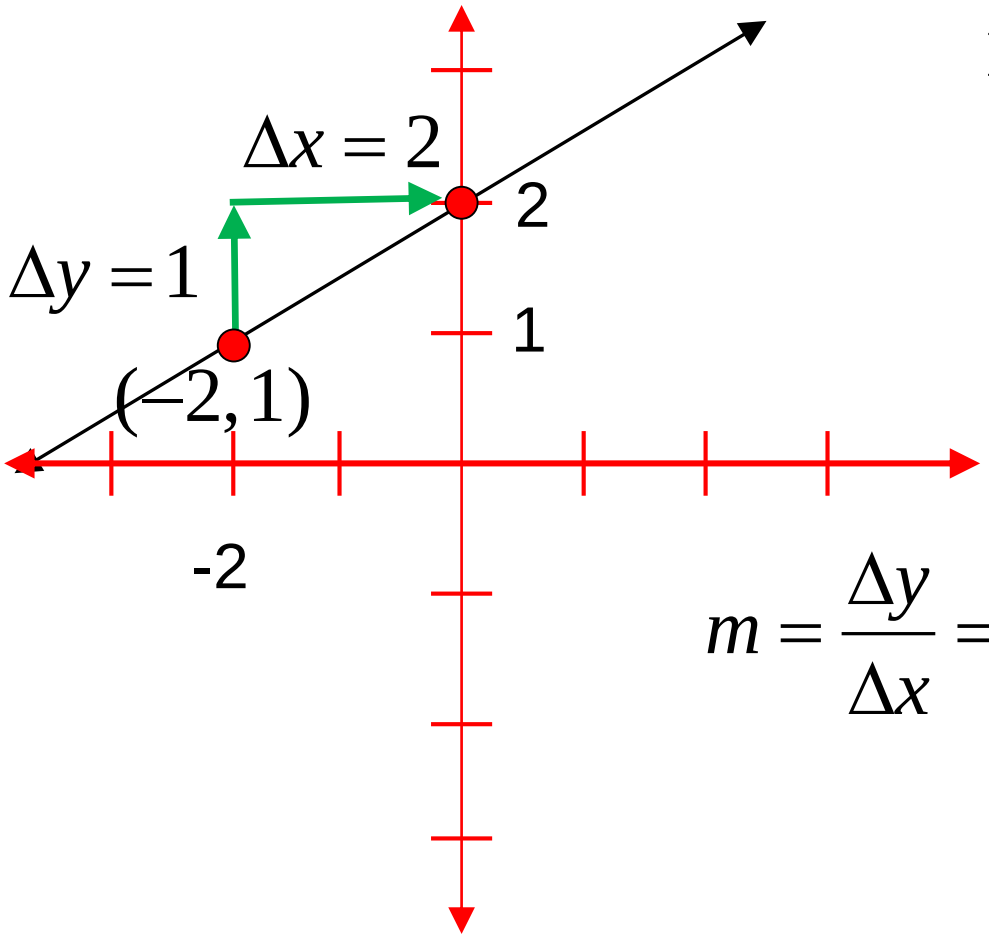
We know 'm' and 'b' → we know the equation that corresponds to the table.

$$y = 2x + 1$$

What is the equation of the line?

