Math-2A
Lesson 11-3

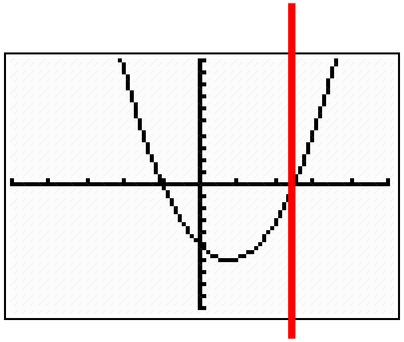
Function Composition

1. Is the following relation a function?

(-2, 5), (5, 6), (-2, 6), (7, 6)

No. Input value -2 has two output values.

2. Is the following relation a function?



Does the graph of the relation pass the “vertical line test” ?

Yes. Each input value has exactly one output value.

Function Notation

y = f(x) "y is a function of x"

'y' equals 'f' of 'x'

A function is a rule that matches input values to out put values.

f(x) = 2x + 1

(Input) x	(rule) 2x + 1	(output) y
2	2(2) + 1	5
3	2(3) + 1	7

f(2) = 5

f(3) = 7

Compositions of Functions

$f(x) = 2x \rightarrow f(3) = ?$

Means: wherever you see an 'x' in the function, replace it with a 3.

1. Replace the 'x' with a set of parentheses.

$$f(3) = 2(\quad)$$

2. Put the input value '3' into the parentheses.

$$f(3) = 2(3)$$

3. Find the output value.

$$f(3) = 6$$

Compositions of Functions

$f(x) = x^2 - 3x + 2$ → f(2) = ?

Means: wherever you see an 'x' in the function, replace it with a '2'.

1. Replace the 'x' with a set of parentheses.

$$f(x) = (\ )^2 - 3(\ ) + 2$$

2. Put the input value '2' into the parentheses.

$$f(x) = (2)^2 - 3(2) + 2$$

3. Find the output value.

f(2) = 0

Cool, we found a zero of the function.

$$f(x) = x^3 - 1 \qquad f(-2) = ?$$

$$f(-2) = (-2)^3 - 1 \qquad f(-2) = -9$$

$$f(x) = 2x^{1/2} \qquad f(9) = ?$$

$$f(9) = 2(9)^{\frac{1}{2}} \qquad f(9) = 6$$

$$f(x) = \frac{2(x-4)}{x^2 + x - 20} \qquad f(-2) = ?$$

$$f(-2) = \frac{2(-2-4)}{((-2)^2 + (-2) - 20)} \qquad f(-2) = \frac{2}{3}$$

Function Notation

$f(x) = 2x + 1$

(Input) x	(rule) $2x + 1$	(output) $f(x)$	
2	$2(2) + 1$	5	$f(2) = 5$
3	$2(3) + 1$	7	$f(3) = 7$
$x – 1$	$2(x – 1) + 1$	$2x – 1$	
$3x$	$2(3x) + 1$	$6x + 1$	

$f(x – 1) = 2x – 1$

$f(3x) = 6x + 1$

If your input is an expression instead of a number:
replace ‘x’ with parentheses and “plug in” the expression
→ parentheses, substitute, simplify

$$f(x) = 3x - 1$$

(Input) x	(rule) 3x - 1	(output) f(x)	
2	3(2) - 1	5	$f(2) = 5$
x^2	3( ) - 1	?	$f(x^2) = 3x^2 - 1$
$x + 2$	3( ) - 1	?	$f(x+2) = 3x + 5$
$3 - 2x$	3( ) - 1	?	$f(3-2x) = 8 - 6x$

Your turn:

input the expressions

$$f(x) = x^2 + 1$$

$$f(2) = ? \quad = 5$$

$$f(x^3) = ? \quad = x^6 + 1$$

$$f(x+2) = ? \quad = (x+2)^2 + 1 \quad = x^2 + 4x + 5$$

$$f(-2x+3) = ? \quad = (-2x+3)^2 + 1$$

$$= 4x^2 - 12x + 10$$

Compositions of Functions

f(x) = 2x and $g(x) = x^2$

Let's use f(x) as the input to g(x) $g(f(x)) = ?$

$$g(..) = (..)^2$$

$$g(2x) = (2x)^2$$

$$g(f(x)) = 4x^2$$

1. Replace the 'x' with a set of parentheses.
2. Put the input value "2x' into the parentheses.
3. Find the output value.

