Math 1010
Lesson 1-8
(Textbook Section 1.15)

How Long Can You Live?
(Simple and Compound Inequalities)

REVIEW

"Math Statement" (like a sentence in English)

A meaningful assertion that is either true or false.

Two types of "statements"

(1) an equation $x + 3 = 5$

(2) an inequality $2x - 1 > 7$

Equivalent Equations: Two different equations that have the same solution (value of 'x' that makes the equation "true."

$x + 2 = 4$

$-2 \quad -2$

$x = 2$

(subtraction property of equality)

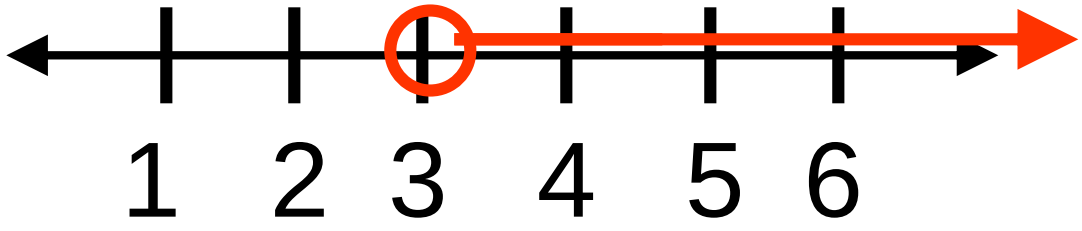
New

Equivalent Inequalities: Two different inequalities that have the same solution.

Simple Inequality: has one letter (variable) and one inequality symbol

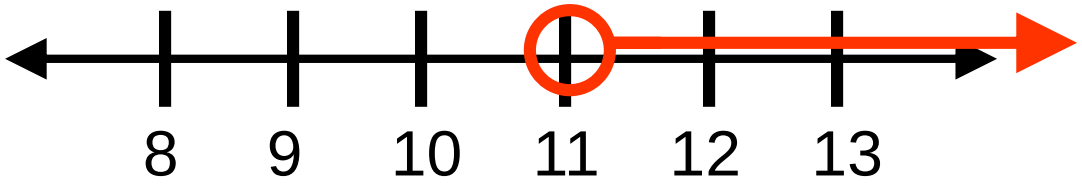
$x > 3$ $3 < x$

All numbers that are greater (bigger) than 3



$x > 11$ “all the numbers that are greater than 11”

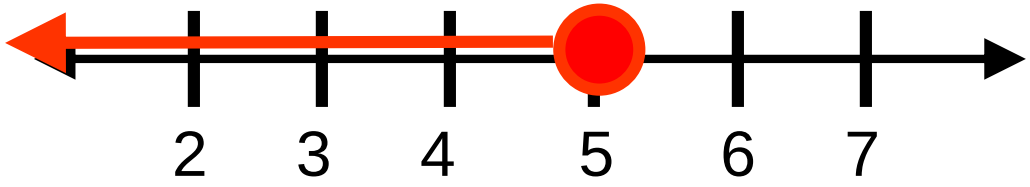
Number line equivalent: (shade all #’s that are solutions)



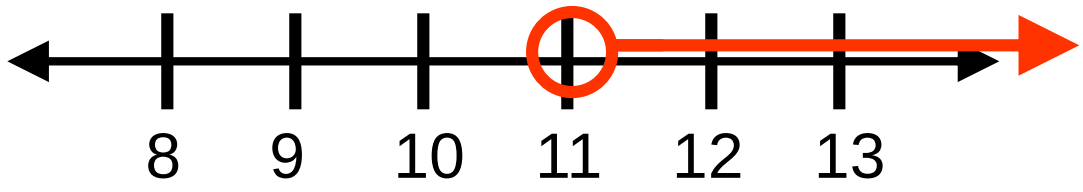
What does the following inequality mean?

$x \leq 5$ “all the number that are less than or equal to 5”

What is the number line equivalent of $x \leq 5$?

