Math 1010 Lesson 1-4 (Textbook 1.7 and 1.8)
Different equations of Lines

1. "slope intercept form" y = mx + b
2. "standard form" Ax + By = C
3. "point slope form" $y - y_1 = m(x - x_1)$

Which one we use depends on what information is given to us in the problem.

$y = 2x + 2$ 'y' is "alone".

$-2x + y = 2$ The constant is "alone".

$y - 4 = 2(x - 1)$ Neither 'x' nor 'y' is "alone".

Slope Intercept form of an equation of a line: the coordinates of all points (x, y) on a line whose average rate of change is "m" and whose y-intercept is ""b".

$$y = mx + b \qquad f(x) = mx + b$$

19) If you substitute x = 0 into the equation above, what is the corresponding value of "y"?

20) What is the slope and y-intercept for the lines given by the following equations? (Give y-intercepts as x-y pairs)

$$f(x) = -2x + 5 \qquad q = 2 - r$$

$$s = 0.75t + 2 \qquad y = \frac{5}{6} - \frac{x}{3}$$

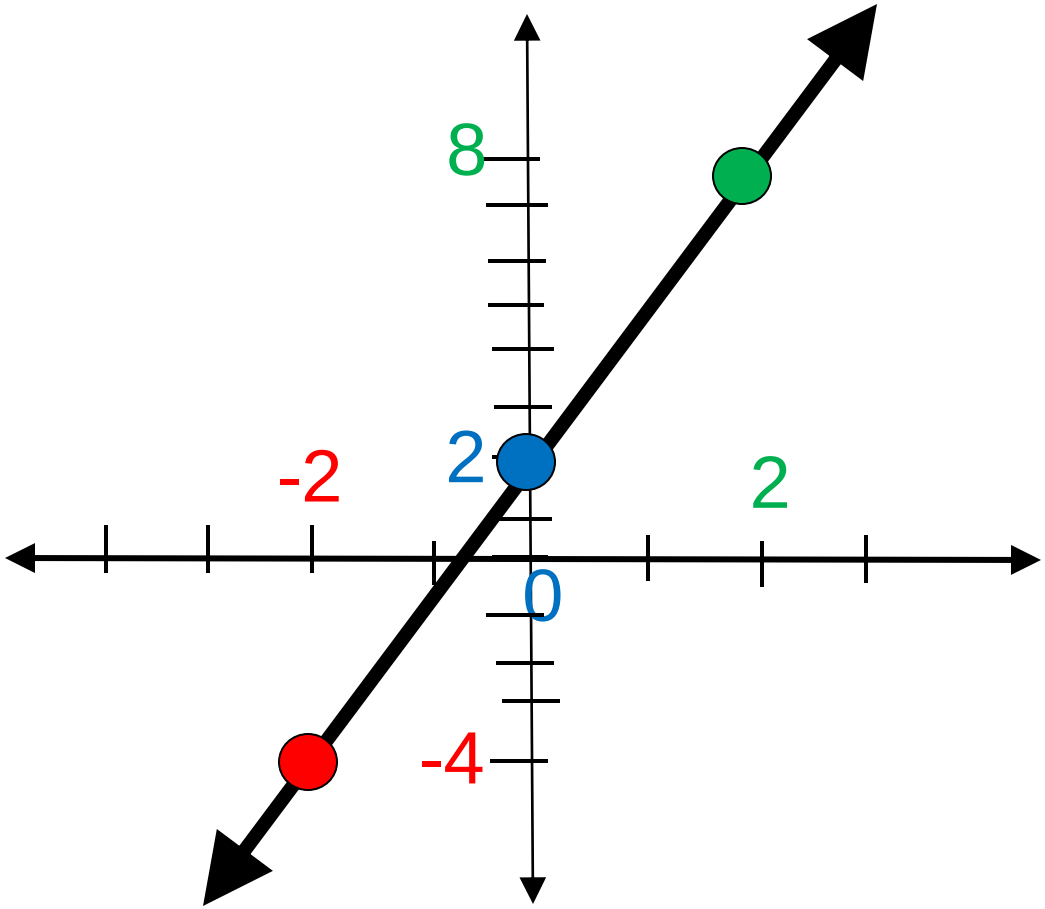
(-2, -4)

$$f(x) = 3x + 2$$

(0, 2)

