Math-2A: Lesson 3-2 (Dividing Radicals)

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

Quotient Property of Radicals

Split the radical of a quotient into the quotient of radicals.

$$\sqrt{\frac{26}{4}} \rightarrow \frac{\sqrt{26}}{\sqrt{4}} \rightarrow \frac{\sqrt{26}}{2}$$

This works great if there is a perfect square in the denominator!

$$\sqrt{\frac{5}{9}} \rightarrow \frac{\sqrt{5}}{\sqrt{9}} \rightarrow \frac{\sqrt{5}}{3}$$

$$\frac{\sqrt{12}}{\sqrt{2}} = \sqrt{\frac{12}{2}} = \sqrt{6}$$

A different way to think about it: factor the numerator

$$\frac{\sqrt{12}}{\sqrt{2}} = \frac{\sqrt{2}*\sqrt{6}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}} * \frac{\sqrt{6}}{1} = 1 * \frac{\sqrt{6}}{1} = \sqrt{6}$$

Any number divided by itself = ?

Factor the numerator!

$$\frac{\sqrt{12}}{\sqrt{2}} \rightarrow \frac{\sqrt{2}*\sqrt{6}}{\sqrt{2}} \rightarrow \frac{\sqrt{2}*\sqrt{6}}{\sqrt{2}}$$

Inverse Property of Multiplication $\rightarrow \sqrt{6}$

$$\frac{\sqrt{48x^3}}{\sqrt{16x}} \rightarrow \frac{\sqrt{16x}*\sqrt{3x^2}}{\sqrt{16x}}$$

Inverse Property of Multiplication $\rightarrow \sqrt{3x^2} \rightarrow x\sqrt{3}$

$$\frac{\sqrt{56x^3y}}{\sqrt{8xy}} \rightarrow \frac{\sqrt{8xy}*\sqrt{7x^2}}{\sqrt{8xy}}$$

Inverse Property of Multiplication $\rightarrow \sqrt{7x^2} \rightarrow x\sqrt{7}$

Simplify

$$\sqrt{\frac{32}{9x^2}}$$

$$\frac{\sqrt{50y^2}}{\sqrt{2y}}$$

$$\frac{\sqrt{49}}{\sqrt{7}}$$

$$\sqrt{\frac{48}{49}}$$

Rationalizing the denominator: using mathematical properties to change an irrational number (or imaginary) in the denominator into a rational number.

We take advantage of the idea:

$$\sqrt{2} * \sqrt{2} = \sqrt{2*2} = \sqrt{4} = 2$$

$$\sqrt{3} * \sqrt{3} = \sqrt{3*3} = \sqrt{9} = 3$$

$$\frac{1}{\sqrt{2}} * \frac{\sqrt{2}}{\sqrt{2}} \rightarrow \frac{\sqrt{2}}{2}$$

Identity Property of Multiplication

multiplying by '1' doesn't change the number.

$$\frac{1}{\sqrt{6}} * \frac{\sqrt{6}}{\sqrt{6}} \rightarrow \frac{\sqrt{6}}{6}$$

$$\frac{2}{\sqrt{6}} * \frac{\sqrt{6}}{\sqrt{6}} \rightarrow \frac{2\sqrt{6}}{6} \rightarrow \frac{2*\sqrt{6}}{2*3} \rightarrow \frac{\sqrt{6}}{3}$$

$$\frac{25}{\sqrt{15}} * \frac{\sqrt{15}}{\sqrt{15}} \rightarrow \frac{25\sqrt{15}}{15} \rightarrow \frac{5*5**\sqrt{15}}{5*3} \rightarrow \frac{5\sqrt{15}}{3}$$

$$\frac{14}{3\sqrt{21}} * \frac{\sqrt{21}}{\sqrt{21}} \rightarrow \frac{14\sqrt{21}}{3*21} \rightarrow \frac{2*7**\sqrt{21}}{3*7*3} \rightarrow \frac{2\sqrt{21}}{9}$$

In all of the previous examples we just multiplied by “one in the form of” the denominator radical over the denominator radical.

$$\frac{3\sqrt{7}}{\sqrt{8}} \rightarrow \frac{3\sqrt{7}}{\sqrt{8}} * \frac{\sqrt{8}}{\sqrt{8}} \rightarrow \frac{3\sqrt{7*2*2*2}}{8} \rightarrow \frac{3*2\sqrt{7*2}}{8}$$

It is always easier to simplify (by factoring) ***BEFORE*** you multiply

$$\rightarrow \frac{3*2\sqrt{14}}{2*4}$$

$$\frac{3\sqrt{7}}{\sqrt{8}} \rightarrow \frac{3\sqrt{7}}{\sqrt{4}*\sqrt{2}} \rightarrow \frac{3\sqrt{7}}{2\sqrt{2}} * \frac{\sqrt{2}}{\sqrt{2}} \rightarrow \frac{3\sqrt{14}}{2*2} \rightarrow \frac{3\sqrt{14}}{4}$$

$$\frac{6\sqrt{5}}{3\sqrt{12}} \rightarrow \frac{3*2*\sqrt{5}}{3*\sqrt{4}*\sqrt{3}} \rightarrow \frac{2*\sqrt{5}}{2*\sqrt{3}} \rightarrow \frac{\sqrt{5}}{\sqrt{3}} * \frac{\sqrt{3}}{\sqrt{3}} \rightarrow \frac{\sqrt{15}}{3}$$

$$\frac{4}{2\sqrt{6}} \rightarrow \frac{2*2}{2\sqrt{6}} \rightarrow \frac{2}{\sqrt{6}} * \frac{\sqrt{6}}{\sqrt{6}} \rightarrow \frac{2\sqrt{6}}{6} \rightarrow \frac{2\sqrt{6}}{2*3} \rightarrow \frac{\sqrt{6}}{3}$$

$$\frac{2}{4\sqrt{6}} \rightarrow \frac{1*2}{2*2\sqrt{6}} \rightarrow \frac{1}{2\sqrt{6}} * \frac{\sqrt{6}}{\sqrt{6}} \rightarrow \frac{\sqrt{6}}{2*6} \rightarrow \frac{\sqrt{6}}{12}$$

$$\frac{25}{15\sqrt{5}} \rightarrow \frac{5*5}{5*3\sqrt{5}} \rightarrow \frac{5}{3\sqrt{5}} * \frac{\sqrt{5}}{\sqrt{5}} \rightarrow \frac{5\sqrt{5}}{3*5} \rightarrow \frac{5*\sqrt{5}}{5*3} \rightarrow \frac{\sqrt{5}}{3}$$

What about variables?

$$\frac{3}{\sqrt{5x}} * \frac{\sqrt{5x}}{\sqrt{5x}} \rightarrow \frac{3\sqrt{5x}}{5x}$$

$$\frac{\sqrt{15}}{\sqrt{5x}} \rightarrow \frac{\sqrt{5} * \sqrt{3}}{\sqrt{5} * \sqrt{x}} \rightarrow \frac{\sqrt{3}}{\sqrt{x}} * \frac{\sqrt{x}}{\sqrt{x}} \rightarrow \frac{\sqrt{3x}}{\sqrt{x^2}} \rightarrow \frac{\sqrt{3x}}{x}$$

Rationalize the denominator

$$\frac{2x}{\sqrt{3x}}$$

$$\frac{3}{\sqrt[4]{x^2}}$$

$$\frac{2\sqrt{3y^3}}{\sqrt{5y}}$$