# Math-1060
# Session #13

## Textbook 5.3

## Graphs of the Other Trigonometric Functions

Domain (of a function): the set of input values that have a corresponding output value.

A single-variable equation is not a function.

Can an expression have a domain?

Rational Expression is a ratio of expressions. In Math-3 we considered ratios of polynomial expressions.

$$\frac{x^2 - 1}{x + 1}$$

Excluded value the value that causes division by zero in the rational expression.

In other words, we exclude the value that causes the expression to be *undefined*.

When x = -1, the expression is *undefined*.

Domain of an expression: the set of all values for ‘x’ that results in the expression being defined.

Consider the following: in a particular city, there are only red cars or white cars. I have a car but it is not white. What color is my car?

We can define a domain two ways:

(1) specifying what the domain is, or

(2) specifying what the domain is not.

Sometimes it is easier so specify what it is not.

$$\frac{x^2 - 1}{x + 1}$$ Specify the domain of the function both ways.

$Domain\colon\ \{x : x = (-\infty, 1) \cup (1, \infty)\}$ (Set-builder notation)

$\boxed{Domain\colon\ x = (-\infty, 1) \cup (1, \infty)}$ (simple definition)

$Domain\colon \{x : x \neq 1\}$

$\boxed{Domain\colon x \neq 1}$

Domain of a single-variable equation. $3x+2=6x+4$

We identify the domain of the expression on the right side of the "=" sign separately from the domain of the left side.

The expression: $6x + 4$ is defined for all real numbers.

The expression: $3x + 2$ is defined for all real numbers.

Domain of Validity: the set of all values for 'x' that results in both sides of the equation being defined.

$$\frac{2x^2-4x}{x-2}=2x$$

Domain of validity: $x \neq 2$

Identity: an equation that is true for all values that are in the domain of both sides of the equation.

Identity: an equation that is true for all values that are in the domain of both sides of the equation.

Is it an Identity? If not, why not?

$$4x + 2 = 6x + 4$$

No. The domain of validity is “all real numbers” but the equation is true only when x = 1.

$$\frac{2x^2 - 4x}{x - 2} = 2x$$

When we factor the left side of the equation, we have $$\frac{2x(x - 2)}{x - 2} = 2x$$

Which simplifies to $2x = 2x$

Which is true for all real numbers.

AND it is true for all values in domain of validity (x ≠ 2). Therefore it is an identity.

Is it an Identity? If not, why not?

$$\sin\theta = \frac{1}{\csc\theta}$$

Working on the right side; the cosecant ratio is defined as hyp/opp.

$$\sin\theta = \frac{1}{\frac{hyp}{opp}}$$

1 divided by hyp/opp is the same as 1 multiplied by opp/hyp

$$\sin\theta = \frac{opp}{hyp}$$

The right side is now the definition of the sine ratio.

Which is true for all real numbers.

AND it is true for all values in domain of validity ($\theta \neq \pi n \; for \; n \in \mathbb{Z}$). Therefore it is an identity.

## Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

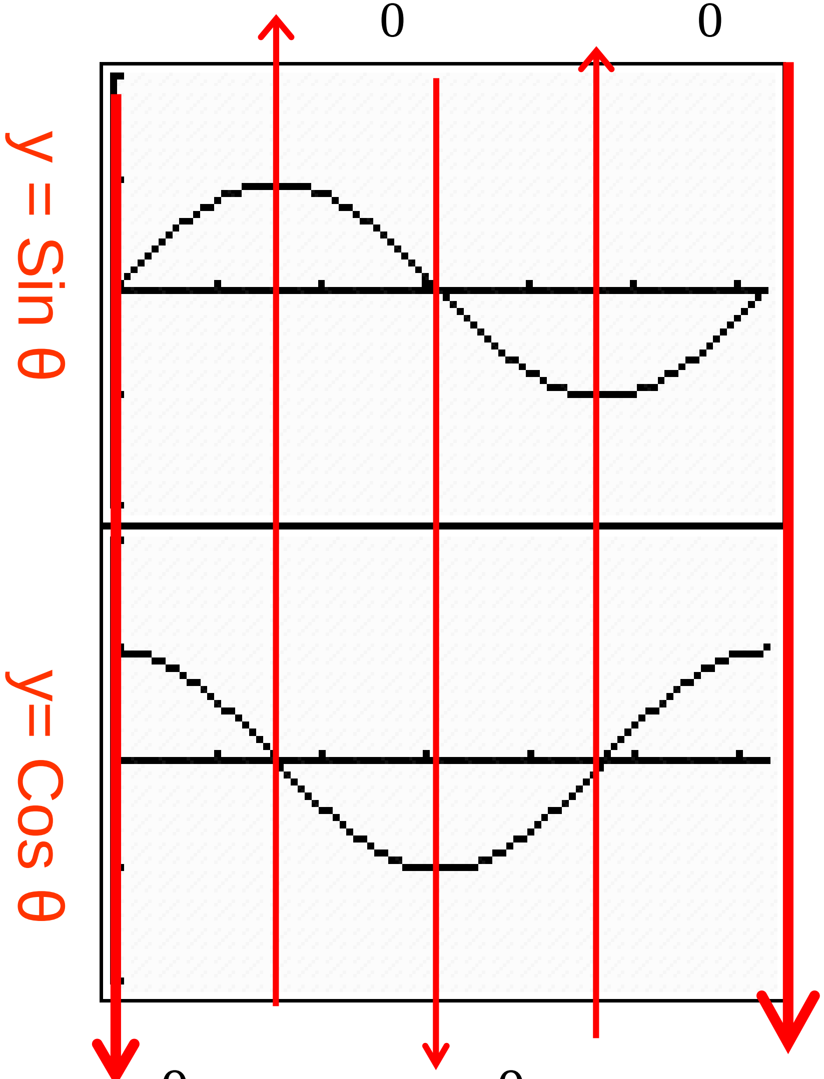
$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

## Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

# Construct the Tangent Function

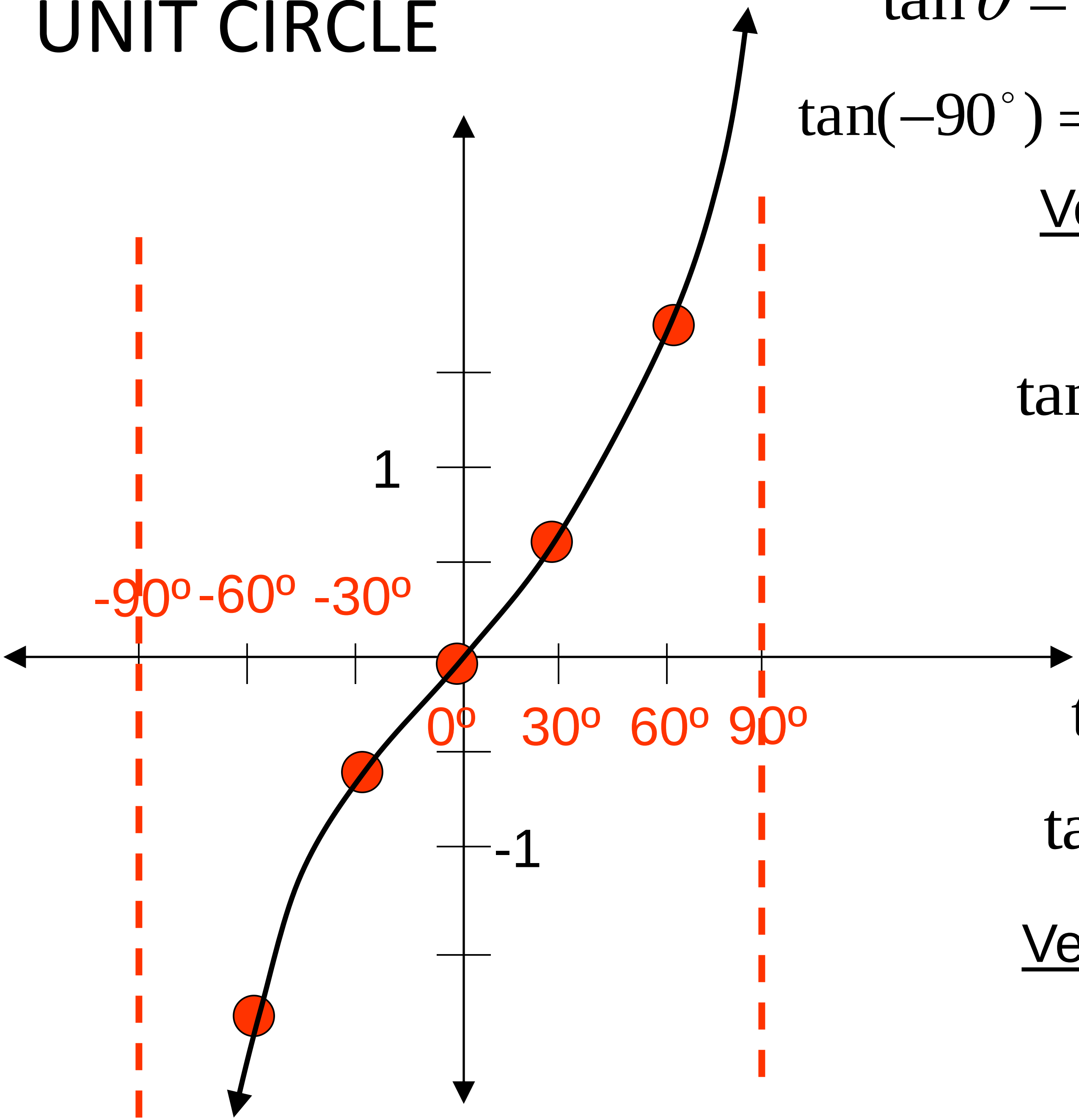
$$\tan 90^\circ = \frac{1}{0} \qquad \tan 90^\circ = \frac{-1}{0} \qquad \tan A = \frac{\sin\theta}{\cos\theta}$$




