

Math-2A
Lesson 10-1
Circles

Transformations of the Square function.

$$f(x) = x^2$$

$$y = (-1)a(x - h)^2 + k$$

Reflection
across x-axis

VSF

left/right

up/down

$$y = -2(x - 3)^2 + 4$$

Reflected across x-axis,
VSF = 2, right 3, up 4

$$y = 3(x + 5)^2 - 6$$

VSF = 3, left 5, down 6

To convert the equation into a graph:

1) Start with the “parent function”

2) Move the vertex left/right and up/down

3) The shape of the graph depends upon VSF and x-axis reflection.

$$y = x^2$$

$$y = (-1)a(x - h)^2 + k$$

Reflection
across x-axis

Vertical
stretch
factor

Shift
left/right

shift up or
down

$$y = |x|$$

$$y = (-1)a|x - h| + k$$

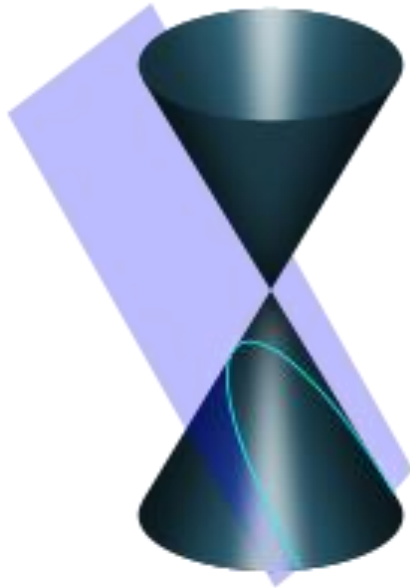
$$f(x) = -5|x - 2| + 3$$

reflected across x-axis

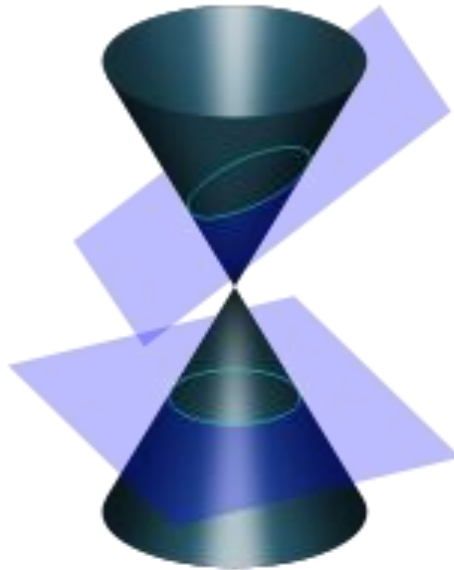
VSF=5,
translated right 2
translated up 3

Conic Sections

Parabola



Ellipse



Circle

Hyperbola



Finding the equation of a circle:

What is the radius of the circle?