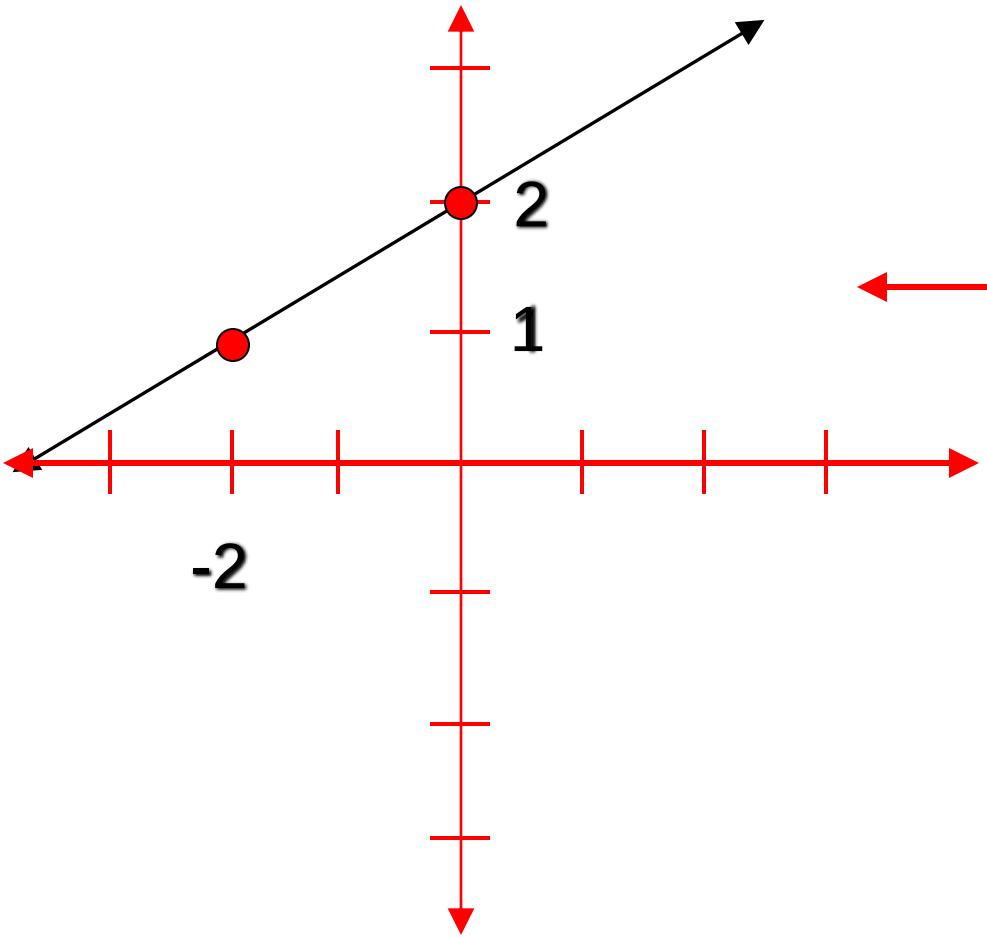
(2, 8)

The constant is the “y-intercept”

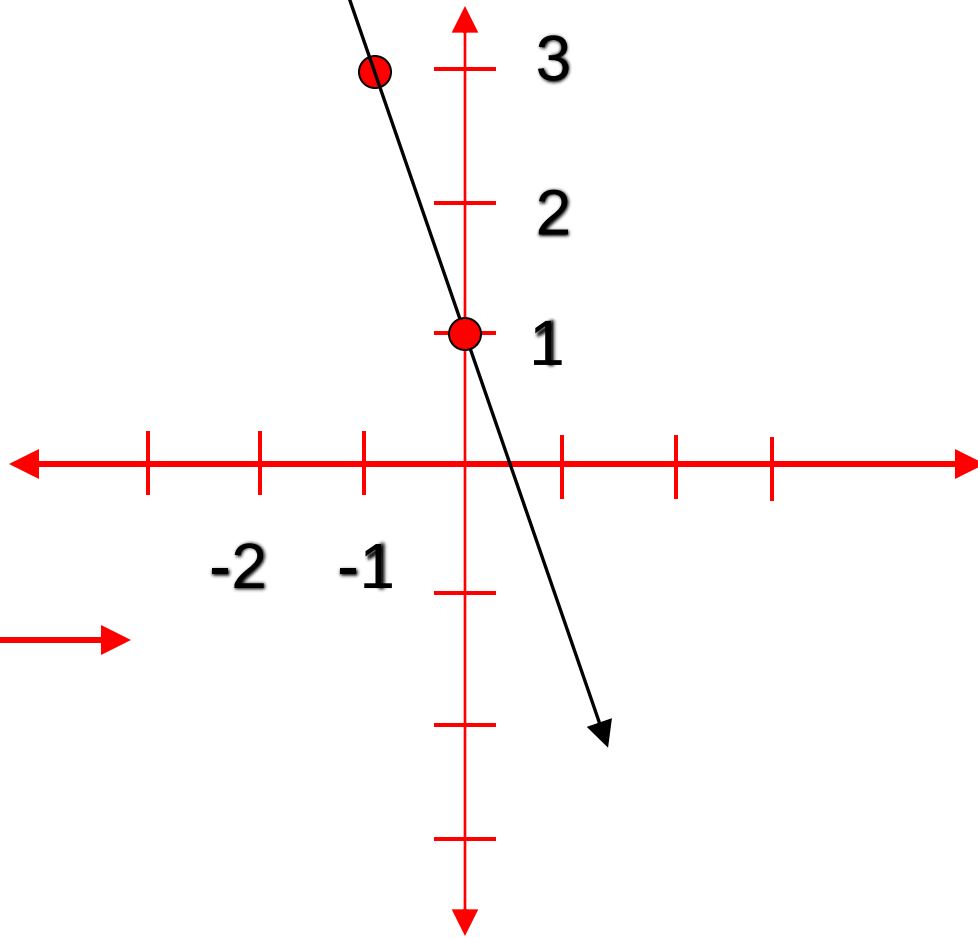


What is the equation of the line?