$$\tan 0 = \frac{0}{1} \qquad \tan 0 = \frac{0}{-1}$$

| θ (radians) | sin θ | cos θ | tan θ |
|---|---|---|---|
| $0$ | $0$ | $1$ | $0$ |
| $\pi/4$ | $\sqrt{2}/2$ | $\sqrt{2}/2$ | $1$ |
| $\pi/2$ | $1$ | $0$ | und |
| $3\pi/4$ | $\sqrt{2}/2$ | $-\sqrt{2}/2$ | $-1$ |
| $\pi$ | $0$ | $-1$ | $0$ |
| $5\pi/4$ | $-\sqrt{2}/2$ | $-\sqrt{2}/2$ | $1$ |
| $3\pi/2$ | $-1$ | $0$ | und |
| $7\pi/4$ | $-\sqrt{2}/2$ | $\sqrt{2}/2$ | $-1$ |
| $2\pi$ | $0$ | $1$ | $0$ |

# UNIT CIRCLE




$$\tan\theta = \frac{y}{x}$$

$$\tan(-90^\circ) = 1/0 = undefined$$

Vertical asymptote

$$\tan(-60^\circ) = -\sqrt{3}$$

$$\tan(-30^\circ) = -\sqrt{3}/3$$

$$\tan 0^\circ = 0$$

$$\tan 30^\circ = \sqrt{3}/3$$

$$\tan 60^\circ = \sqrt{3}$$

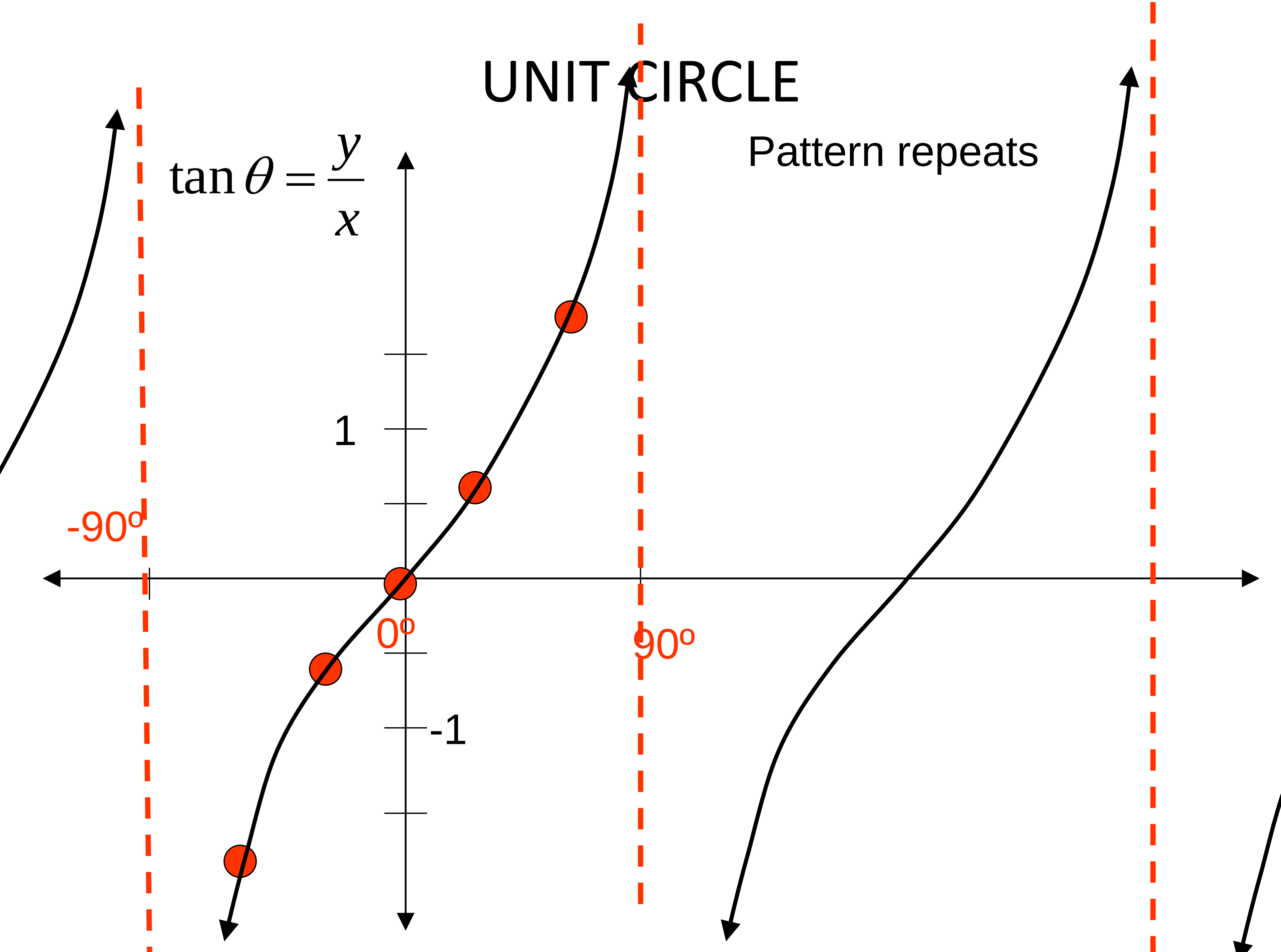
$$\tan 90^\circ = 1/0$$

Vertical asymptote

# UNIT CIRCLE

$$\tan\theta = \frac{y}{x}$$

Pattern repeats

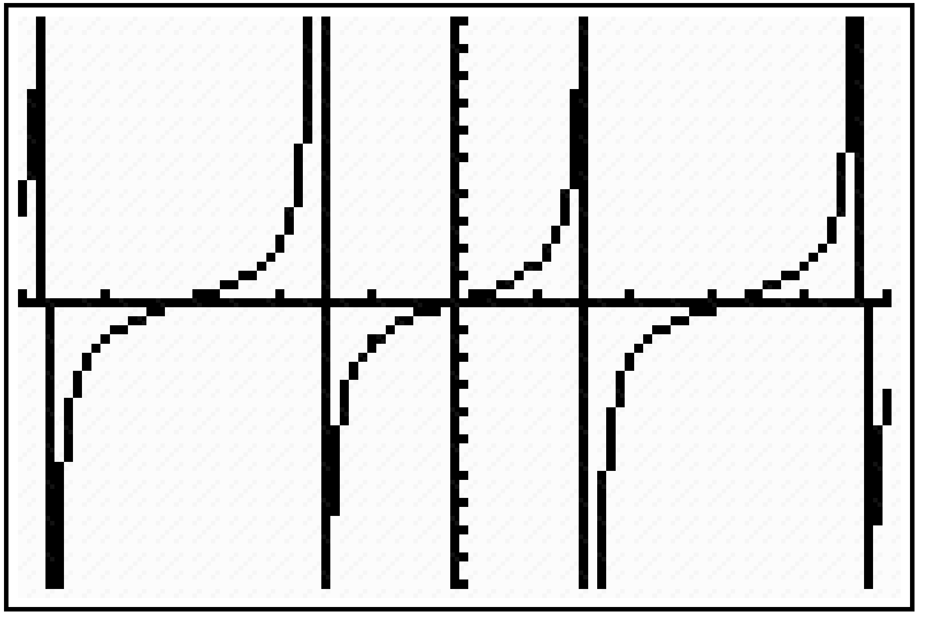


1. What are the “zeroes” of the tangent function?

(θ = ?)

