Math-2A

Lesson 11-1
Linear Combinations of Functions

Combining Functions Algebraically

Multiplication by a number

$$f(x) = x^2 - 1$$

$2f(x) = ?$ Replace "f(x)" with parentheses

2 ( )

Substitution step → $f(x) = x^2 - 1$

$2f(x) = 2(x^2 - 1)$ Simplify →

$f(x) = x^2 - 1$ $2f(x) = 2x^2 - 2$

Perform the indicated operation:

$f(x) = 4x + 2$ $\quad$ $g(x) = -5x + 3$

3f(x) = ? $= 3(4x + 2) = 12x + 6$

-2g(x) = ? $= -2(-5x + 3) = 10x - 6$

5g(-3) = ? $\Rightarrow g(-3) = -5(-3) + 3$

$g(-3) = 18$

$\Rightarrow 5g(-3) = 5(18) = 90$

"Linear combinations of functions"

$$f(x) = 4x + 2 \qquad g(x) = -5x + 3$$

(1) Replace with parentheses, (2) plug in, (3) Simplify.

f(x) - g(x) = ? $= (\quad) - (\quad)$

$= (\ 4x + 2\ ) - (-5x + 3\ )$

$= 4x + 2 + 5x - 3$

f(x) - g(x) $= 9x - 1$

Perform the indicated operation:

$f(x) = 4x + 2$ $\qquad$ $g(x) = -5x + 3$

(1) Replace with parentheses, (2) plug in, (3) Simplify.

g(x) – 2 f(x) = ( ) – 2( )

$= (-5x + 3) - 2(4x + 2)$

$= -5x + 3 - 8x - 4$

g(x) – 2 f(x) $= -13x - 1$

Perform the indicated operation:

$f(x) = 4x + 2$ $\qquad$ $g(x) = -5x + 3$

(1) Replace with parentheses, (2) plug in, (3) Simplify.

-3 f(x) – 2 g(x) = -3( ) – 2( )

= -3($4x + 2$) – 2($-5x + 3$)

$= -12x - 6 + 10x - 6$

-3 f(x) – 2 g(x) $= -2x - 12$

Perform the indicated operation:

$$f(x) = x^2 + 4x - 21 \qquad g(x) = x - 3$$

(1) Replace with parentheses, (2) plug in, (3) Simplify.

f(x) ÷ g(x) $= \dfrac{(\qquad\qquad)}{(\qquad\qquad)} = \dfrac{(x^2 + 4x - 21)}{(x - 3)}$

$$= \frac{(x+7)(x-3)}{(x-3)}$$

$$f(x) - g(x) = x - 3$$

Perform the indicated operation:

$$f(x) = x^2 - 5x - 14 \qquad g(x) = x + 3$$

(1) Replace with parentheses, (2) plug in, (3) Simplify.

g(x) ÷ 2 f(x) $= \dfrac{(\qquad\qquad)}{(\qquad\qquad)}$

$$= \frac{(x+3)}{2(x^2-5x-14)} = \frac{(x+3)}{2(x-7)(x+2)}$$

New Notation

$f(x) - g(x) = (f - g)(x)$

Rewrite the following in the new notation.

$g(x) - 2f(x) = (g - 2f)(x)$

$-3f(x) - 2g(x) = (-3f - 2g)(x)$

$f(x) \div g(x) = \left(\frac{f}{g}\right)(x)$

Perform the indicated operation:

$$j(x) = 15x^2 \qquad k(x) = -5x^3$$

(2j – 3k)(x) $= 2(15x^2) - 3(-5x^3)$

$$= 15x^3 + 30x^2$$

(2j – 3k)(-1) $= 15(-1)^3 + 30(-1)^2$

(2j – 3k)(-1) = 15

$$\left(\frac{2k}{j}\right)(x) = ? \quad = \frac{2(-5x^3)}{15x^2}$$

$$= \frac{5*(-2)*x*x*x}{5*3*x*x} \quad = \frac{-2x}{3} \quad = -\frac{2}{3}x$$

$j(x) = 15x^2$ $k(x) = -5x^3$

(j + k)(-2) What does this mean?

$$(j+k)(x) = j(x) + k(x) = 15x^2 - 5x^3$$

$$j(x) + k(x) = 15x^2 - 5x^3$$

$$(j+k)(-2) = 15(\quad)^2 - 5(\quad)^3$$

$$= 15(-2)^2 - 5(-2)^3$$

$$= 60 + 40$$

$$= 100$$

OR, you can do it this way

$$(j+k)(-2) = j(-2) + k(-2)$$

$$j(-2) + k(-2) = 15(-2)^2 - 5(-2)^3$$

$$= 60 + 40$$

$$= 100$$

Perform the indicated operation:

$f(x) = x^2 + 4x - 21 \qquad g(x) = x - 3$

(f – 2g)(-1)

f(-1) – 2g(-1)

$f(-1) = (-1)^2 + 4(-1) - 21 \quad = -24$

$g(-1) = (-1) - 3 \quad = -4$

f(-1) – 2g(-1) $= -24 - 2(-4)$

$= -24 + 8$

$= -16$

Combining Functions Algebraically

Product: (fg)(x) = f(x) * g(x)

$$f(x) = x + 2 \qquad g(x) = x - 2$$

$$(fg)(x) = (x + 2)(x - 2)$$

$$(fg)(x) = x^2 - 4$$

$f(x) = 3x$ $g(x) = x - 4$

$(f * g)(2) = ?$

$(f * g)(x) = 3x(x - 4)$

$(f * g)(2) = 3(2)[(2) - 4]$

$(f * g)(2) = 6(-2)$

$(f * g)(2) = -12$

OR

$(f * g)(2) = f(2) * g(2)$

$f(2) = 3(2)$

$f(2) = 6$

$g(2) = (2) - 4$

$g(2) = -2$

$(f * g)(2) = (6) * (-2)$

$(f * g)(2) = -12$

Which way is easier for you?

$$f(x) = 3x \qquad g(x) = x - 4$$

$$(2f + 3g)(-1) = ?$$

Method 1: Write as two separate functions then combine the result

$$(2f + 3g)(-1) = 2f(-1) + 3g(-1)$$

$$= 2[3(-1)] + 3[(-1) - 4]$$

$$= 2(-3) + 3(-5)$$

$$= -6 - 15$$

$$= -21$$

$f(x) = 2x - 3$ $\quad$ $g(x) = 4x - 5$

$(3f + 2g)(-1) = ?$

Method 2: Find linear combination of the functions THEN plug in.

$$(3f + 2g)(x) = 3(2x - 3) + 2(4x - 5)$$

$$= 6x - 9 + 8x - 10$$

$$= 14x - 19$$

$$(3f + 2g)(x) = 14x - 19$$

$$(3f + 2g)(-1) = 14(-1) - 19$$

$$= -14 - 19 \quad = -33$$