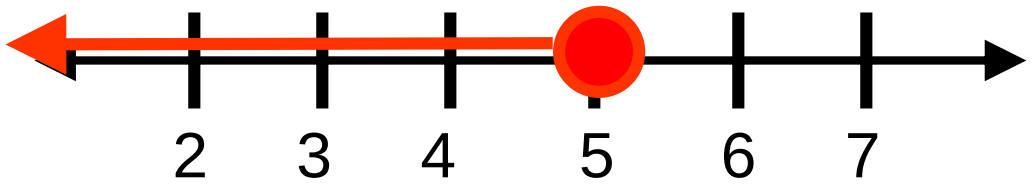


$x > 11$



$x = 11$ is NOT a solution. We DO NOT shade 11.

$x \leq 5$



$x = 5$ IS a solution. We shade 5.

Your turn: Start with the following:

$5 > 3$ True or false?

+2 +2 Add 2 left and right (of the “>” symbol) and rewrite

$7 > 5$ True or false?

This will always work for addition.

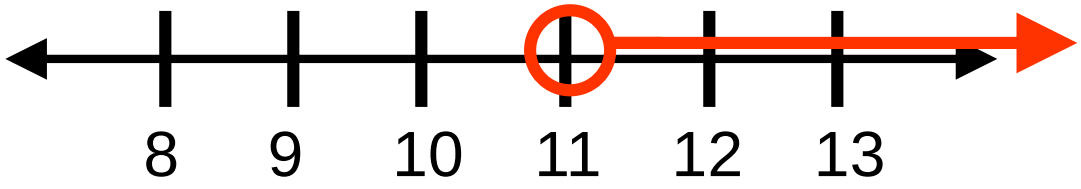
<u>Addition Property of Inequality</u>: adding the same number left and right of the “<, >, ≤, or ≥” symbol will result in an equivalent inequality.

$x - 4 > 7$

+4 +4

$x > 11$

Number line:



Your turn: Start with the following:

$2 < 6$ True or false?

$-1 \quad -1$ Subtract "1" left and right (of the "<" symbol) and rewrite

$1 < 5$ True or false?

This will always work for subtraction.

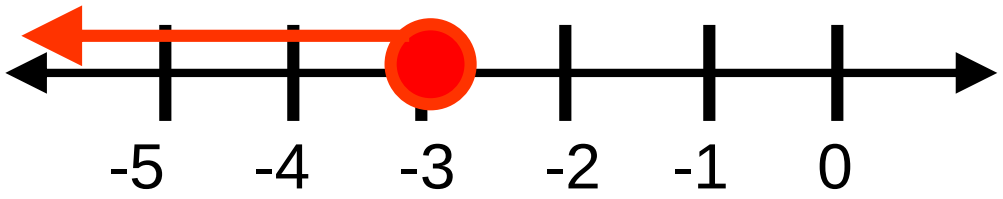
Subtraction Property of Inequality: subtracting the same number left and right of the "<, >, ≤, or ≥" symbol will result in an equivalent inequality.

$x + 5 \leq 2$

$-5 \quad -5$

$x \leq -3$

Number line:



Your turn: Start with the following:

$3 < 5$ True or false?

*2 *2 Multiply by “2” left and right (of the “<” symbol), rewrite

$6 < 10$ True or false?

This works for multiplication of positive numbers, BUT…..

$6 < 8$ True or false?

-1 -1 Multiply by "-1" left and right (of the "<" symbol), rewrite

$-6 < -8$ True or false? $-6 > -8$

This will work for multiplication of negative numbers if we reverse the direction of the inequality.