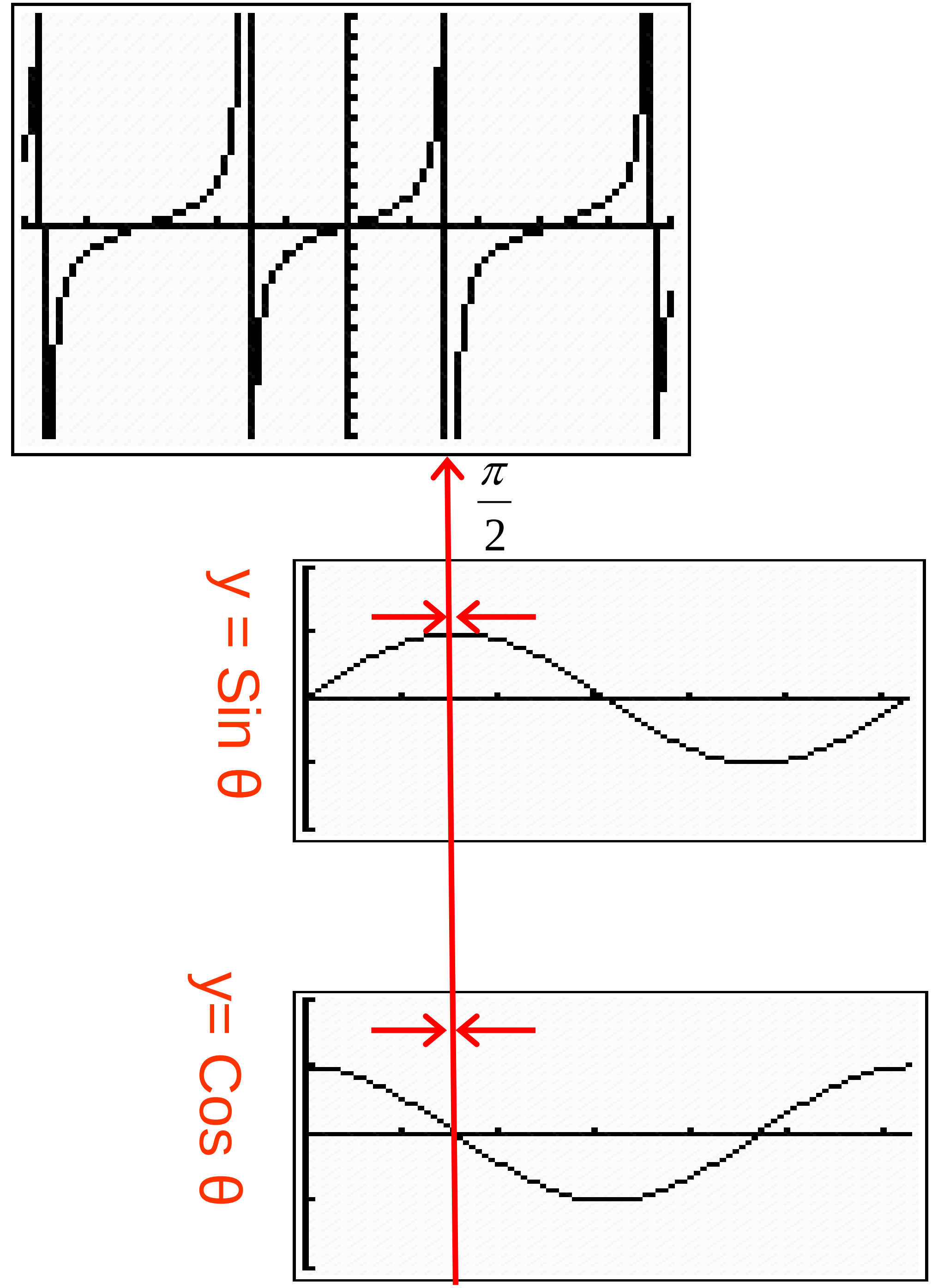
2. What are the vertical asymptotes of the tangent function?