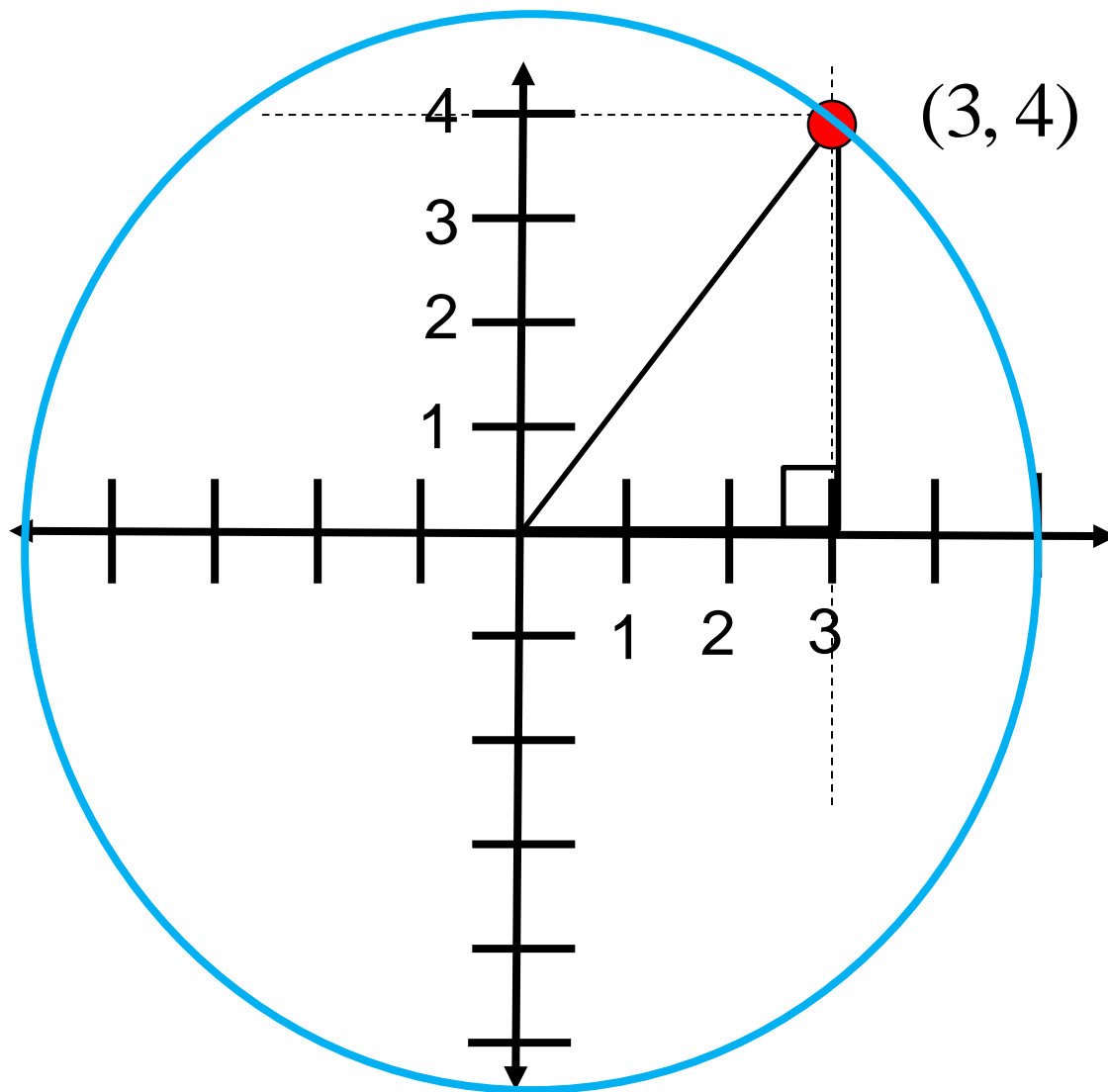
$$a^2 + b^2 = c^2$$

$$3^2 + 4^2 = r^2$$

$$9 + 16 = r^2$$

$$25 = r^2$$

$$r = 5$$

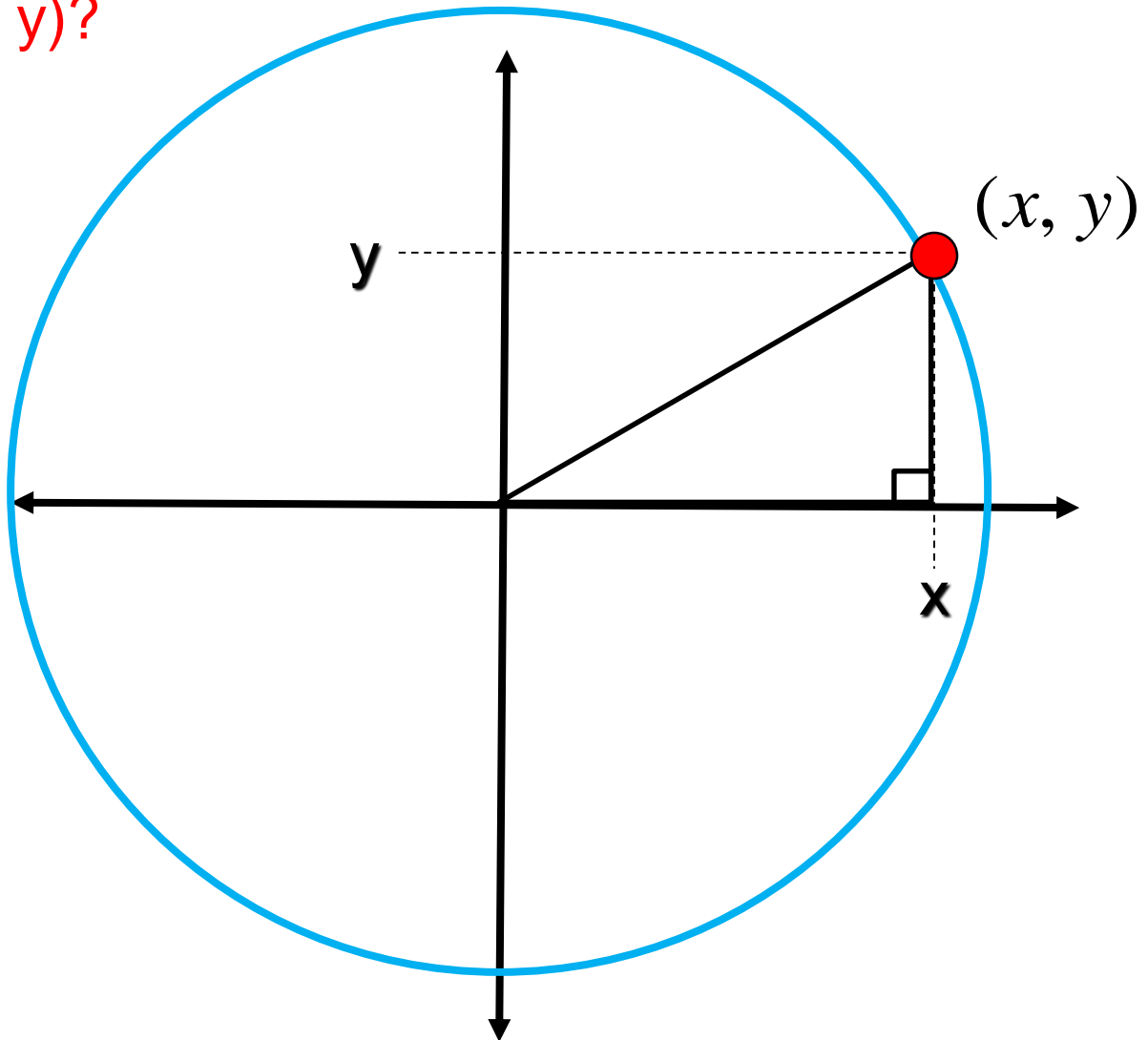


Now I will pick a random point on the circle.

What is the equation for the radius of the circle
when the point is (x, y) ?

$$x^2 + y^2 = r^2$$

This is the general
equation for a circle
centered at $(0, 0)$!!!

