Math-1010

Lesson 2-7 (textbook 5.8)

Radical Equations

Mathematical Modeling: representing a real-world phenomenon or quantity with an equation or inequality.

"Height as a function of time" $h(t) = -16t^2 + v_0 t + h_0$

v_0 Initial upward (or downward) velocity

What is the initial velocity of something you drop? $v_0 = 0$

$$h(t) = -16t^2 + h_0$$

What is the height of ground level? $h(t) = 0$

$$0 = -16t^2 + h_0$$

Solve for "t". $16t^2 = h_0$ $t^2 = \frac{h_0}{16}$ $t = \sqrt{\frac{h_0}{16}}$

$$t = \frac{\sqrt{h_0}}{4}$$

$$t = \frac{\sqrt{d}}{4}$$

Example Problem: How long does it take for a dropped object to fall 100 feet?

$$t = \frac{\sqrt{d}}{4}$$

$$t = \frac{\sqrt{100}}{4} = \frac{10}{4} = 2.5\sec$$

Example Problem: How long does it take for a dropped object to fall 50 feet?

$$t = \frac{\sqrt{50}}{4} = 1.77\sec$$

Question: Why doesn't the object take twice as long take twice as long to fall 100 feet as it does to fall 50 feet?

The time to fall a certain distance.

Which is it: t = f(d) or d = f(t) ?

What is the input variable? …the output variable?

$$t = \frac{\sqrt{d}}{4}$$

Build a table of values for the following distances.

$$\mathrm{d} = \{0,\ 100,\ 200,\ 300,\ 500,\ 750,\ 1000\}$$

Graph your results. Properly label your axes.

Radical equation: an equation that contains a variable in the radicand. $\sqrt{2x+1}=5$

To Solve:

a) Single unknown equation: to find the unknown value that makes the equation "true".

$$\sqrt{2x+1}=5$$

$$\left(\sqrt{2x+1}\right)^2=5^2$$

$$2x+1=25$$

-1 -1

$$2x=24$$

÷2 ÷2

$$x=12$$

To Solve:

a) Two variable equation: to "isolate" the specified variable on one side of the equation.

Solve for x $y-\sqrt{2x+1}=5$

$$y-5=\sqrt{2x+1}$$

$$(y-5)^2=\left(\sqrt{2x+1}\right)^2$$

$$y^2-10y+25=2x+1$$

$$y^2-10y+24=2x$$

$$\frac{1}{2}y^2-5y+12=x$$

There are several versions of radical equations, two of which are:

1) Single radical term

$$6+\sqrt{3x+2}=11$$

$$\sqrt{3x+2}=5$$

$$\left(\sqrt{3x+2}\right)^2=5^2$$

$$3x+2=25$$

-2 -2

$$3x=23$$

÷3 ÷3

$$x=\frac{23}{3}$$

2) Two radical terms that can be set equal to each other

$$2\sqrt{3x}-\sqrt{5x+7}=0$$

$$2\sqrt{3x}=\sqrt{5x+7}$$

$$\left(2\sqrt{3x}\right)^2=\left(\sqrt{5x+7}\right)^2$$

$$4*3x=5x+7$$

$$12x=5x+7$$

$$7x=7$$

$$x=1$$

A beach ball and a marble are dropped at the same time from 250 feet. Which one will take longer to fall to the ground? Why?

Assume the marble has no air resistance. Find the time it takes to fall 250 feet.

$$t = \frac{\sqrt{d}}{4} \qquad t = 0.25\sqrt{250} \qquad t = 3.95\,\text{sec}$$

The coefficient 0.25 models the effect of the acceleration of gravity with <u>no air resistance.</u>

We can find the coefficient "k" that models the effect of gravity and the air resistance acting on the falling beach ball experimentally.

$$t = k\sqrt{250}$$

t = 4.11 sec.