$$\tan x = \frac{\sin x}{\cos x}$$



$\lim_{x \to -\frac{\pi}{2}} f\,(x) = ?$

$\lim_{x \to +\frac{\pi}{2}} f\,(x) = ?$



# Asymptotes and Zeros of the Tangent Function

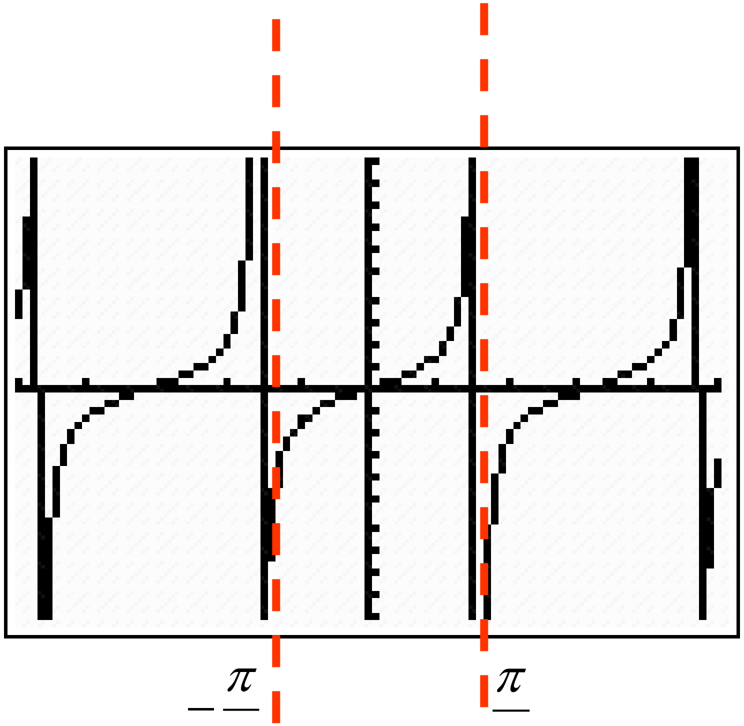
$$\tan x = \frac{\sin x}{\cos x}$$

X-intercepts → zeroes of the numerator: sinx = 0

Θ = ? for sin θ = 0

$0 \quad \pi \quad 2\pi \quad 3\pi$

$\theta = n\pi$ for $\theta \in \mathbb{Z}$




Vertical Asymptotes → zeroes of the denominator: cos $x$ = 0

Θ = ? for cos θ = 0

$\frac{\pi}{2} \quad \frac{3\pi}{2} \quad \frac{5\pi}{2} \quad \frac{7\pi}{2}$

Odd-numbered multiples of pi/2

$$\theta = n\pi + \frac{\pi}{2} \qquad \theta = (n + \frac{1}{2})\pi$$

$$\theta = \frac{(2n+1)\pi}{2} \quad for\ \theta \in \mathbb{Z}$$

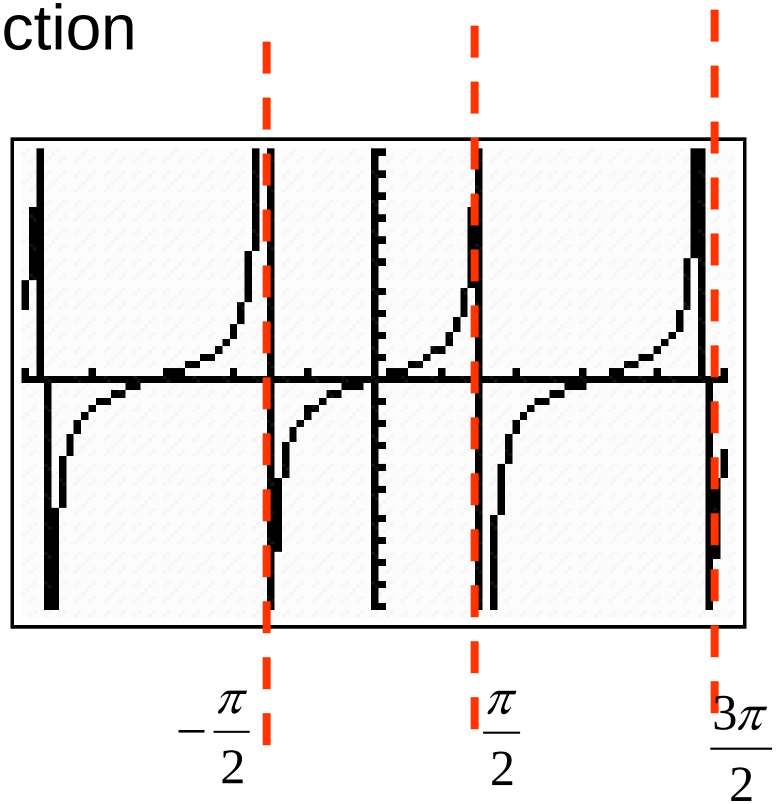
# The Tangent Function

$$\tan x = \frac{\sin x}{\cos x}$$

X-intercepts: sin $x$ = 0

Asymptotes: cos $x$ = 0

$$f(x) = \tan(bx - c) + d$$




Amplitude: doesn't have one (look at the graph)

Period: $\frac{\pi}{b}$ A full period of tan(x) occurs between 2 asymptotes which occur at odd multiples of $\frac{\pi}{2}$

Horiz. stretch factor causes asymptotes to be odd multiplies of: $\frac{\pi}{2b}$

# Tangent Function

$$f(x) = a\tan(bx - c) + d$$

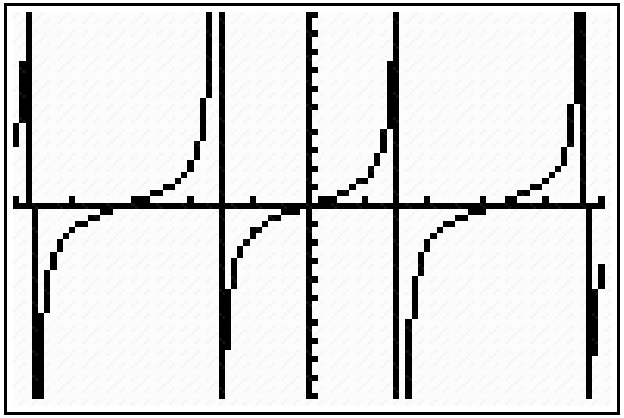
a = ±1 (always, why?)

$$\tan x = \frac{a\sin x}{a\cos x}$$

If a < 0: reflection across x-axis

Period: $\frac{\pi}{b}$

A full period occurs between 2 consecutive asymptotes.