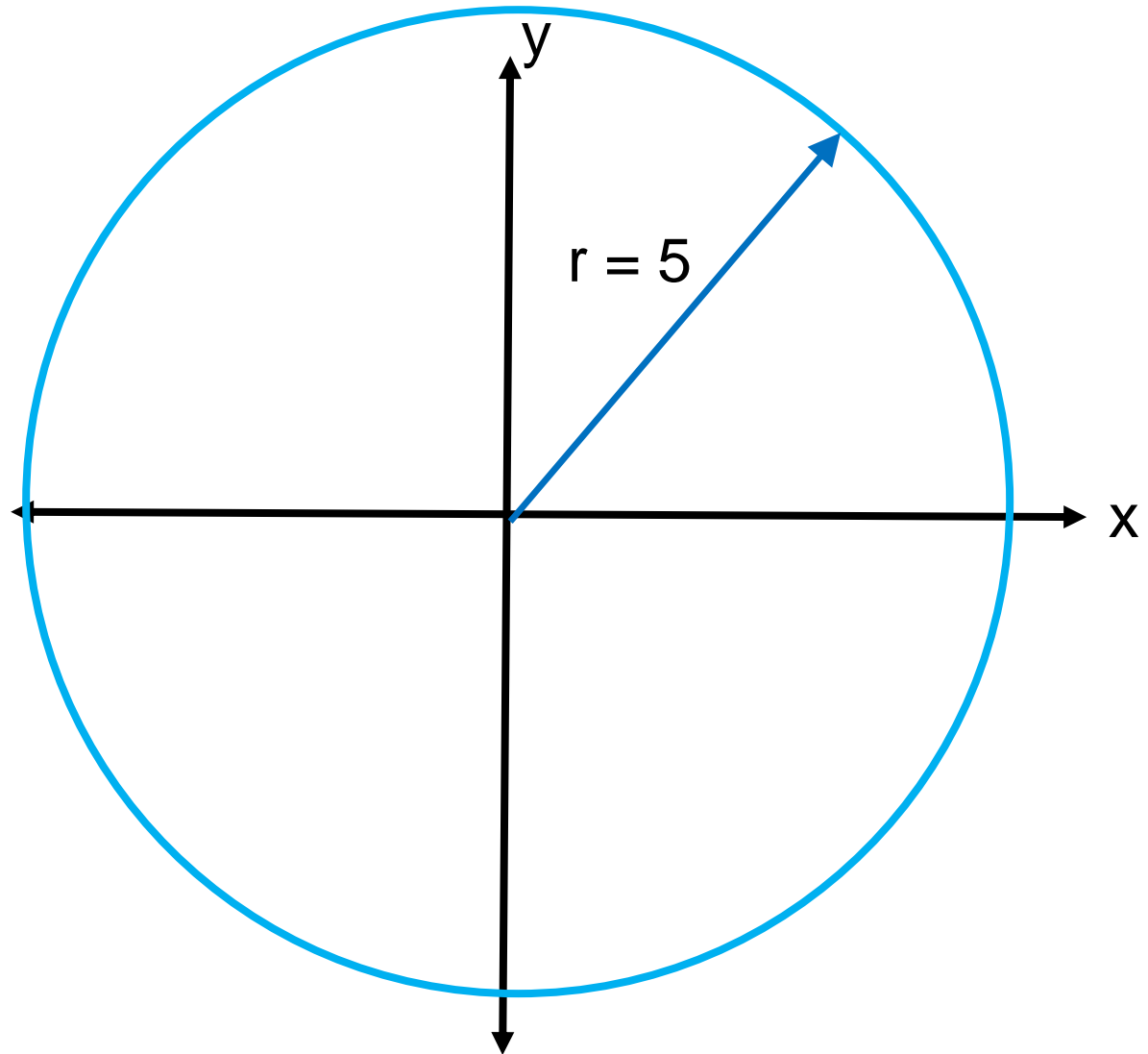


This is the equation of a circle centered
at the origin whose radius is 5.

$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = 5^2$$

$$x^2 + y^2 = 25$$



What is the equation of this circle?

$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = 3^2$$

$$x^2 + y^2 = 9$$

What is the y-value
when $x = 3$?

$$(3)^2 + y^2 = 9$$

$$9 + y^2 = 9$$

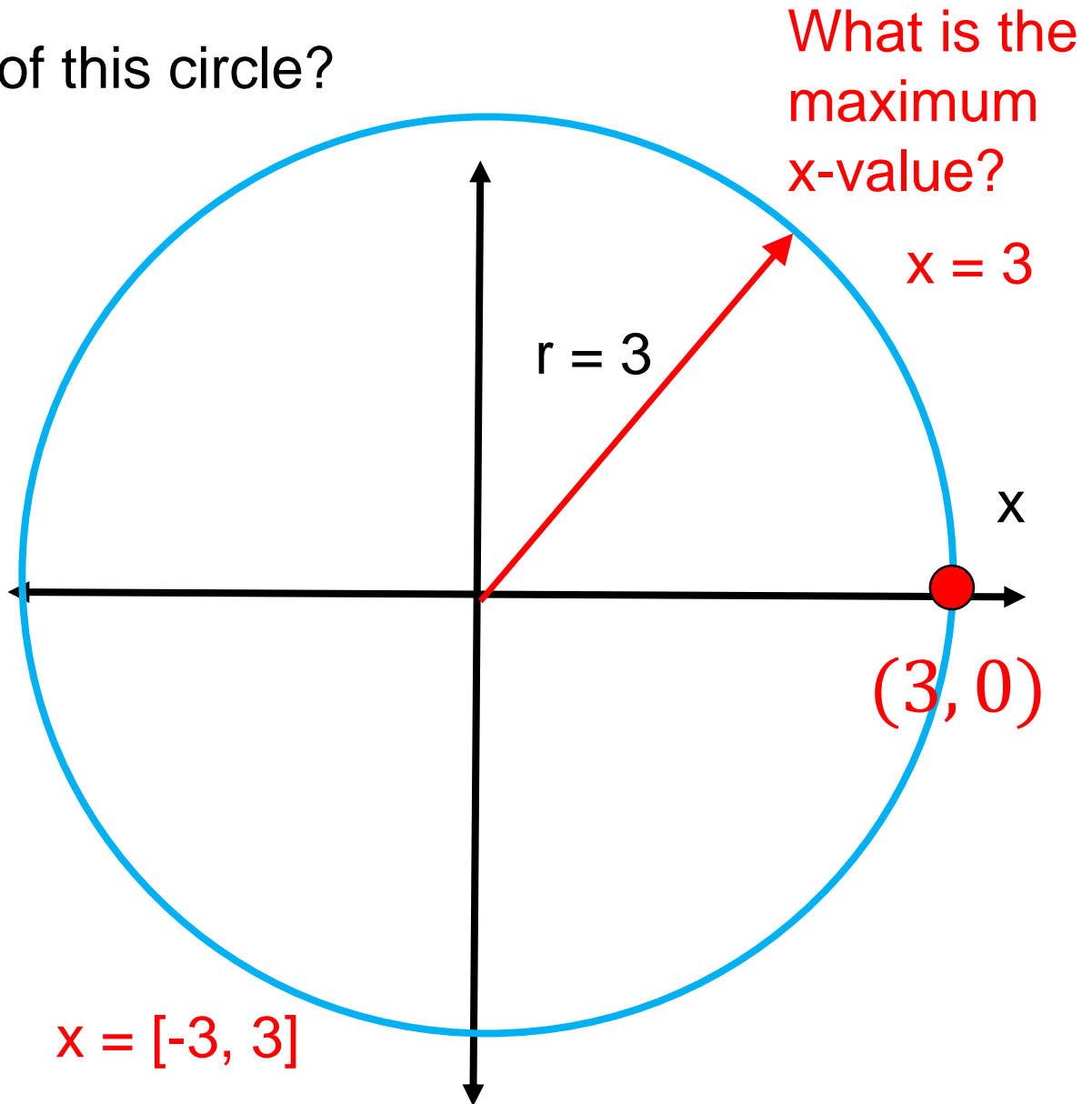
$$y = 0$$

What is the domain?