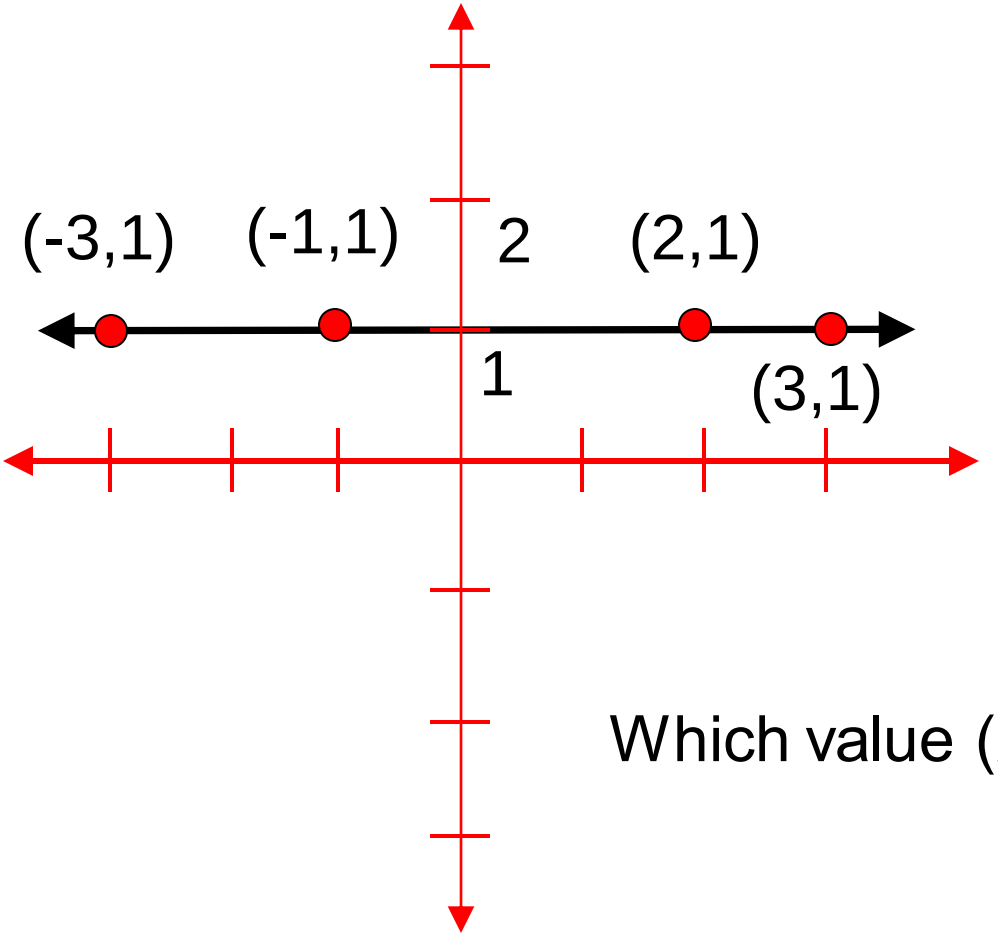
$y = \frac{1}{2}x + 2$



$y = -2x + 1$



What is the equation of the line?