Compositions of Functions

f(x) = 2x + 3 and $g(x) = x^2$

f(g(x)) = ?

1. The input value to f(x) is g(x).

$$f(..) = 2(..) + 3$$

2. Replace the 'x' in f(x) with a set of parentheses.

$$f(x^2) = 2(x^2) + 3$$

3. Put the input value (g(x)) into the parentheses.

$$f(g(x)) = 2x^2 + 3$$

4. Find the output value.

Function “composition”

$$f(x) = x^2 + 1 \qquad g(x) = x^2$$

$$f(2) = ?$$ **What does this mean?**

“Substitute ‘2’ in for ‘x’ in the function f(x).”

$$f(g(x)) = ?$$ **What does this mean?**

“Substitute ‘g(x)’ in for ‘x’ in the function f(x).”

$$f(g(x)) = (g(x))^2 + 1$$

“Which means the same as…”

$$f(x^2) = (x^2)^2 + 1 \qquad = x^4 + 1$$

Composition of Functions

f(x) = 2x + 1 g(x) = 3x + 2 h(x) = x + 5

$$f(g(x)) = ? = 2(\qquad) + 1 = 2(3x + 2) + 1$$

$$h(g(x)) = ? = (\qquad) + 5 = (3x + 2) + 5$$

$$h(f(x)) = ? = (\qquad) + 5 = (2x + 1) + 5$$

$$g(h(x)) = ? = 3(\qquad) + 2 = 3(x + 5) + 2$$

$$f(f(x)) = ? = 2(\qquad) + 1 = 2(2x + 1) + 1$$

New Notation for the Composition of Functions

$(f \circ g)(x) = f(g(x))$ "g" plugged into rule "f"

$f(x) = 4x - 1 \qquad g(x) = -5x + 3$

$(f \circ g)(x) = ? \quad = 4(\qquad) - 1 \; = 4(-5x + 3) - 1$

"g" plugged into rule "f" $\quad (f \circ g)(x) = -20x + 11$

$(g \circ f)(x) = ? \; = -5(\qquad) + 3 \; = -5(4x - 1) + 3$

"f" plugged into rule "g" $\quad (g \circ f)(x) = -20x + 8$

$(f \circ f)(x) = ? \quad = 4(\qquad) - 1 \quad = 4(4x - 1) - 1$

"f" plugged into rule "f" $\quad (f \circ f)(x) = 16x - 5$

$(g \circ g)(x) = ? \; = -5(\qquad) + 3 = -5(-5x + 3) + 3$

"g" plugged into rule "g" $\quad (g \circ g)(x) = 25x - 12$

One more layer!

$$f(x) = 3x \qquad g(x) = x^2$$

$$f(g(4))$$

4

$$g(\ ) = (\ )^2$$

$$g(4) = (4)^2$$

16

$$f(\ ) = 3(\ )$$

$$f(16) = 3(16)$$

48

$$f(g(x)) = 3(g(x)) = 3x^2$$

$$f(g(4)) = 3(g(4)) = 3(4)^2$$ **= 48**

One more layer. $g(x) = x^2$ $f(x) = 3x$

$(g \circ f)(-1) = ?$

Rewrite in "old" notation

$g(f(-1)) = ?$

The input to f(x) is -1.

$f(-1) = 3(-1)$

$f(-1) = -3$

The output of f(-1) is -3.

The input to g(x) is -3.

$g(-3) = 9$

$g(f(-1)) = 9$