$$x = [-3, 3]$$

What is the range?

$$y = [-3, 3]$$



$$x^2 + y^2 = 9$$

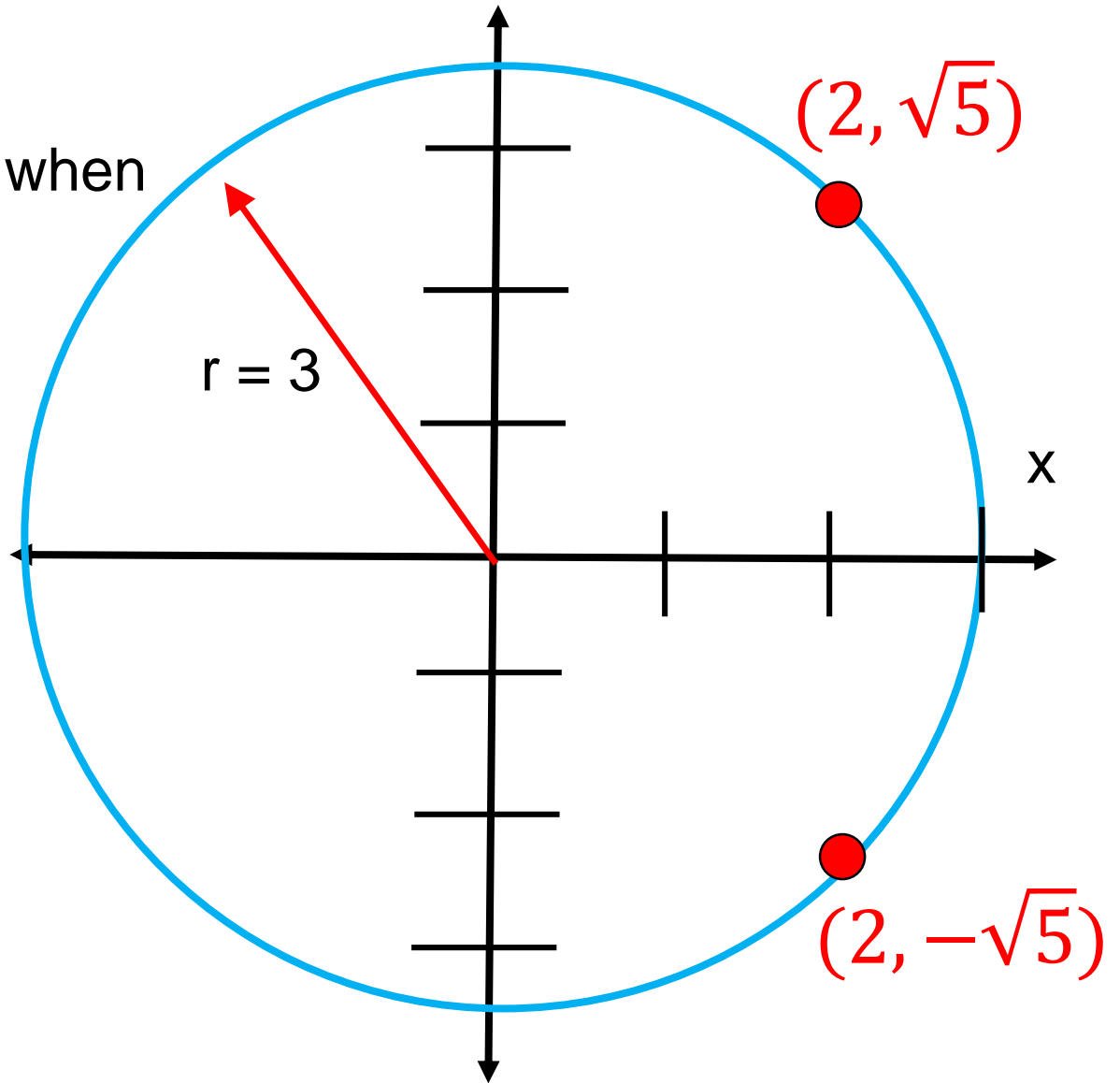
What are the y-values when
 $x = 2$?

$$(2)^2 + y^2 = 9$$

$$4 + y^2 = 9$$

$$y^2 = 5$$

$$y = \pm\sqrt{5}$$



How an Equation Makes a Circle.

$$x^2 + y^2 = 25$$

How do you graph this on your calculator??!!!

$$y^2 = 25 - x^2$$

Your turn, solve for 'y'

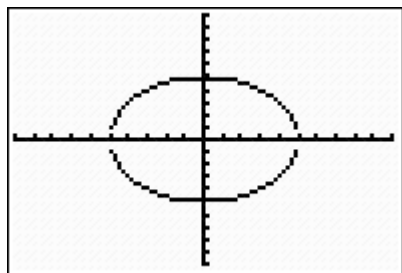
$$y = \sqrt{25 - x^2}$$

Graph this equation on your calculator.

Why do you only get the top half?

$$y = -\sqrt{25 - x^2}$$

Also Graph this equation on your calculator.



Why is the circle “squished”?

The physical width of the calculator window is wider than the calculator window height so you can't use the same “window” numbers for both.

To reflect the physical dimensions of the calculator window, use “Zoom Square”.