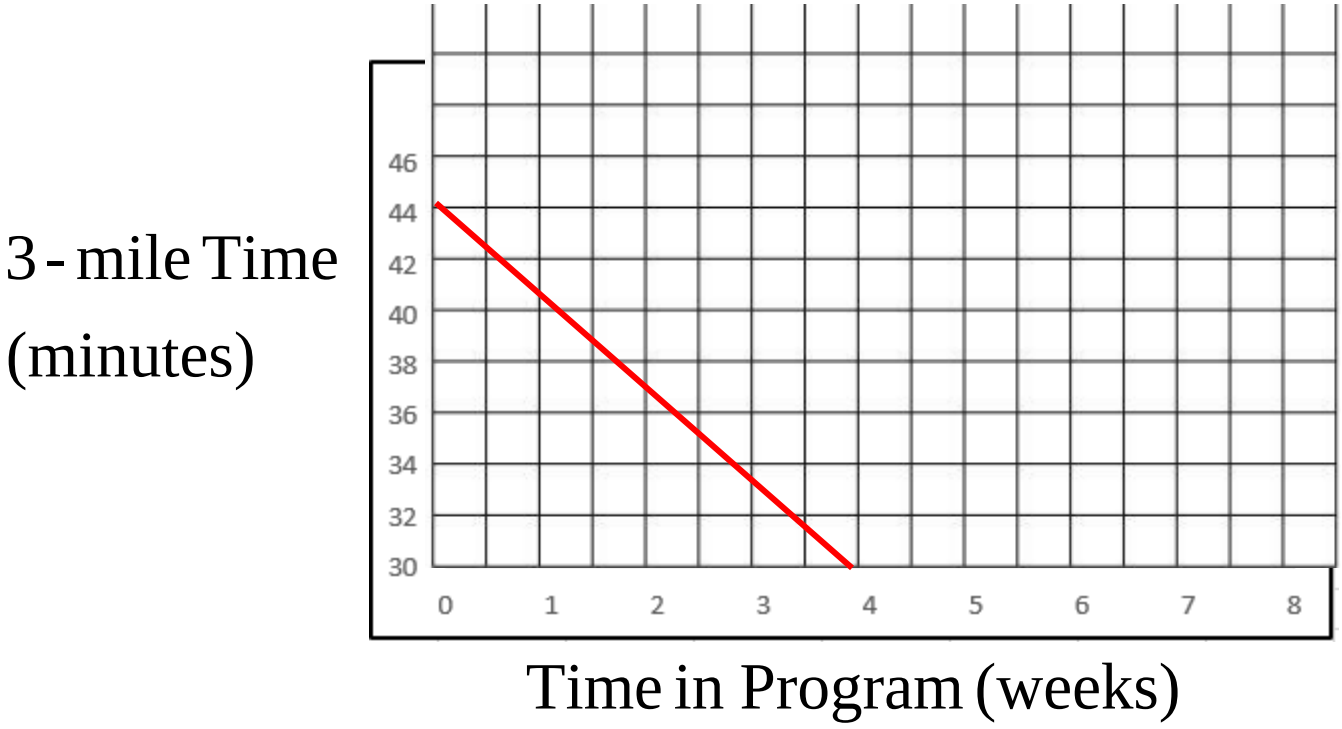
$y = 1$

or

$x = 1$

?

Which value (x or y) is always ‘1’?



Horizontal intercept (x-intercept), the point where the graph crosses the input axis. It always has (what value?) ____ as the y-value of the point. Numerically it is always: (a, 0) or (x, 0)

In function notation it is always: $f(x) = 0$

Example: Find the x-intercept for each of the following equations. ***One step—rewrite—show your work!!!***

$f(x) = -2x + 5$

0 = -2x + 5

+2x +2x

2x = 5

÷2 ÷2

$x = \frac{5}{2}$

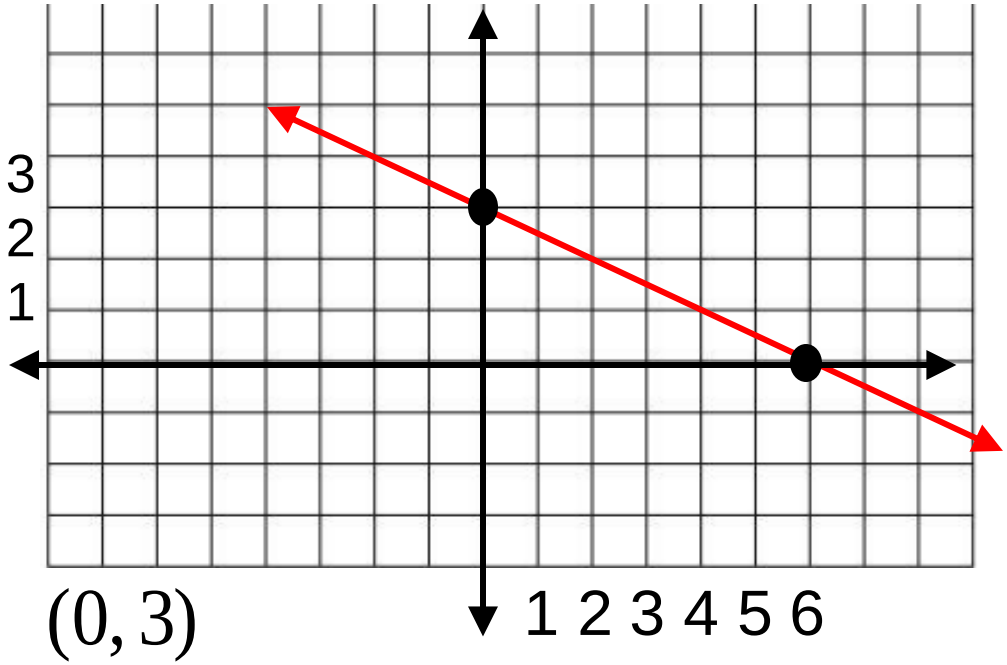
$q = 2 - r$

$s = 0.75t + 2$

$y = \frac{5}{6} - \frac{x}{3}$

Graph an equation of a line:

$$y = -\frac{1}{2}x + 3$$



1) Calculate the intercepts:

y-intercept: f(0) = y

$$y = -\frac{1}{2}(0) + 3 \qquad y = 3 \qquad (0, 3)$$

x-intercept: f(x) = 0

$$0 = -\frac{1}{2}x + 3$$

-3 -3

$$6 = x$$

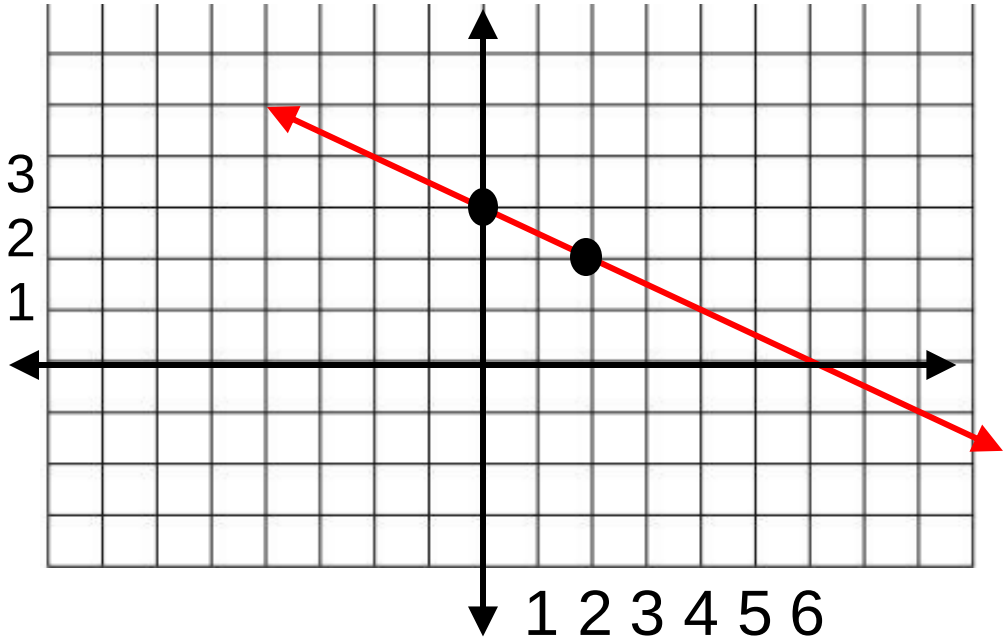
$$(6, 0)$$

$$-3 = -\frac{1}{2}x$$

*(-2) *(-2)

Graph an equation of a line:

$$y = -\frac{1}{2}x + 3$$



1) y = mx + b

y-intercept: f(0) = b

$(0, 3)$

slope: $m = -\frac{1}{2} = \frac{-1}{2}$

From y-intercept,
move down 1, right 2. $(2, 2)$

Point—Slope Equation of a Line

Slope is given by: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Multiply by $(x_2 - x_1)$ $\quad m(x_2 - x_1) = \dfrac{y_2 - y_1}{(x_2 - x_1)} * (x_2 - x_1)$

$$m(x_2 - x_1) = y_2 - y_1$$

$$y_2 - y_1 = m(x_2 - x_1)$$

If we know one x-y pair (for example: (1, 3)) and the slope (for example: m = 2) this becomes: $y_2 - 3 = 2(x_2 - 1)$

We can drop the subscripts: $y - 3 = 2(x - 1)$

Convert to slope intercept form: $y - 3 = 2x - 2$

+3 +3

$$y = 2x + 1$$

Point—Slope Equation of a Line $y_2 - y_1 = m(x_2 - x_1)$

Convert the following equations into slope—intercept form (one step—rewrite).

$y - 4 = 3(x - 2)$

$y - 4 = 3x - 6$

+4 +4

$y = 3x - 2$

$y + 6 = \frac{3}{2}(x - 4)$

$y + 6 = \frac{3}{2}x - 6$

-6 -6

$y = \frac{3}{2}x - 12$

Point Slope Form of a Linear Equation

What is the equation of a line that passes through the point (3, 4) and has a slope of -2 ?

Step 1: write the general form of the equation.

$$m = \frac{y - y_1}{x - x_1} \quad \text{or} \quad y - y_1 = m(x - x_1)$$

Step 2: substitute numbers into the equation.

$(x_1, y_1) = (3,4)$ m = -2

$y - 4 = -2(x - 3)$

$y = -2x + 10$

Step 3: Solve for 'y' (slope/int form).

Time (min)	0	1	2	3	4
Height (ft)	36,000	32,000	28,000	24,000	20,000

What is happening?

Write an Equation

→ what is the slope?

$$m = \frac{rise}{run} = \frac{-4000\ ft}{\min}$$

What is the y-intercept ? ⇒ (0, b) (0, 36000)

write equation: y = mx + b

y = -4000x + 36000

Time (min)	8:03 AM	8:04 AM	8:05 AM	8:06 AM	8:07 AM
Height (ft)	36,000	32,800	29,600	26,400	23,200

Notice how this time doesn't start at zero.

To write an equation, you need a y-intercept.

It is often easier to change the time scale to read "time since" some reference point.

Time (min) (since 8:03 AM)	0	1	2	3	4
Height (ft)	36,000	32,800	29,600	26,400	23,200

Year	1990	1992	1994	1996	1998
Imports (Billions $)	52	55	58	61	64

Write an Equation

→ what is the slope? $m = \frac{rise}{run} = \frac{3}{2}$

What is the y-intercept ? → (0, b)

Change scale for years to “years since 1990) (0, b)

Years since 1990	0	2	4	6	8
Imports (Billions $)	52	55	58	61	64

write equation: y = 1.5x + 52

Finding an equation of a line that passes through to given points.

(3, 4) and (7, 0)

Which form will work the easiest?

slope-intercept ?

standard form ?

point-slope ?

Equations of Parallel Lines

How do the slopes of parallel lines compare?

Parallel → same slope

Find the equation of a line that is parallel to the line $y = 2x + 1$ and passes through the point (4, -1)

What equation form will you use?