$y = mx + b$ $\quad b = 2$ $\quad y = mx + 2$

$(x, y) = (-2, 1)$



$1 = m(-2) + 2$

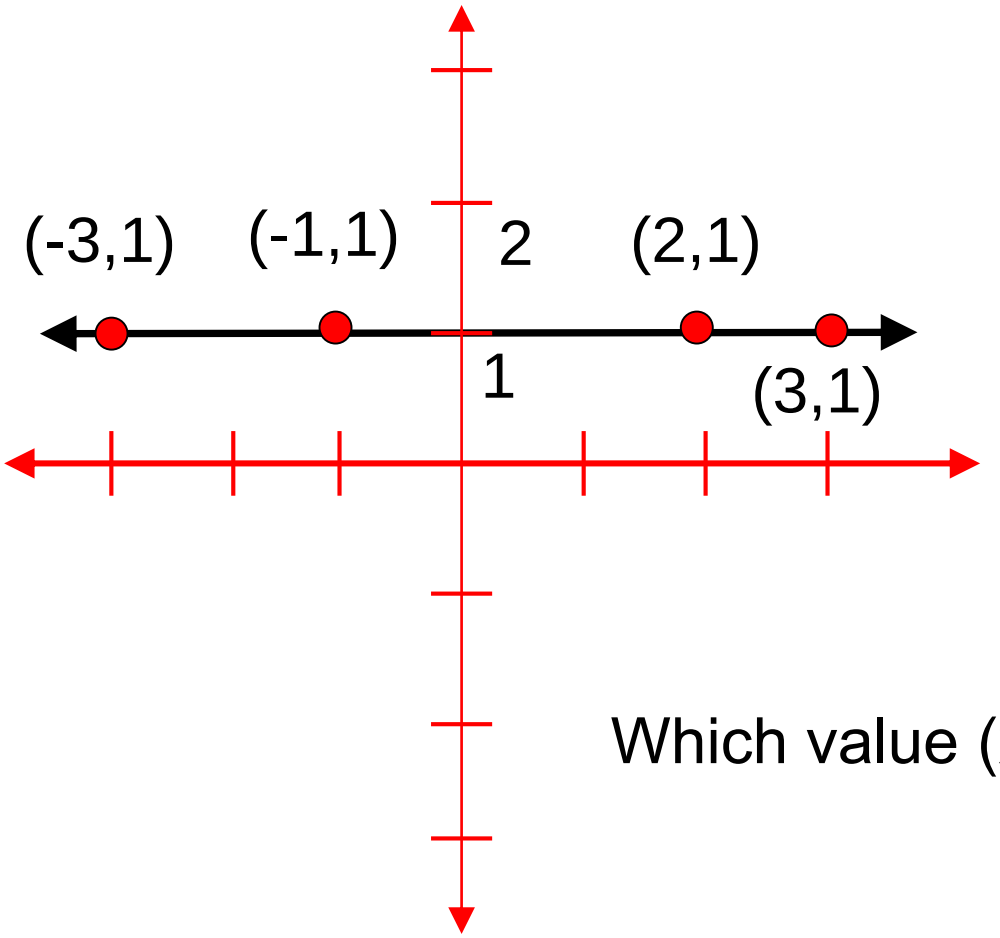
$-1 = m(-2)$

$m = \dfrac{-1}{-2} = \dfrac{1}{2}$

$m = \dfrac{\Delta y}{\Delta x} = \dfrac{1}{2}$

$y = \dfrac{1}{2}x + 2$

What is the equation of the line?



$y = 1$

or

$x = 1$

?

Which value (x or y) is always ‘1’?

Your turn: What is the equation that fits the data?