Now we just plug in number to both equations.

$$x^2 + y^2 = 4$$

$$y = \sqrt{4 - x^2}$$

$$y = -\sqrt{4 - x^2}$$

$$y = -\sqrt{4 - 0^2} = -2$$

$$y = \sqrt{4 - 0^2} = 2$$

$$y = \sqrt{4 - 1^2} \approx 1.7$$

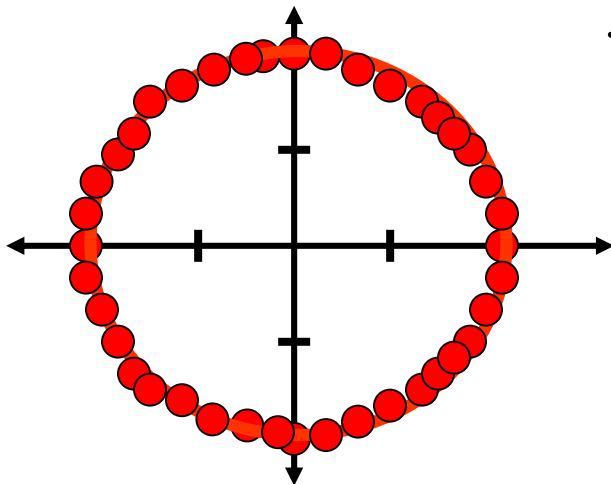
$$y = -\sqrt{4 - 1^2} \approx -1.7$$

Each input will give
a “+” and a “-” output.

$$y = \sqrt{4 - \frac{1}{3}^2} \approx 1.97$$

$$y = -\sqrt{4 - \frac{1}{3}^2} \approx -1.97$$

x	y
0	-2
0	2
1	$\sqrt{3} \approx 1.7$
1	$-\sqrt{3} \approx -1.7$
$\frac{1}{3}$	≈ 1.97
$\frac{1}{3}$	≈ -1.97
2	0
-2	0



More inputs
result in more
outputs.

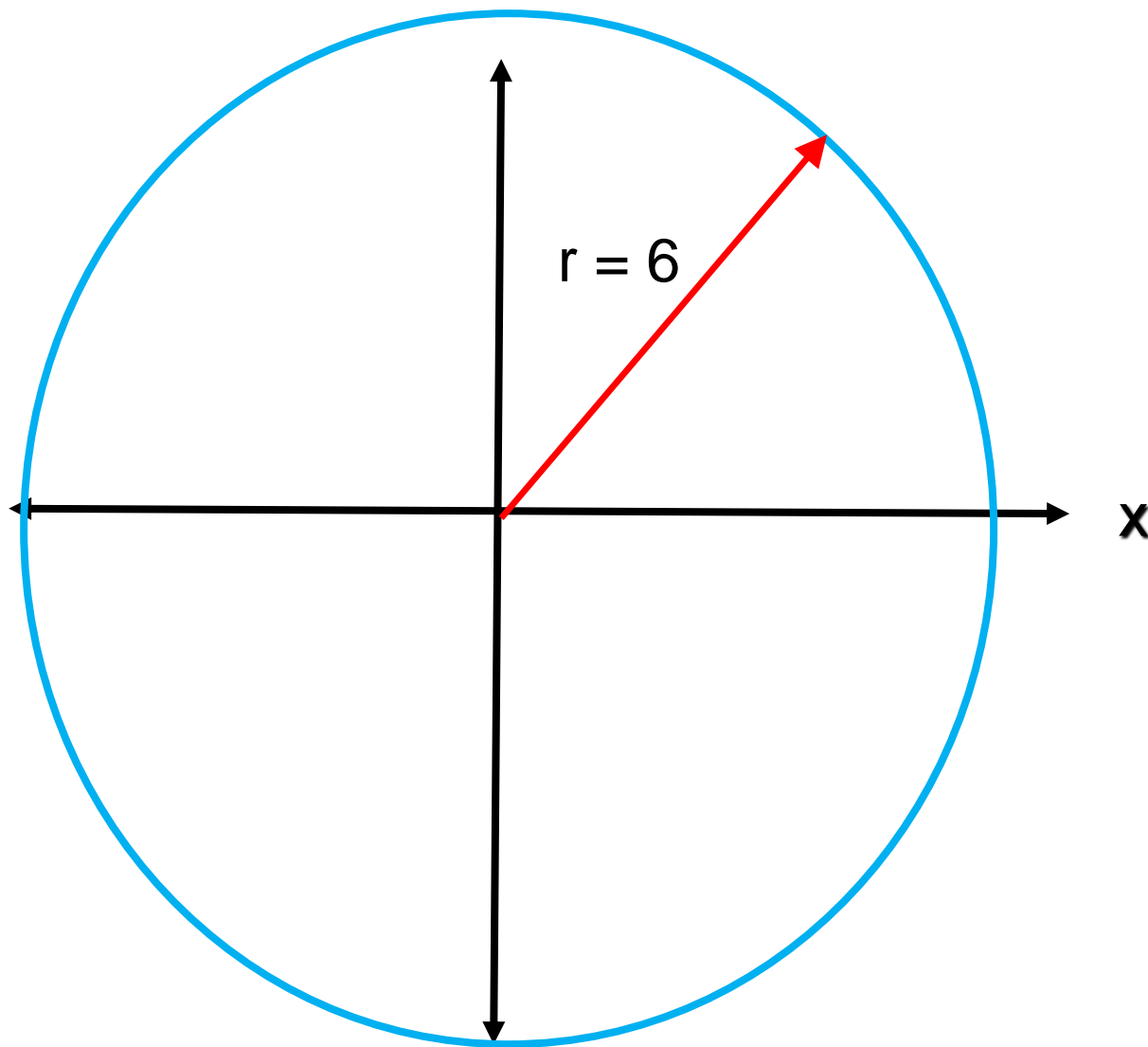
(except for the
input 2 and -2)

What is the equation of the circle?