Slope = 2 $(x_1, y_1) = (4, -1)$

$$y - y_1 = m(x - x_1)$$

$$y - (-1) = 2(x - 4)$$

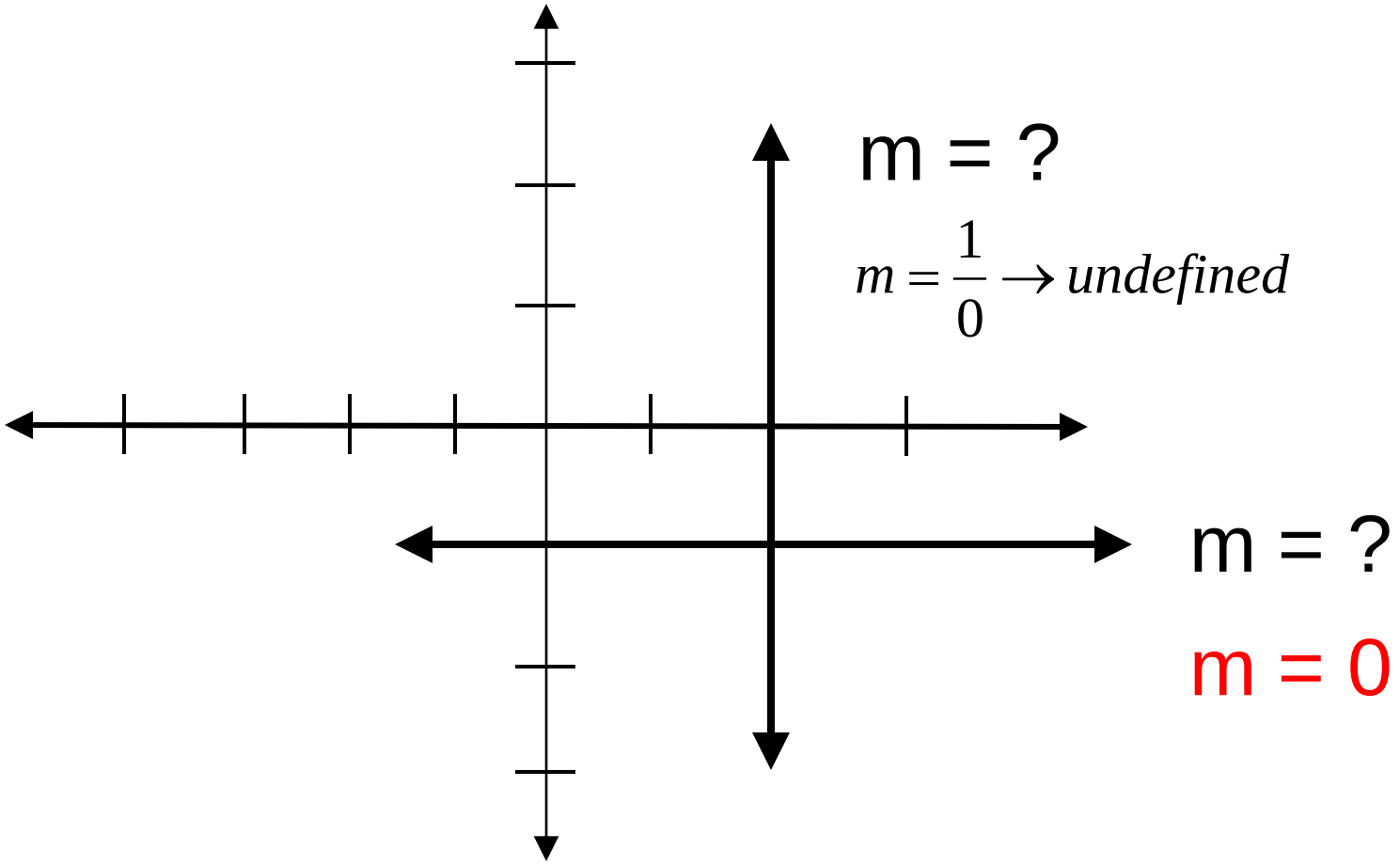
$$y + 1 = 2x - 8$$

$$y = 2x - 9$$

Perpendicular Lines

How do the slopes of perpendicular lines compare?