$$y = mx + b \qquad b = -3$$

x	f(x)
-4	-9
-2	-6
0	-3
2	0
4	3
6	6
8	9
10	12
12	15

$$y = mx - 3$$

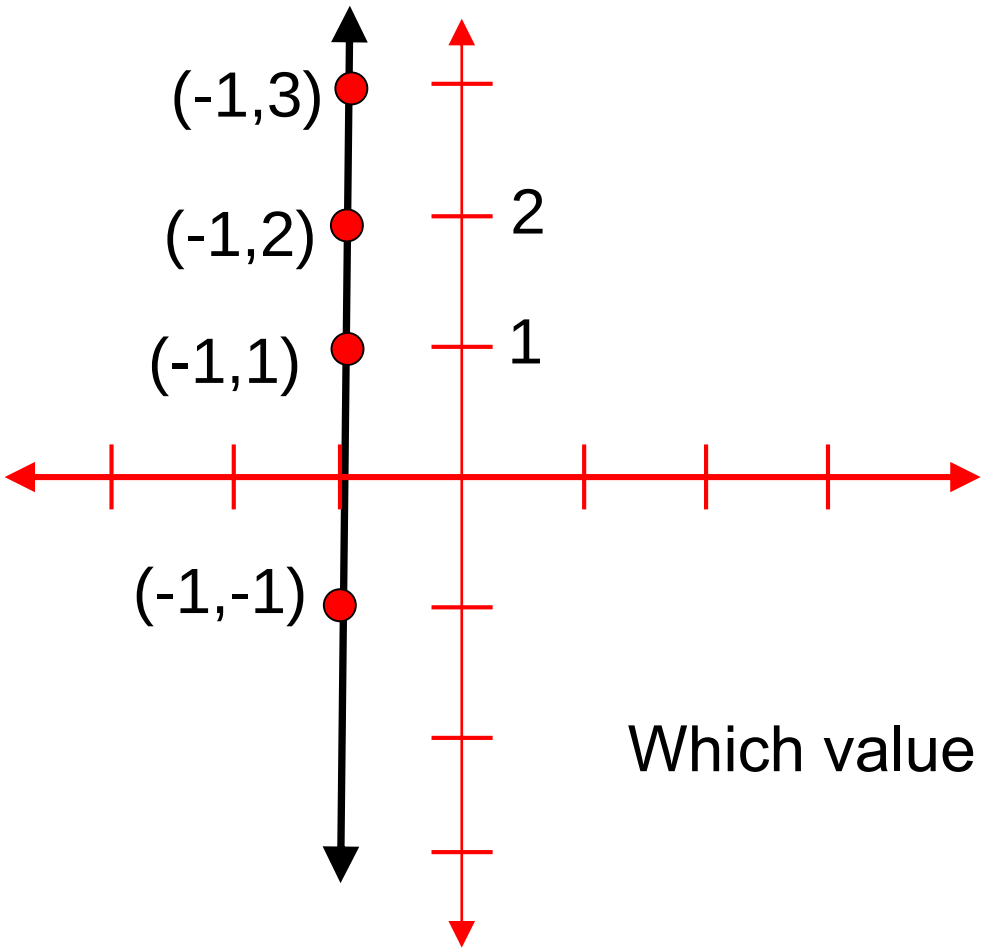
$$0 = m(2) - 3 \qquad m = \frac{3}{2}$$

$$y = \frac{3}{2}x - 3$$

$\Delta x = 2$ $\Delta y = 3$

$$m = \frac{\Delta y}{\Delta x} = \frac{3}{2}$$

What is the equation of the line?



$y = -1$

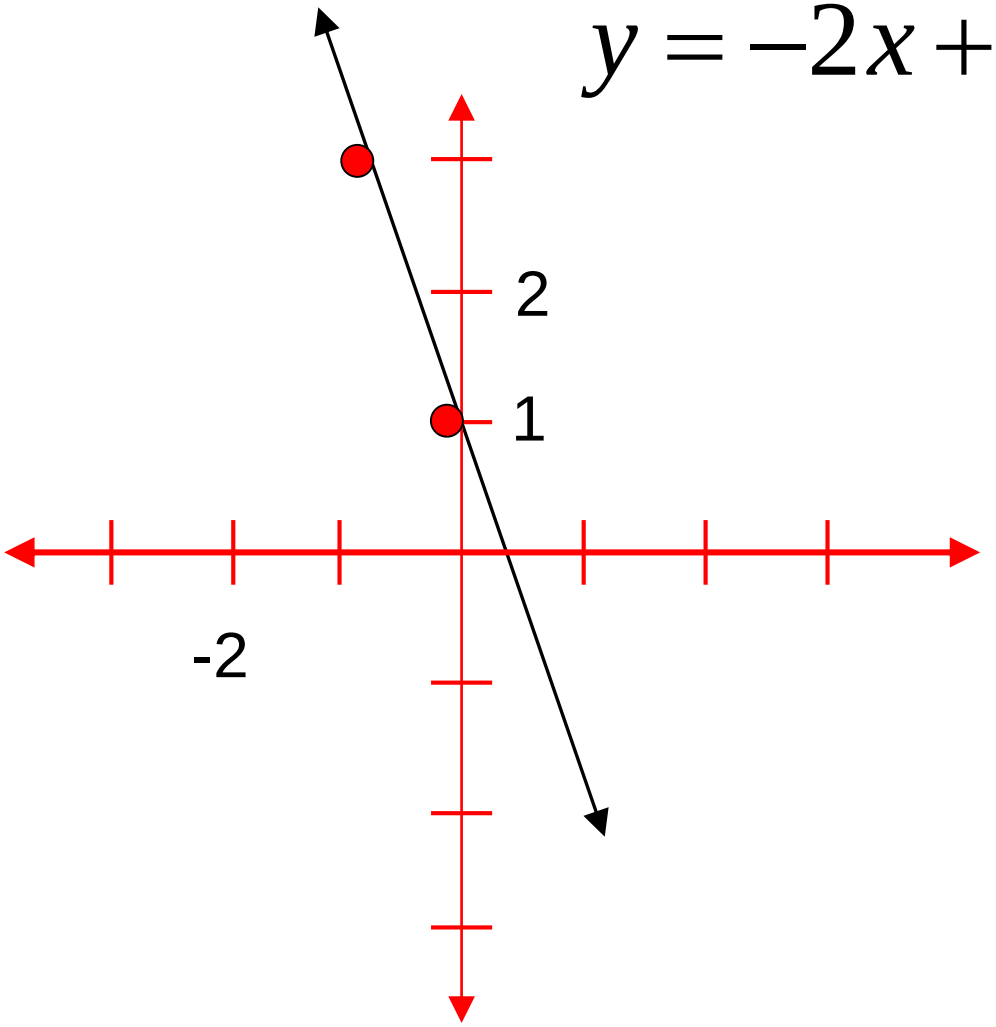
or

$x = -1$

?

Which value (x or y) is always ‘-1’?

What is the equation of the line?



$$y = -2x + 1$$