$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = 6^2$$

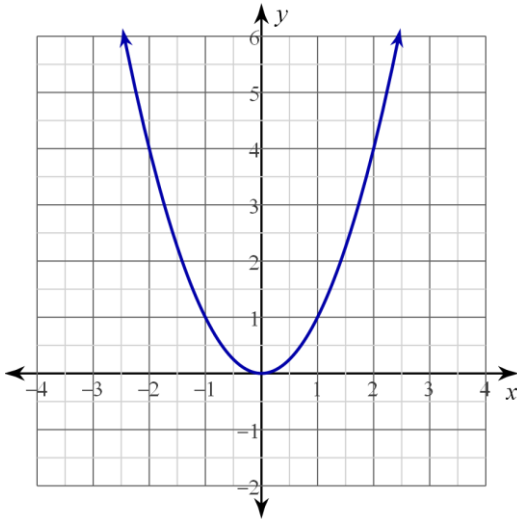
$$x^2 + y^2 = 36$$



Graphical Transformations

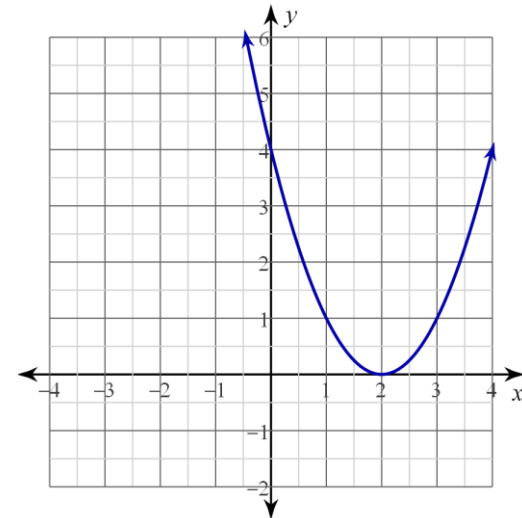
Parent Function: The simplest function in a family of functions (lines, parabolas, cubic functions, etc.)

$$f(x) = x^2$$



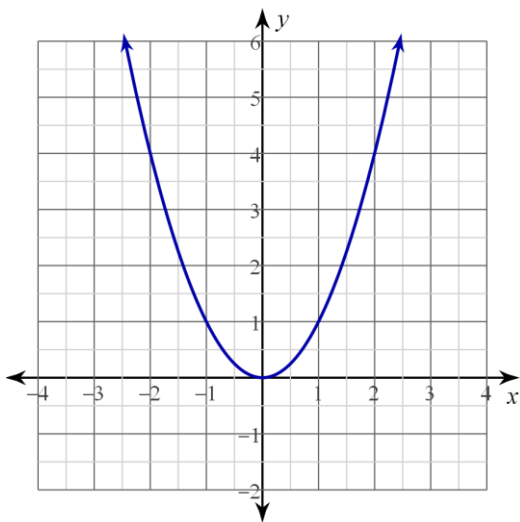
How is $g(x)$ a transformation of $f(x)$?

$$g(x) = (x - 2)^2$$



Replacing 'x' with 'x - 2' translates the parent function right by 2.

$$f(x) = x^2$$



How is $h(x)$ a transformation of $f(x)$?

Up 2

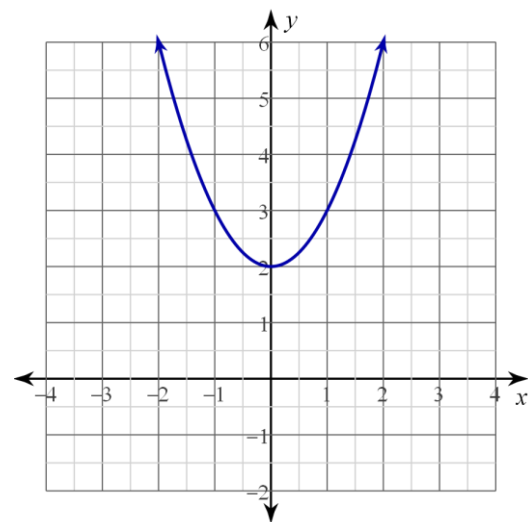
Notice: Subtract 2 from both sides yields:

$$y - 2 = x^2$$

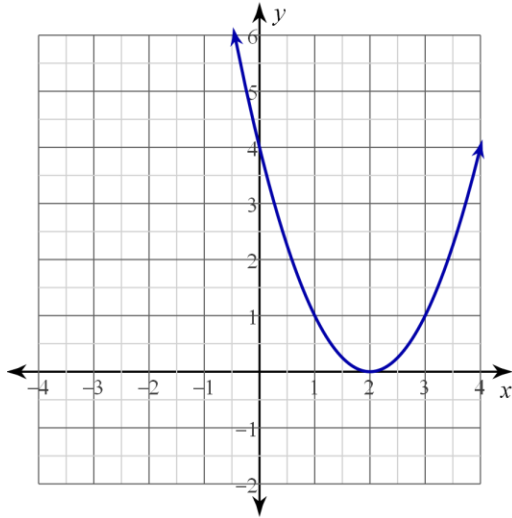
$$(y - 2) = x^2$$

Replace “y” in the parent function with $(y - 2)$ moves the graph up 2.

$$h(x) = x^2 + 2$$

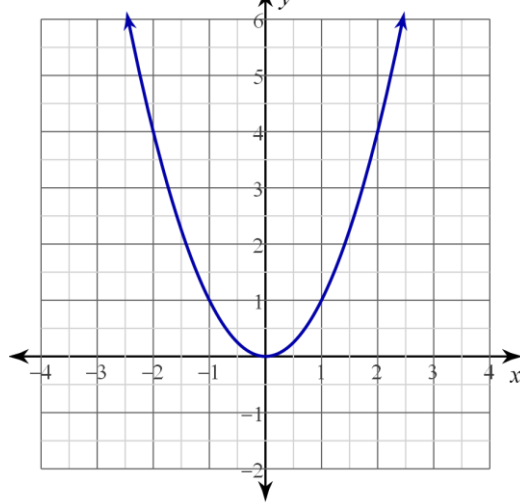


$$y = (x - 2)^2$$

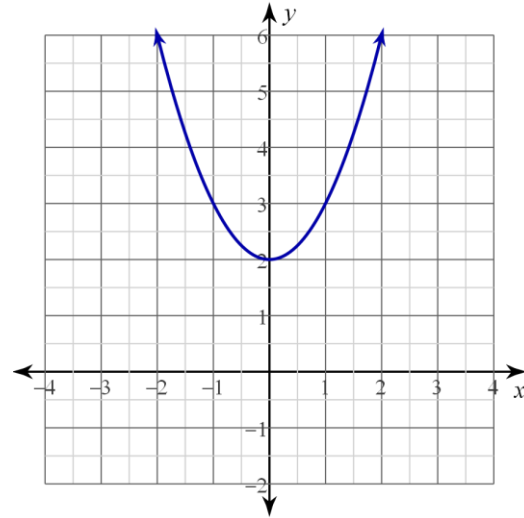


Replacing 'x' with $(x - 2)$ moves the parent function right by 2.

$$y = x^2$$



$$(y - 2) = x^2$$



Replace "y" with $(y - 2)$ moves the parent function up by 2.