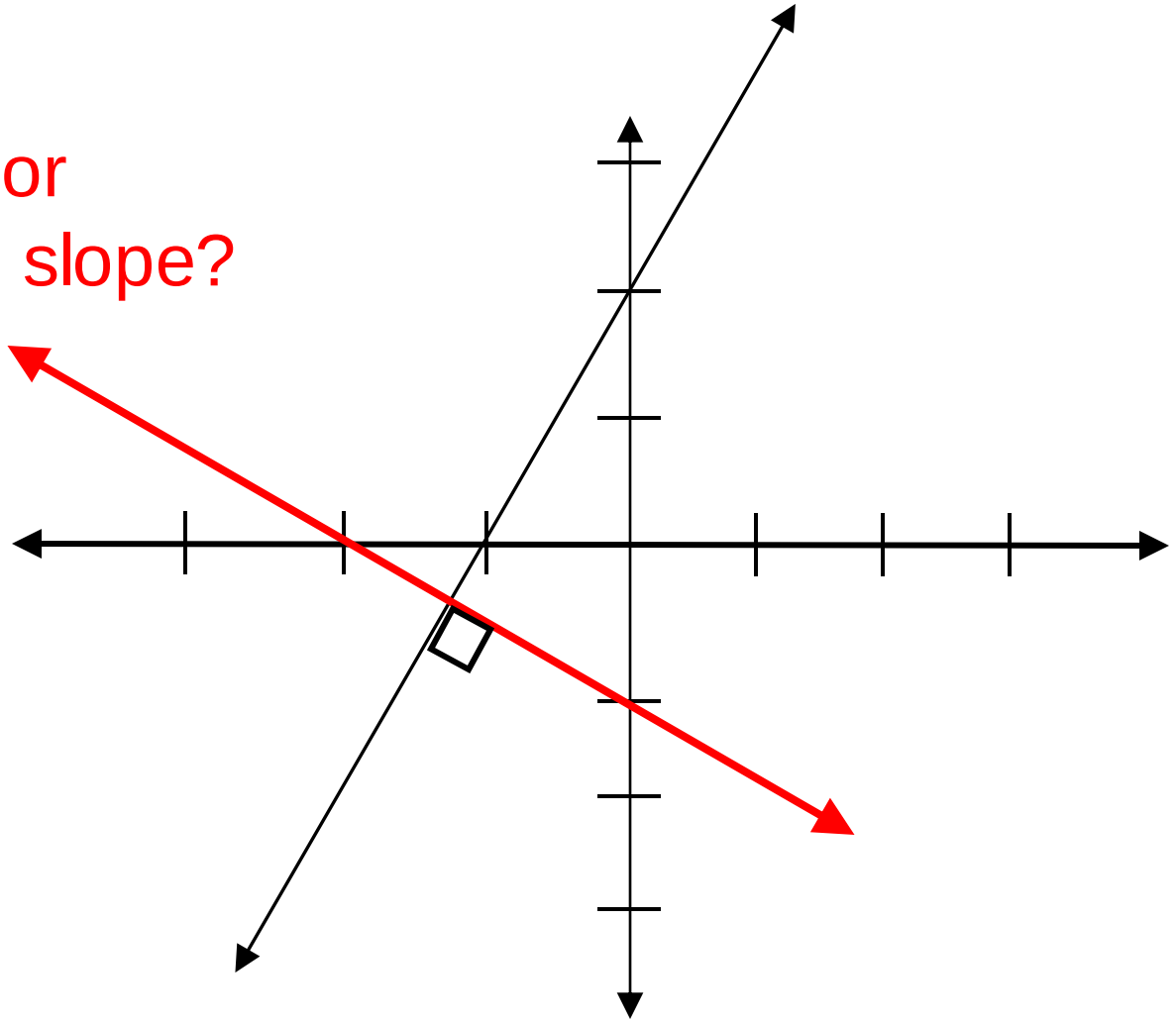
Slopes are reciprocals of each other.



Perpendicular Lines

Positive or negative slope?

Positive or negative slope?



Perpendicular Lines:

The slopes are reciprocals of each other.

The slopes have opposite signs of each other.

m = -3 What is the slope of a line that is perpendicular?

Find the slope intercept form of a line that is perpendicular to the line:

$y = 2x - 6$

and passes through the point (7, 1)

You decide to buy a new Honda Civic, but you are concerned about the value of the car depreciating over time. You search the Internet and obtain the following information.

Suggested Retail Price: $20,905

Depreciation per year: $1750 (assume constant)

1) What does this mean?

2) Complete the table.

"V" is the value of the car after "n" years of ownership

n (years)	0	1	2	3	5	8
V, ($)	20,905	19,155	17405	15,655	12,155	6905

3) Is the value of the car a function of years of ownership? Explain why or why not.

4) Write the relation in “function notation.”

5) What is the input?

6) What is the output?

7) Select 2 ordered pairs: determine the average rate of change

8) What are the units of the average rate of change?

9) What is the practical meaning of the sign (+/-) of average rate of change?

10) Select 2 other ordered pairs: determine the average rate of change

11) Select 2 other ordered pairs not used in #7 and #10: determine the average rate of change

12) Using your results from questions #7, #10, and #11, what can you infer about the average rate of change for any interval?

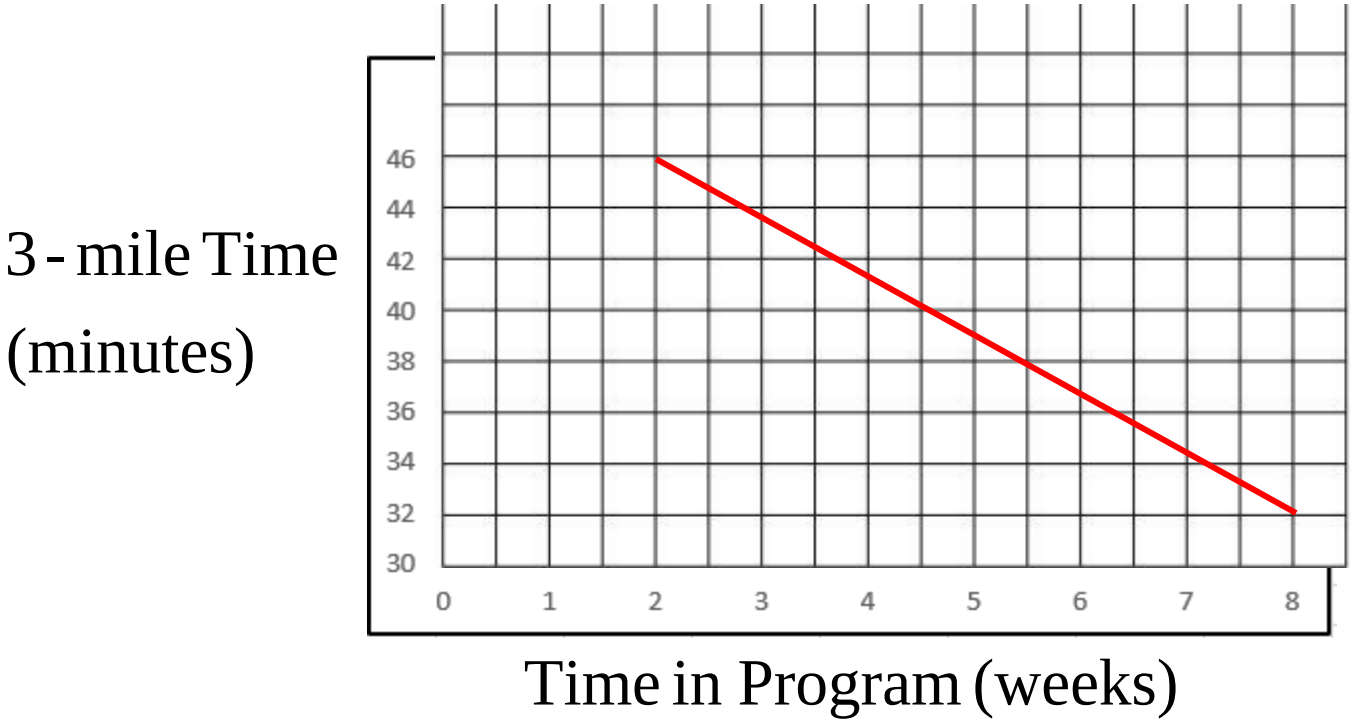
Average Rate of change: a comparison between the change in output values to the change in the input values using a ratio.

