Math-1010
Lesson 1-6

Textbook 1-11
(Systems of Linear Equations)

College Finals are over. You're moving back home for the summer. You need to rent a truck to move your possessions from the dorm back to your home. You check two truck rental companies and obtain the following information for the rental of a truck that will meet your needs..

Company A: $60 per day plus $0.75 per mile

Company B: $30 per day plus $1.00 per mile

1) What quantity is the input?

2) What quantity is the output?

3) Determine a relation between the two quantities and write it in function notation.

Company A: $60 per day plus $0.75 per mile

$$Cost = f(miles)$$

Company B: $30 per day plus $1.00 per mile

$$C_A(m) = 60 + 0.75m \qquad C_B(m) = 30 + m$$

Fill in the remainder of the table.

How would you define the "break-even" point?

"m" miles	Total Cost-A	Total Cost-B
40	90	70
60	105	90
80	120	110
100	135	130
120	150	150
140	165	170
160	180	190

The input value that results in the same output value for both functions.

If your home is 98 miles from school, which company will you use?

If your home is 125 miles from school, which company will you use?

"2 x 2" System of linear equations: Two equations (of lines) that each have the same two variables.

$ax + by = c$ (equation 1) $y = ax + b$ (equation 1)
$dx + ey = f$ (equation 2) $y = cx + d$ (equation 2)
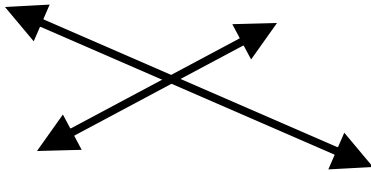
$3x + y = 7$
$5x - 2y = -3$

$y = 2x + 3$
$y = -3x - 4$

Solution of a System of linear equations: the ordered pair (x, y) that makes BOTH equations true.
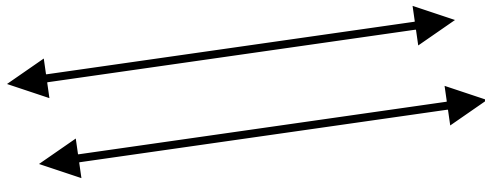
Consistent System of linear equations: has exactly one solution.

Categories of Solutions:

Ways 2 lines can be graphed:

Cross → one solution →
the system is consistent.

Parallel →no solution →
the system is inconsistent

Same line → infinite number of solutions
(all the x-y pairs that make up the line)
→ the system is Dependent

$$C_A(m) = 60 + 0.75m \qquad C_B(m) = 30 + m$$

How did we find the solution?

"m" miles	Total Cost-A	Total Cost-B
40	90	70
60	105	90
80	120	110
100	135	130
120	150	150
140	165	170
160	180	190

This is called a <u>numerical method</u> (we just compared numbers in a table)

Is the system
(1) Consistent,
(2) Inconsistent, or
(3) Dependent ???

What other methods can we use to find the solution to a system?

<u>Algebraic</u> solution method: using algebra to solve a system.

<u>Graphical</u> solution method: using a graph to solve a system.

How do you know how many solutions there are? (1, 0, or infinite #)

$$y = 3x + 1$$
$$y = 2x + 1$$

Not same line, not parallel → one solution.

$$y = -2x + 3$$
$$y = -2x - 4$$
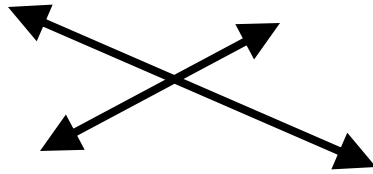
parallel → no solutions

$$2x + 2y = 2$$
$$x + y = 1$$

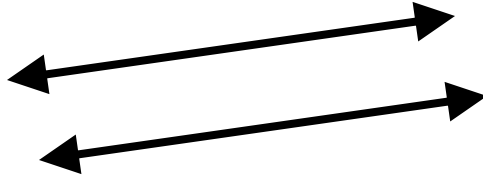1st equation is a multiple of the 2nd equation → same line

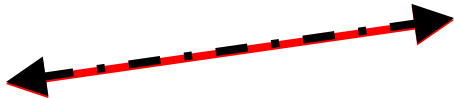→ infinite # of solutions.

