Math-1010
Lesson 1-7

Textbook 1.12 and 1.13

(More Systems of Linear Equations)

You have volunteered to serve on the committee formed to handle the 4th of July fireworks in your town. Terminology:

(1) "cake": an item with a single fuse to light several tubes in sequence. The have a variety of intricate aerial effects, including spinners, fish, flower bouquets, comets, and other elements. Cakes are the most popular consumer fireworks item outside of sparklers and firecrackers.

(2) "peony" is an aerial effect that looks like a spherical ball of colored lights in the sky.

You are put in charge of buying cakes and peonies for the first 10 minutes of the show. You determine you need 30 cakes and peonies to cover the time frame. Peonies cost $44 each and a group of cakes cost $47 each. Your budget is $1350. Let 'c' be the number of cakes and 'p' be the number of peonies.

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What are the constraints of the problem?

(1) total number of fireworks = 30. $C + P = 30$

(2) total budget for the fireworks = $1350. $47C + 44P = 1350$

Write an equation for each constraint.

Solve the system of equations using substitution.

(1) total number of fireworks = 30. $C+P=30$

(2) total budget for the fireworks = $1350. $47C+44P=1350$

Solve the system of equations using substitution
(by solving each equation for "P" first).

$$P=30-C \qquad P=\frac{1350-47C}{44}$$

Substitution step: P = P

Solve for 'C"

$$44*\left(30-C\right)=\frac{1350-47C}{44} \quad *44$$

$$44(30-C)=1350-47C$$

$$1320-44C=1350-47C$$

+47C +47C

$$1320+3C=1350$$

$$1320+3C=1350$$

-1320 -1320

$$3C=30$$

÷3 ÷3

$$C=10 \qquad C+P=30$$

$$P=20$$

10 cakes, 20 Peonies

Algebraic Methods of Solving Systems of Equations

Substitution: Solve one equation for one of the variables. Substitute the equivalent expression for the variable into the other equation. This results in one equation with one variable.

Elimination: Add the equations (or multiples of the equations) to eliminate one of the variables. Then solve the single variable equation and “back substitute” the result.

Vocabulary

Elimination Method: Eliminate one of the variables by adding the equations together.

$$x - 3y = 5$$
$$-x + 5y = 3$$

What property allows me to add equations together?

"Property of Equality"

Adding these equations will eliminate the 'x' variable.

$$2x - 3y = 5$$
$$-4x + 3y = 3$$

Adding these equations will eliminate the 'y' variable.

What variable will be eliminated if I add the following equations?

1\.

$$2x + y = -2$$
$$-2x + 3y = -8$$

2\.

$$4x - 3y = -2$$
$$-2x + 3y = -8$$

3\.

$$3x + y = -1$$
$$2x + 3y = 18$$

Eliminate one of the variables by adding the equations together.

$$x - 3y = 5$$
$$-x + 5y = 3$$

$$x - x - 3y + 5y = 5 + 3$$

$$2y = 8$$

$$y = 4$$

$$x - 3(4) = 5$$

$$x = 17$$

Solution: (17, 4)

Replace ‘y’ with 4 in either of the original equations, then solve for ‘x’.

Check the solution: (using substitution)

If your work indicated the solution to be (17, 4), replace 'x' with 17 and 'y' with 4 in <u>both</u> of the original equations, to see if the ordered pair (17, 4) is a solution to the system of equations.

$$x - 3y = 5$$
$$-x + 5y = 3$$

$$(17) - 3(4) = 5$$

$$-(17) + 5(4) = 3$$

Checks!

Checks!

Solution: (17, 4)

Solve $2x - 5y = 6$

$-x + 5y = 2$

$2x - x - 5y + 5y = 6 + 2$

$x = 8$

$-(8) + 5y = 2$

$5y = 10$

$y = 2$

Replace 'x' with 8 in either of the original equations, then solve for 'y'.

Solution: (8, 2)

Solve the equation using "elimination"

4x – 3 y = -2

-2x + 3y = -8

$$2x = -10$$

$$x = -5$$

$$-2(-5) + 3y = -8$$

$$10 + 3y = -8$$

$$3y = -18$$

$$y = -6$$

Least common multiple (of 2 numbers) is the smallest number that is divisible by those two numbers.

2 and 4	LCM = 4
4 and 6	LCM = 12
4, 9	LCM = 36
3, 5	LCM = 15
4, 5	LCM = 20

What if the coefficients are <u>not</u> the same?

$$5x - y = -2$$
$$-2x + 3y = -8$$

What is the LCM for the coefficients of 'y'?