$$m = \frac{\text{change in output}}{\text{change in input}}$$

Linear Function: any function where the average rate of change between any pair of points is constant.

13) Is the value "V" of the car a linear function of the number of years of ownership "n"? Explain using the definition of a linear function.

Treadmill times to walk, jog, or run 3 miles has been graphed as a function of weeks on an exercise program.



14) Describe in words what the graph is saying.

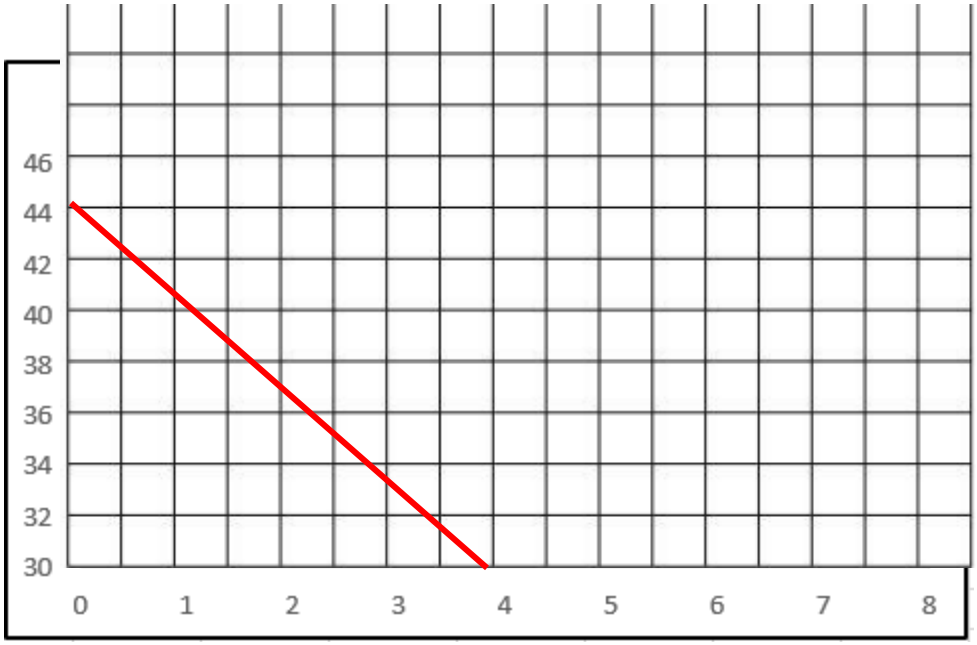
15) Calculate the Average Rate of change:

Slope: the average rate of change of a linear function.

$$\text{m} = \frac{\text{change in output}}{\text{change in input}} = \frac{\Delta y}{\Delta x}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

15) What is the significance of the numbers ‘1’ and ‘2’ in the formula above?



Vertical intercept (y-intercept), the point where the graph crosses the vertical axis. It always has (what value?) ____ as the x-value of the point. Numerically it is always: (0, b) or (0, y)

In function notation it is always: $f(0) = y_{\text{intercept}}$

16) What is the practical meaning of the y-intercept for the graph above?

"V" is the value of the car after "n" years of ownership

n (years)	0	1	2	3	5	8
V, ($)	20,905	19,155	17405	15,655	12,155	6905

17) Once the initial value of the car was known, how did you calculate the value for the other years?

18) Write an equation for the function. $V(n) = 20{,}905 - 1750n$

The slope of the graph was the average rate of change (the yearly depreciation rate): m = -1750 $V(n) = 20{,}905 - 1750n$

The y-intercept (the value of the car at n = 0) was: (0, 20,905)

$V(n) = 20{,}905 - 1750n$