Which Category ?

$$y = 2x + 6$$

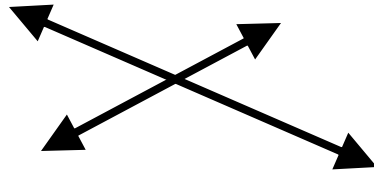$$y = 4x - 2$$

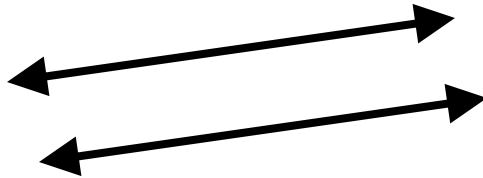Cross → one solution

Parallel → no solutions

Same line → infinite
number of
solutions

WHICH CATEGORY ?

$$y = 2x + 4$$

$$y = 2x - 7$$

Cross → one solution

Parallel → no solutions
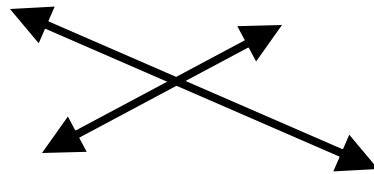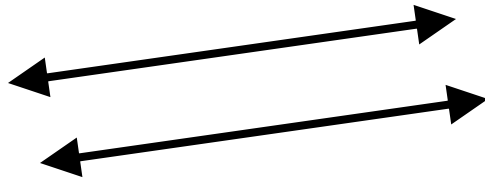
Same line → infinite number of solutions

WHICH CATEGORY ?

$$2x + 3y = 6$$

$$4x + 6y = 12$$

Cross → one solution

Parallel → no solutions

Same line → infinite
number of
solutions

College is finally over. You want to buy your "I love me" car to celebrate. You have narrowed the choice to two cars;

1) Honda Accord XL: $20,025
2) VW Passat GLX: $23,900

You're concerned about depreciation of the two cars over time.

1) Accord's depreciation: $1385 per year
2) Passat's depreciation: $1790 per year

What quantity is the input?   What quantity is the output?

Determine a relation between the two quantities and write it in function notation.

$$V_A(t) = 20025 - 1385t \qquad V_P(t) = 23900 - 1790t$$

Find the "break even" point using the numerical method.

"V" is the value of the car after "n" years of ownership

n (years)	0	1	2	3	10
V, (Accord) ($)	20,025	18,640	17,255	15,870	6175
V, (Passat) ($)	23,900	22,110	20,320	18,530	6000

$$V_A(t) = 20025 - 1385t \qquad V_P(t) = 23900 - 1790t$$

Will the Passat ever have a lower value than the Accord? Explain your answer.

Passat has a higher depreciation rate

Passat is loosing value faster than the Accord.

Classify the system of equations.

Algebraic Methods of Solving Systems of Equations

Substitution: Solve one equation for one of the variables. Substitute the equivalent expression for the variable into the other equation. This results in one equation with one variable.
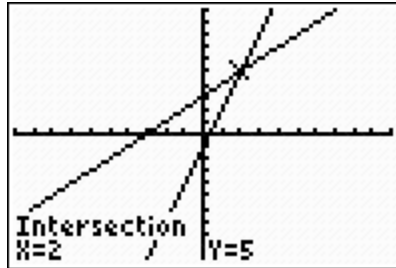
Elimination: Add the equations (or multiples of the equations) to eliminate one of the variables. Then solve the single variable equation and "back substitute" the result.

Solving by graphing:

Is <u>very easy</u> if you use a graphing calculator.

$$y = x + 3$$

$$y = 3x - 1$$

Intersection
X=2   Y=5

Unfortunately, in this class you are not allowed to use a graphing calculator.

Substitution Method

$$2x + y = 8$$

$$-x + 3y = 3$$

1. Solve one equation for one of the variables.

$$y = -2x + 8$$

2. Substitute the value of the variable into the other equation.

$$-x + 3(-2x + 8) = 3$$

3. Solve for the single variable.

Substitution Method $-x + 3(-2x + 8) = 3$

3. Solve for the single variable.

$$-x - 6x + 24 = 3$$

$$-7x = -21$$

$$x = \frac{-27}{-7} = 3$$

4. Substitute the value of the variable back into <u>either</u> initial equation and solve for the other variable.

$$2x + y = 8$$

$$-x + 3y = 3$$

$$2(3) + y = 8 \qquad y = 2$$

6. The solution is: $(3, 2)$

Which system is easier to solve using substitution? Why?

$$y = 2x - 6$$
$$y = -3x + 9$$

$$-2x + y = -6$$
$$3x + y = 9$$

Slope intercept form!!!

The x-y pair where the graphs cross is the solution of both equations—it makes both equations true, it's where the graphs cross.

The value of 'y' will be the same for both equations. Substitute the expression that equals 'y' from one equation into the 'y' position in the other equation.

$$-3x + 9 = 2x - 6$$

Solve using substitution.

$$V_A = 20025 - 1385t \qquad V_P = 23900 - 1790t$$

$$20025 - 1385t = 23900 - 1790t$$

+1790t +1790t

$$20025 + 405t = 23900$$

-20025 -20025

$$405t = 3875$$

÷405 ÷405

$$t \approx 9.6$$

Practically speaking, what does this value of "t" mean?

$$V_A = 20025 - 1385(9.6)$$

$$V_A = 6770.55$$

Practically speaking, what does this value of "V" mean?