Horizontal translation (phase shift)

<u>Careful</u>: if there is a Horizontal stretch factor, it will affect horizontal translation (phase shift) so we separate them *<u>by factoring</u>*

Vertical translation

Transformations of the tangent function: Describe how tan(x) is transformed to graph: -tan(2x)

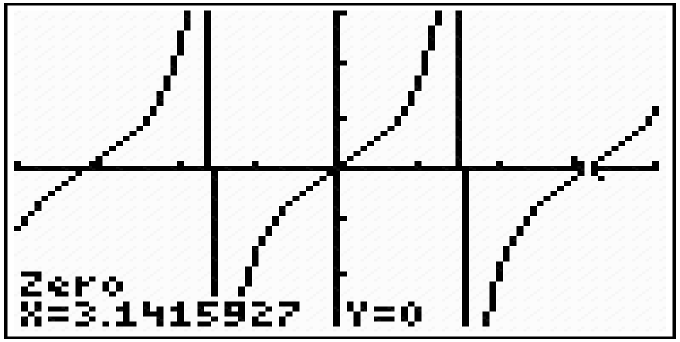
$$f(x) = \tan(bx - c) + d$$

1. Amplitude: n/a

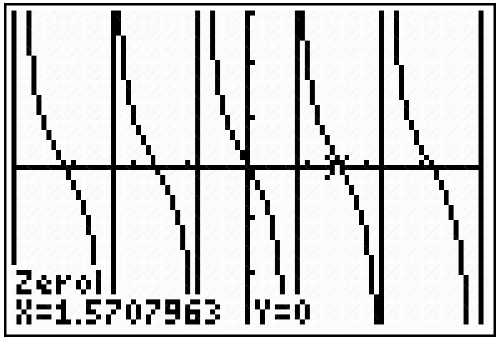
2. Period: $\frac{\pi}{b} = \frac{\pi}{2}$

3. Vertical Asymptotes: odd multiples of $\frac{\pi}{2b} = \frac{\pi}{4}$

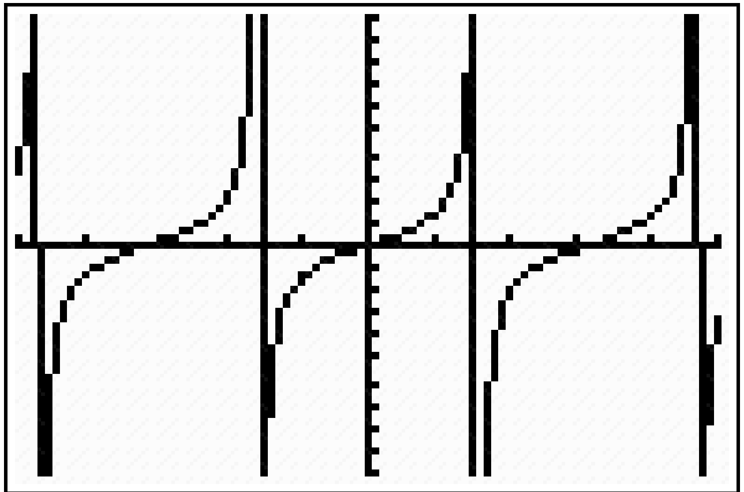
4. -1 coefficient: causes reflection across x-axis.




Tan(x)



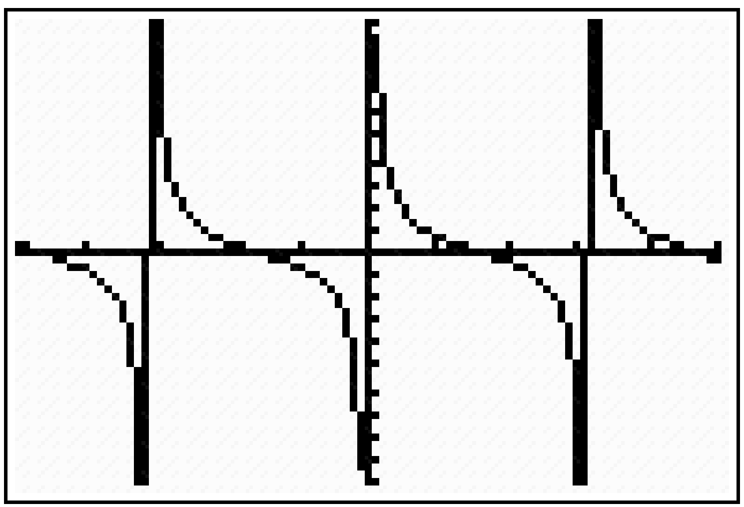

-tan(2x)