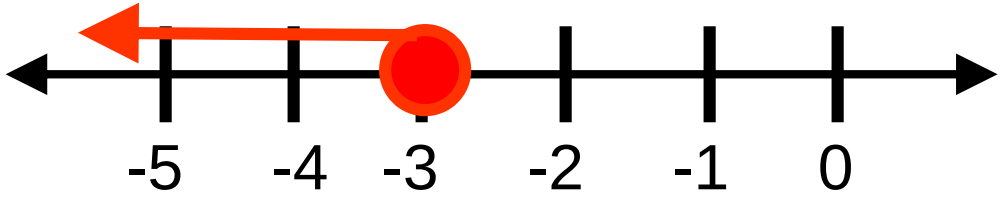
Multiplication Property of Inequality: multiplying the same positive number left and right of the "<, >, ≤, or ≥" symbol will result in an equivalent inequality. We reverse the direction of <, >, ≥, and ≤ *if the factor is negative*.

$-2x \geq 6$

$\div(-2)$ $\div(-2)$

$x \leq -3$

Number line:



Solving inequalities requiring the Distributive Property

Review:

(Distributive Property of Multiplication "over" Addition

$2(x + 4) \rightarrow \underline{2x + 2(4)} \rightarrow \underline{2x + 8}$

$2(x - 4) < 4x + 6$

$2x - 8 < 4x + 6$

$-2x \qquad -2x$

$-8 < 2x + 6$

$-6 \qquad -6$

$-14 < 2x$

$(\div 2) \qquad (\div 2)$

$-14 < 2x$

$-7 < x$

$x > -7$

Your turn: Solve the inequalities (one step-rewrite)

$2x + 2 \leq 6$

$2(x - 3) \geq 8$

$-14 < -5x + 6$

Draw the equivalent number line for each solution.

Compound Inequality: the result of combining two simple inequalities with the logical words “and” or “OR”.

$x \leq -3$ or $x > 2$

$x > 5$ and $x < 7$

If you are wearing a red shirt OR if you are wearing blue jeans, you will be awarded $100. Which of the girls below will get $100?.

Who gets $100?

Logical Word “OR:” two or more required conditions are given. If either of the conditions is met then the statement is true.

"OR" type compound inequalities.

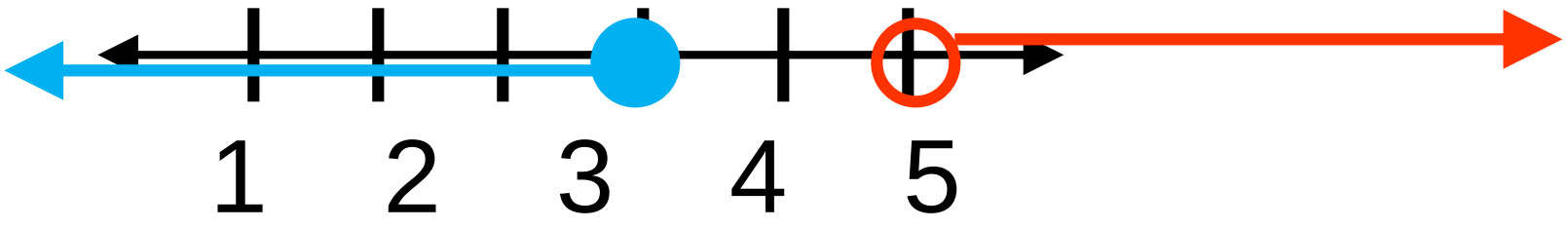
$x \leq 3$ or $x > 5$

Is -2 a solution to the compound inequality?

Or means: the numbers that satisfy either condition

Graph $x \leq 3$?

Graph $x > 5$?



Hint: inequality with "OR" looks like: ← →

Solve and graph the compound inequality:

Solve each simple inequality separately.

