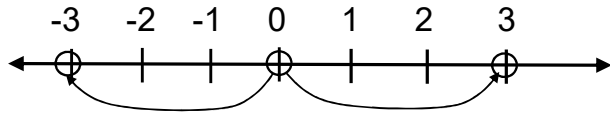Math -2: Lesson 1-6 (Absolute Value Equations)

-3   -2   -1   0   1   2   3

3 And -3 are the same distance from zero.

-3 is the "opposite" of 3

Absolute Value of a number: $|x|$ The distance the number is from zero on the number line.

$$|3| = 3 \qquad |-3| = 3$$

Absolute Value of a number: $|x|$ The distance the number is from zero on the number line.

$$|x| = 3 \quad \rightarrow \quad |3| = 3 \qquad |-3| = 3 \quad \rightarrow \quad x = 3, -3$$

Means: "what numbers are a distance of _three units_ from zero on the number line?"
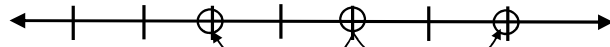
What is the solution to the equation?

$$|x| = -5$$ Means: "what numbers are a distance of _negative five_ units from zero on the number line?"

What is the solution?

Is distance ever negative?
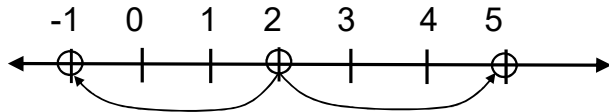
$$|x| = -5$$ Has no solution.

What two numbers are 2 units away from the number 1 on the number line?   -3   -2   -1   0   1   2   3

-1 And 3 are 2 units away from 1 (the same distance).

-1 and 3 are on opposite sides of 1

What two numbers are 3 units away from the number 2 on the number line?

-1   0   1   2   3   4   5

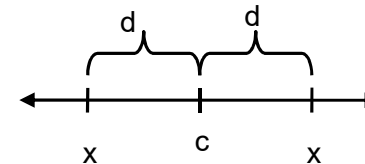-1 And 5 are 3 units away from 2 (the same distance).

-1 and 5 are on opposite sides of 2

Absolute Value: $|x - c| = d$ The number "d" is the distance between "x' and "c" on the number line.

$$|x - (c)| = d$$

"c" is the "center number" and
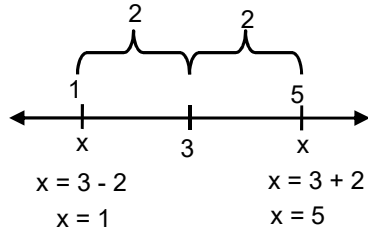"d" is the distance from the center number.

d   d

x   c   x

1

Panel 1:

$|x - 3| = 2$

(English): What numbers are exactly 2 units from the center number "3"?

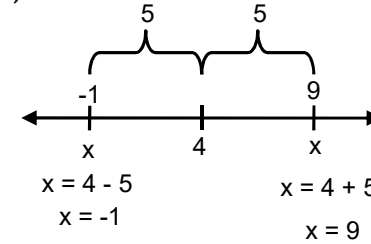$x = 3 \pm 2$

$|x - (+3)| = 2$

$x = 1, 5$

x = 3 - 2
x = 1

x = 3 + 2
x = 5

Panel 2:

$|x - 4| = 5$

(English): What numbers are exactly 5 units from the center number "4"?

$x = 4 \pm 5$
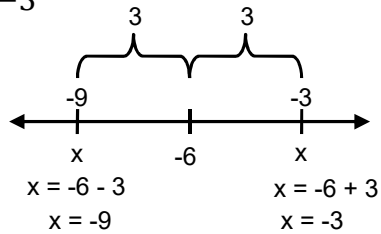
$|x - (+4)| = 5$

$x = -1, 9$

x = 4 - 5
x = -1

x = 4 + 5
x = 9

Panel 3:

$|x + 6| = 3$

(English): What numbers are exactly 3 units from the center number "-6"?

$x = -6 \pm 3$

$|x - (-6)| = 3$

$x = -9, -3$

x = -6 - 3
x = -9

x = -6 + 3
x = -3

Panel 4:

Solve the equations. Draw a picture if necessary.

$|x + 1| = 3$

$|x - 4| = 5$

$|x - 5| = 1$

Another way to think about it

$$|-1| = 1 \qquad |+1| = 1$$

$$|x-5| = 1 \qquad |x-5| = 1$$

$$x - 5 = -1 \qquad x - 5 = 1$$

$$x = 4 \qquad x = 6$$

Another way to think about it.

$$|x+4| = 5$$

$$|-5| = 5 \qquad |5| = 5$$

$$x + 4 = -5 \qquad x + 4 = 5$$
$$x = -9 \qquad x = 1$$

For some problems, this is a better way to think about it.

Solve algebraically

$$|2x-1| = 5$$

$$|-5| = 5 \qquad |5| = 5$$

$$2x-1 = -5 \qquad 2x-1 = 5$$
$$+1 \quad +1 \qquad +1 \quad +1$$
$$2x = -4 \qquad 2x = 6$$
$$\div 2 \quad \div 2 \qquad \div 2 \quad \div 2$$
$$x = -2 \qquad x = 3$$

Solve $|x-10| = -4$

This distance between 'x' and '10' is <u>negative</u> 4.

Distances are NOT negative.

The absolute value <u>always</u> equals a <u>positive number.</u>

<u>No solution!!!!</u>