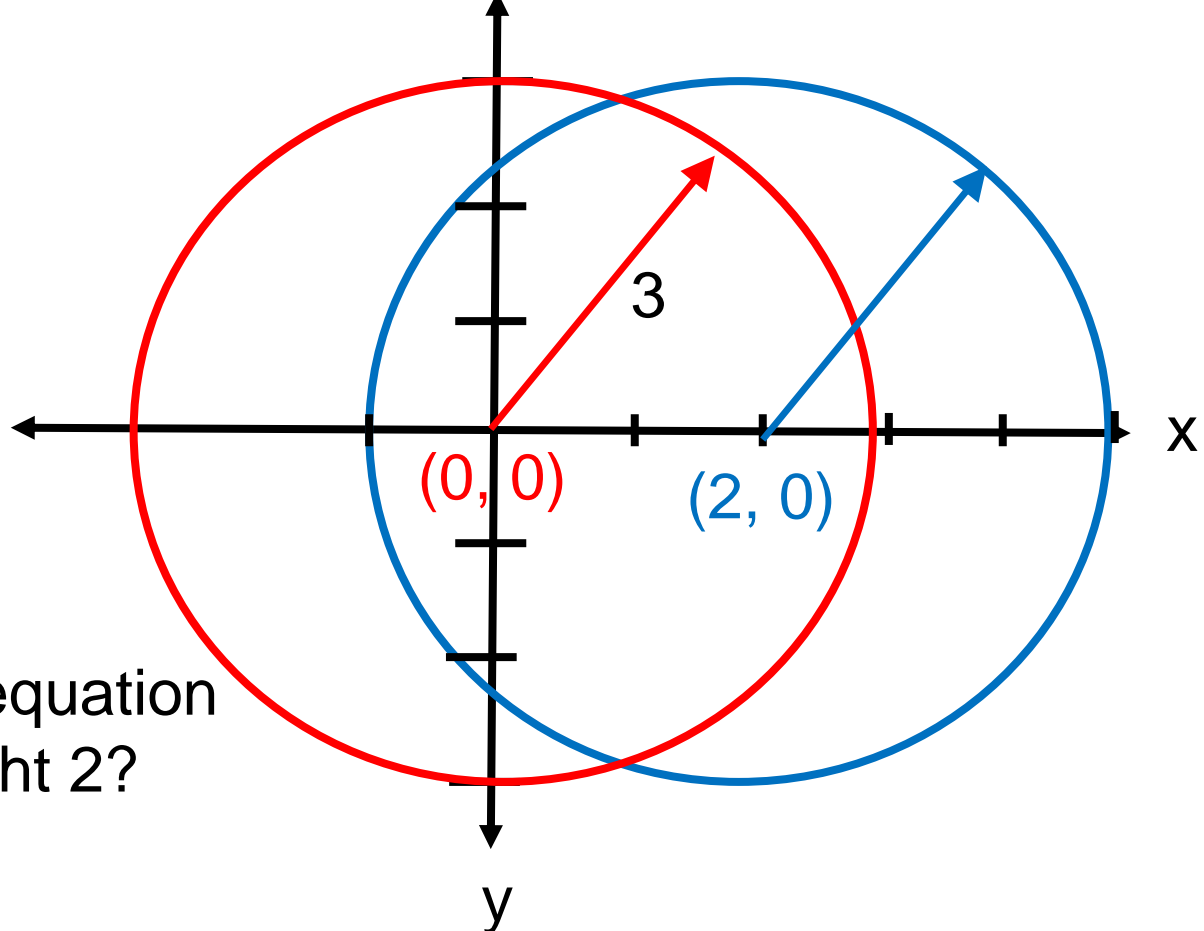
$$x^2 + y^2 = 9$$

What is the domain?

$$x = [-3, 3]$$

What is the range?

$$y = [-3, 3]$$



How do we change the equation to translate the graph right 2?

Replace 'x' with $(x - 2)$

$$(x - 2)^2 + y^2 = 9$$

What is the domain?

$$x = [-1, 5]$$

What is the range?

$$y = [-3, 3]$$

How do we change the equation to translate the graph down 2?