Find the value of 'k'.

$$(4.11)^2 = k^2(250) \qquad \frac{(4.11)^2}{250} = k^2$$

$$0.0676 = k^2$$

$$0.26 = k \qquad t_{beachball} = 0.26\sqrt{d}$$

In general, if we drop the beach ball from a height that is 50 feet lower than the height we drop the marble, the beach ball equation becomes: $t_{beachball} = 0.26\sqrt{d-50}$

In general the marble equation remains: $t_{marble} = 0.25\sqrt{250}$

Find the height we should drop the marble from so that the time for the marble to reach the ground is the same as the beachball (which is dropped from a height that is 50 feet lower).

$$0.25\sqrt{d} = t_{marble} = t_{beachball} = 0.26\sqrt{d-50}$$

$$0.25\sqrt{d} = 0.26\sqrt{d-50}$$

$$\left(0.25\sqrt{d}\right)^2 = \left(0.26\sqrt{d-50}\right)^2$$

$$0.0625d = 0.0676(d-50)$$

$$0.0625d - 0.0676d = -3.38$$

$$-0.0051d = -3.38$$

$$d = 662.75\,\text{ft}$$

Solve:

Check your solution.

$$\sqrt{x+3}+5=0$$

$$\sqrt{22+3}+5=0$$

$$\sqrt{x+3}=-5$$

$$\sqrt{25}+5=0$$

$$x+3=25$$

$$5+5\neq 0$$

Extraneous solution.

$$x=22$$

$$\sqrt{2-x}=-x$$

$$x=-2,\ 1$$

Check your solutions.

$$2-x=(-x)^2$$

$$\sqrt{2-(-2)}=-(-2)$$

$$2-x=x^2$$

$$\sqrt{4}=2$$

Checks.

$$0=x^2+x-2$$

$$0=(x+2)(x-1)$$

$$\sqrt{2-(1)}=-(1)$$

$$\sqrt{1}\neq -1$$

Extraneous solution.

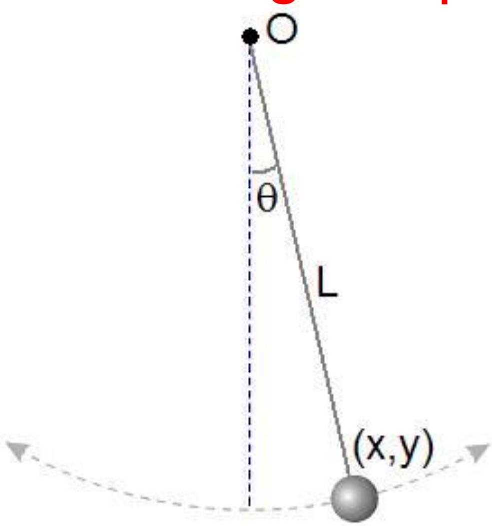
$$x=-2$$

$$x\neq 1$$

The time (in seconds) that it takes a pendulum to complete one cycle (called the ***period of the pendulum***) is given by:

$$t = 2\pi\sqrt{\frac{L}{32}}$$ where *'L'* is the length of the pendulum.

How long is a pendulum that has a period of 1.95 seconds?



$$1.95 = 2\pi\sqrt{\frac{L}{32}}$$

$$\frac{1.95}{2\pi} = \sqrt{\frac{L}{32}}$$

$$\left(\frac{1.95}{2\pi}\right)^2 = \left(\sqrt{\frac{L}{32}}\right)^2$$

$$\left(\frac{1.95}{2\pi}\right)^2 = \frac{L}{32}$$

$$L = 32\left(\frac{1.95}{2\pi}\right)^2$$

$$L \approx 3.08\, ft$$

In a certain population, there are 28,520 births on a particular day. The number ***N,*** of those surviving to age ***'x'*** can be modeled by the function: $N = 2850\sqrt{100 - x}$

a) How many will survive until age 5? $N = 2850\sqrt{100 - 5}$

$\boxed{N = 27{,}778}$

b) What is the practical domain of this function?

$x = [0, 112 - ish)$

c) When only 5000 of these people are still alive, how old are they? $5000 = 2850\sqrt{100 - x}$

$1.7544 = \sqrt{100 - x}$ $(1.7544)^2 = 100 - x$ $\boxed{x = 97}$

d) How many years will pass until half of the 28,520 people are still alive? $14260 = 2850\sqrt{100 - x}$

The velocity of wind can be measured using a pressure gauge whose units of measure are ***pounds force per square foot.*** The relationship between wind velocity (in miles per hour) and pressure is given by:

$$V = \sqrt{\frac{1000P}{3}}$$

a) What is the wind speed if the pressure gage reads 10 psf?

b) What is the wind pressure if the wind is blowing at 70 mph?

c) Rewrite the formula as pressure as a function of wind velocity.