LCM = 3

You only have to fix one!

$$3*(5x - y) = -2 *3$$
$$-2x + 3y = -8$$

$$15x - 3y = -6$$
$$-2x + 3y = -8$$

What if the coefficients are <u>not</u> the same?

$$5x - y = -2$$
$$-2x + 3y = -8$$

What is the LCM for the coefficients of 'x'?

LCM = 10 You have to fix both!

$$2*(5x - y) = -2 *2 \qquad 10x - 2y = -4$$
$$5*(-2x + 3y) = -8*5 \qquad -10x + 15y = -40$$

$$3x - 4y = -10$$
$$6x + 3y = -42$$

(-2)3x – (-2)4y = -10(-2)
6x + 3y = -42

-6x + 8y = 20
6x + 3y = -42

$$11y = -22$$

$$y = -2$$

$$6x + 3(-2) = -42$$

$$6x - 6 = -42$$

$$6x = -36$$

$$x = -6$$

$$3x + 2y = 6$$
$$x - 4y = -12$$

$$3(0) + 2y = 6$$
$$(0) - 4y = -12$$

$$(2)3x + (2)2y = 6(2)$$
$$x - 4y = -12$$

$$6x + 4y = 12$$
$$x - 4y = -12$$

$$5x = 0$$

$$x = 0$$

$$2y = 6$$
$$-4y = -12$$

$$y = 3$$

Solution is (0, 3)

Linear Equation in 3 Variables:

$Ax + By + Cz = D$ $3x + 2y - z = 5$

System of Linear Equations: 3 equations, each with the same 3 variables

(3 equations in 3 unknowns)

$$Ax + By + Cz = D$$
$$Ex + Fy + Gz = H$$
$$Jx + Ky + Lz = M$$

Solving by Elimination

$$Ax + By + Cz = D$$

$$Ex + Fy + Gz = H$$

$$Jx + Ky + Lz = M$$

Pick two equations and remove one of the variables.

$$x + y = G$$

Pick two other equations and remove the same variable.

$$x + y = G$$

Solve the system of 2 equations in 2 variables.

Eq#1: $x + 2y - 2z = -15$

Eq#2: $2x + y - 5z = -21$

Eq#3: $x - 4y + z = 18$

Eq#1: $-2(x + 2y - 2z) = (-15)(-2)$

Eq#2 $2x + y - 5z = -21$

$-2x - 4y + 4z = 30$

Eq#1/#2 $-3y - z = 9$

Eq#1: $-1(x + 2y - 2z) = (-15)(-1)$

Eq#3: $x - 4y + z = 18$

$-x - 2y + 2z = 15$

Eq#1/#3 $-6y + 3z = 33$

Eq#1/#2 $-3y - z = 9$

$\div 3(-6y + 3z) = (33)(\div 3)$

Eq#1/#3 $-2y + z = 11$

$-3y - z = 9$

$-5y = 20$

$\boxed{y = -4}$

$-3(-4) - z = 9$

$12 - z = 9$

$\boxed{z = 3}$

$x - 4(-4) + (3) = 18$

$x + 16 + 3 = 18$

$\boxed{x = -1}$

You start your own company to make smartphones. You decide on 3 models; basic, 3G model, and the 4G model.

The basic model is for people who do not have a lot of disposable income. The 3G model has the speed and download capability that most people want. The 4G model has all of the “bells and whistles” and is expandable to meet future needs.

You hire and train your employees to perform all of the basic tasks; assembly, testing, and packaging of each phone.

You analyze your process and employees and decide you have 260 man-hours for assembly in a week, 170 man-hours for testing, and 120 man-hours for packaging.

The table below shows the man-hour totals required for each of the three tasks.

	Basic Model	3G Model	4G Model
Assembly	1 man-hour	3 man-hours	4 man-hours
Testing	1 man-hour	2 man-hours	2 man-hours
Packaging	1 man-hour	1 man-hour	2 man-hours

What are your three constraints?

260 man-hours for assembly $x + 3y + 4z = 260$

170 man-hours for testing, and $x + 2y + 2z = 170$

120 man-hours for packaging. $x + y + 2z = 120$

Write an equation for each of the constraints. Your goal is to figure out how many phones of each type you should build.

Let 'x' be the number of basic phones, 'y' be the number of 3G, and 'z' be the number of 4G phones you will build.