$$x^2 + y^2 = 9$$

Replace 'y' with $(y + 2)$

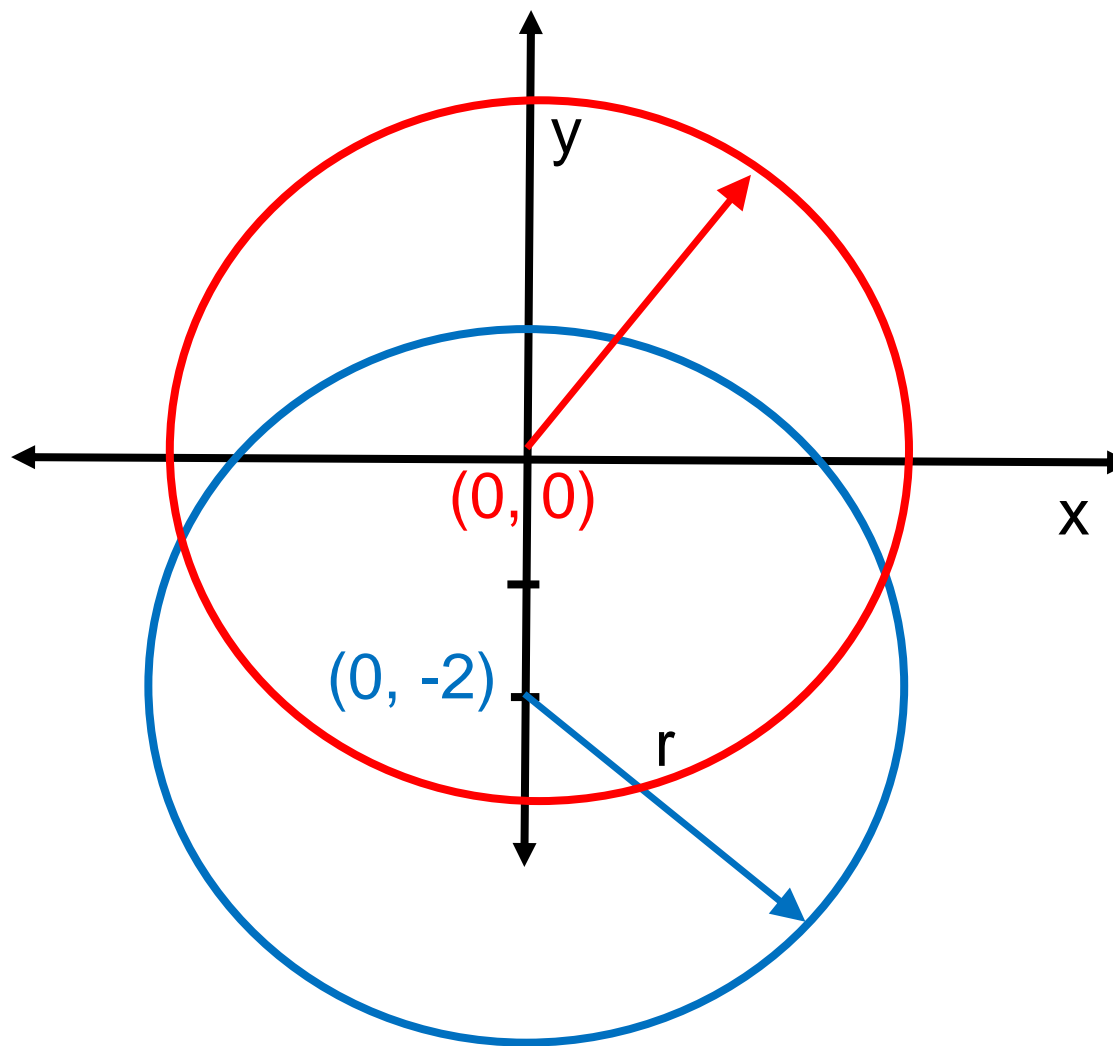
$$x^2 + (y + 2)^2 = 9$$

What is the domain?

$$x = [-3, 3]$$

What is the range?

$$y = [-5, 1]$$



What is the radius and center of the circle?

$x^2 + y^2 = 25$ radius is 5 No left/right or up/down shift
→ center is (0, 0).

$(x + 3)^2 + y^2 = 25$ Left 3 shift → center is (-3, 0)

$(x - 5)^2 + (y + 2)^2 = 25$ Right 5, down 2 shift
→ center is (5, -2)

$(x - 7)^2 + y^2 = 49$ radius is 7
Right 7 shift → center is (7, 0)

$(x + 3)^2 + (y - 4)^2 = 5$ *radius* = $\sqrt{5}$
Left 3, up 4 shift → center is (-3, 4)

Equations of Circles

$$r^2 = (x - h)^2 + (y - k)^2$$

'r' is the radius
of the circle.

(h, k) is the center
of the circle.

$$9 = (x - 3)^2 + (y + 4)^2$$

'r' = ? (h, k) = ?

r = 3 center is (3, -4)

$$3 = (x + 2)^2 + (y - 1)^2$$

'r' = ? (h, k) = ?

r = $\sqrt{3}$ center is (-2, 1)

Equations of Circles

$$r^2 = (x - h)^2 + (y - k)^2$$

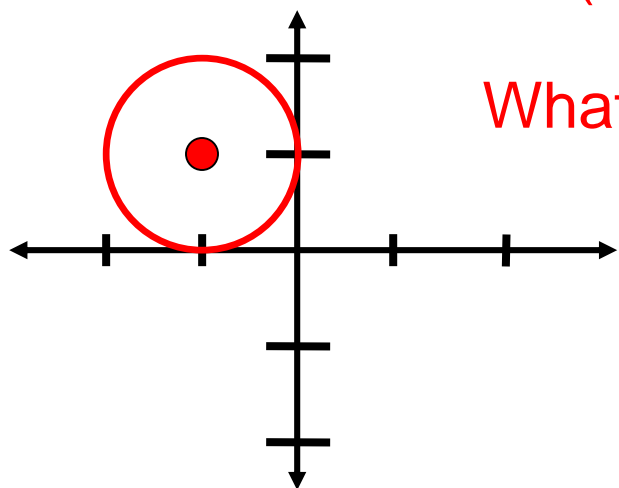
'r' is the radius of the circle.

(h, k) is the center of the circle.

$$'r' = ? \quad 'r' = 1$$

$$(h, k) = ? \quad (h, k) = (-1, 1)$$

What is the equation of the circle?



$$1^2 = (x - (-1))^2 + (y - 1)^2$$

$$1 = (x + 1)^2 + (y - 1)^2$$

Write the equation of a circle centered at (0, 3) with a radius of 4.

$$x^2 + (y - 3)^2 = 16$$

Write the equation of a circle centered at (3, -2) with a radius of 6.

$$(x - 3)^2 + (y + 2)^2 = 36$$

Write the equation of a circle centered at (-2, -4) with a radius of $\sqrt{5}$.

$$(x + 2)^2 + (y + 4)^2 = 5$$

$$(x - 5)^2 + (y - 7)^2 = 25$$

Is the point $(9, 4)$ on the circle?

Plug in $x = 9$, $y = 4$

$$(9 - 5)^2 + (4 - 7)^2 = 25$$

$$(4)^2 + (-3)^2 = 25$$

$$16 + 9 = 25$$

Yes $(9, 4)$ is on the circle.