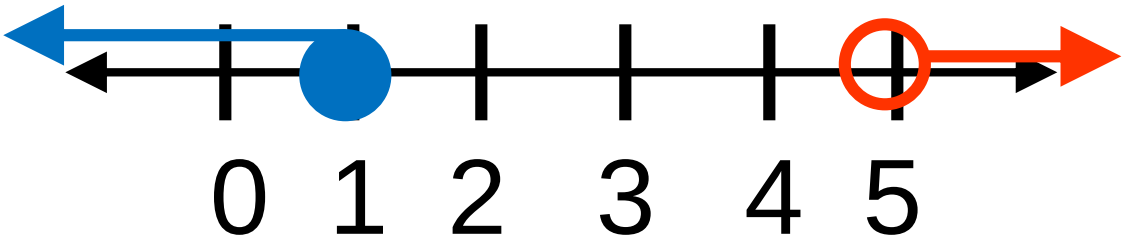
$2x + 3 \leq 5$ or $x - 3 > 2$

$-3 \quad -3 \qquad +3 \quad +3$

$2x \leq 2$ or $x > 5$

$\div 2 \quad \div 2$

$x \leq 1$ or $x > 5$



If you are under the age of 15 AND are walking a dog, then you are pretty cool.

Which picture shows a person(s) who is(are) “pretty cool?”

Logical Word “AND:” two or more required conditions are given. If BOTH of the conditions are met then the statement is true.

"AND" type compound inequalities.

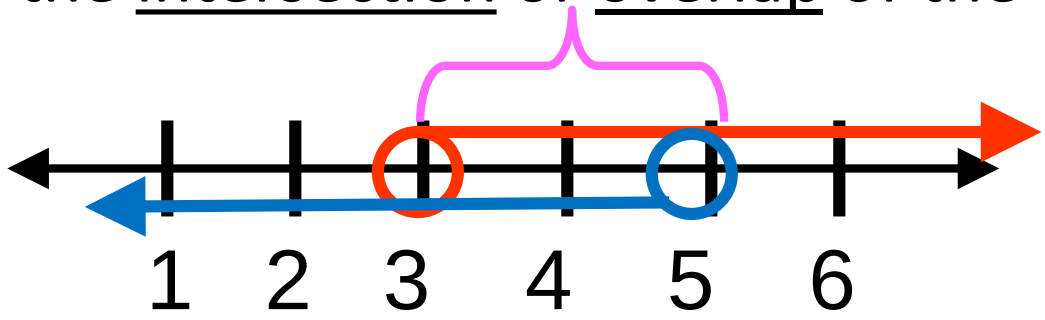
$x > 3$ and $x < 5$

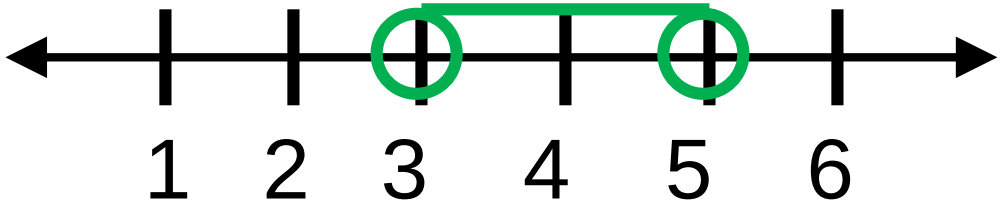
Is -2 a solution to the compound inequality?

And means both conditions must be met

What part is $x > 3$? What part is $x < 5$?

What is the intersection or overlap of the two?



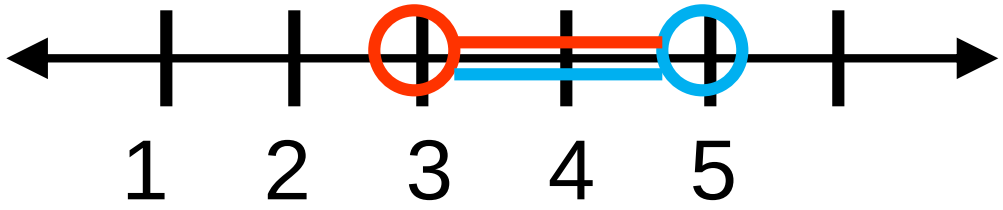


Vocabulary

Compound inequality $x > 3$ and $x < 5$

Hint: This can also be written as: $3 < x < 5$

Hint: Inequality with “and” looks like: → ←



Verbal Inequalities

The cost of a car is at most $20,000. $c \leq \$20{,}000$

It takes Joe no less than 5 minutes to run a mile.

$t \geq 5 \text{ min}$

It takes between 3 and 8 months to build a house.

$3 \text{ months} \leq t \leq 8 \text{ months}$

The cost of a loaf of bread is less than $2

$0 \leq c < \$2$

You can't buy a car for less than $8000.

$c \geq \$8000$

Your turn: (a) Write in inequality notation
(b) Graph the inequality

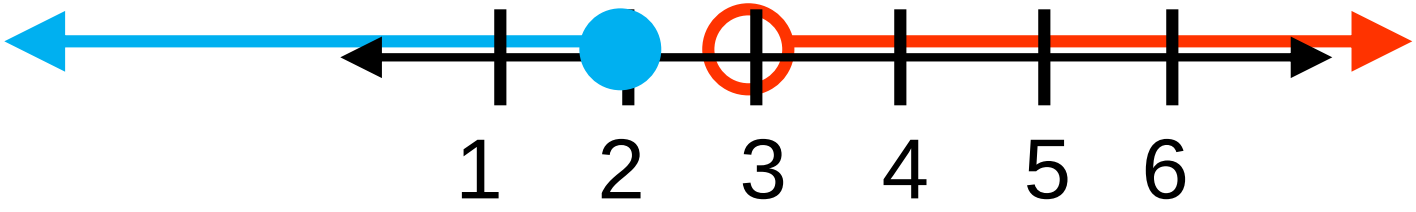
There are least 65,000 spectators at the game.

It never gets above 100 degrees in Huntsville.

You can fit, at most, 5 cars in your garage.

Three Ways to show an Inequality

1. Inequality: $x > 3$ $x \leq 2$
2. Interval Notation: $(3, \infty)$ $(-\infty, 2]$
3. Number line Notation:



Life expectancy in the U.S. is steadily increasing, and the number of Americans aged 100 and older will exceed 850,000 by the middle of the century. Medical advancements are the primary reason for longer life expectancy. Another factor has been the increased awareness of maintaining a healthy lifestyle.

Life expectancy is dependent upon gender. For Americans born after 1980 it be modeled as follows:

$$W(x) = 0.101x + 77.5$$

$$M(x) = 0.192x + 70.0$$

What is the input variable?

Birth year since 1980.

What is the output variable?

Expected person's age when he/she dies.

What is the practical meaning of the "y-intercept" ?

Life expectancy of a person born in 1980.

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What is the practical meaning of the "y-intercept" ?

Life expectancy of a person born in 1980.

What is the practical meaning of the "slope" ?

The number of rate of change of life expectancy for each additional year a person is born after 1980.

At what birth year will the life expectancy of men and women be the same? 'x' means the # of years SINCE 1980.

$$W(x) = 0.101x + 77.5$$

$$M(x) = 0.192x + 70.0$$

Solve by substitution.

$$0.101x + 77.5 = 0.192x + 70.0$$

-0.101x -0.101x

$$77.5 = 0.091x + 70.0$$

-70 -70

$$7.5 = 0.091x$$

÷0.091 ÷0.091

$$82.4 = x$$

$$\text{birth year} = 1980 + 82.4$$

$$\text{birth year} = 2062.4$$

$$W(x) = 0.101x + 77.5$$

$$M(x) = 0.192x + 70.0$$

For what birth years will the life expectancy of men be <u>greater than</u> for women?

$$M(x) > W(x)$$

$$0.192x + 70.0 > 0.101x + 77.5$$

-0.101x -0.101x

$$0.091x + 70.0 > 77.5$$

-70 -70

$$0.091x > 7.5$$

÷0.091 ÷0.091

$$x > 82.4$$

$$\text{birth year} > 1980 + 82.4$$

$$\text{birth year} > 2062.4$$