Tan θ

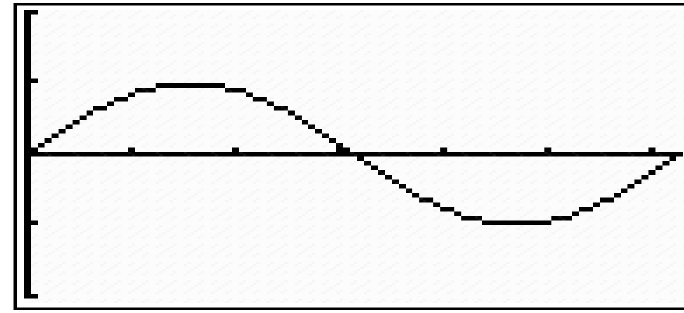
$\frac{\pi}{2}$

$$\cot\theta = \frac{1}{\tan\theta}$$

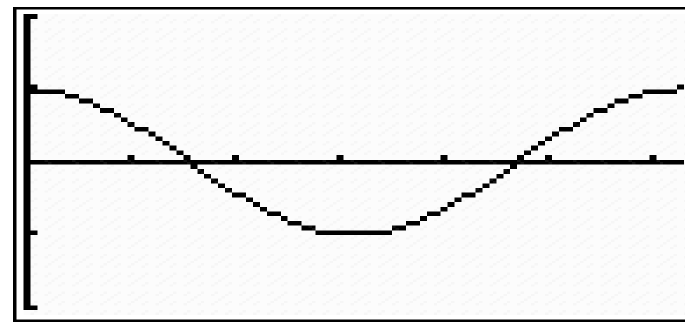


Cot θ

y = Sin θ

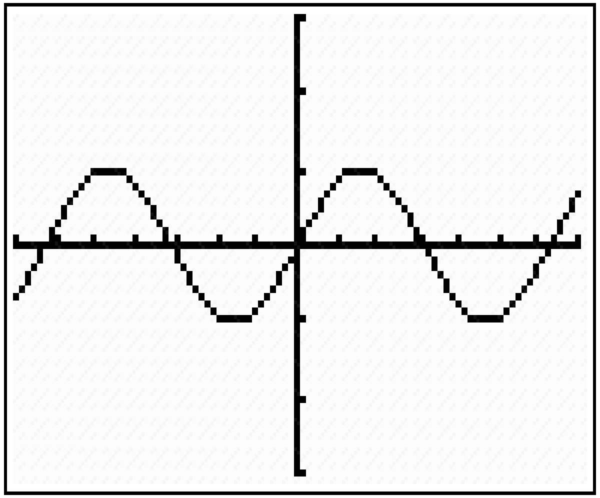


y= Cos θ



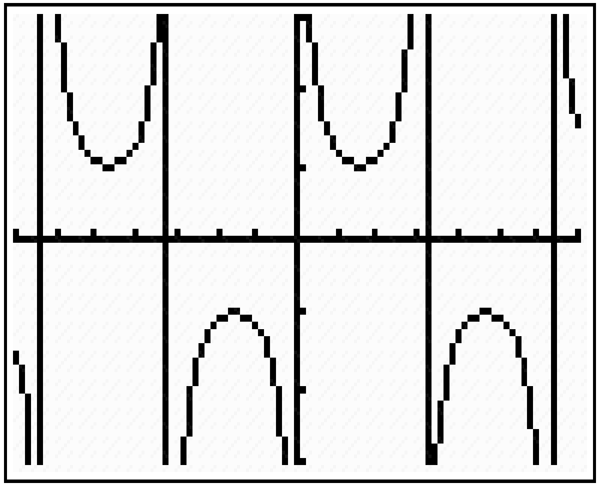
# The Sine and Cosecant Functions

Sine (x)



$$\csc\theta = \frac{1}{\sin\theta}$$

Cosecant (x)



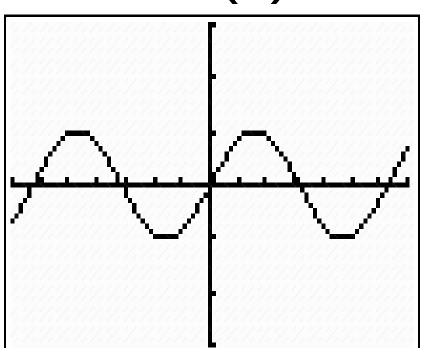
# The <u>Cosecant Function</u>: $y = 2 \csc 2x$

To graph transformations, rewrite the equation as it's reciprocal, then find zero's, max/mins (amp), and period.

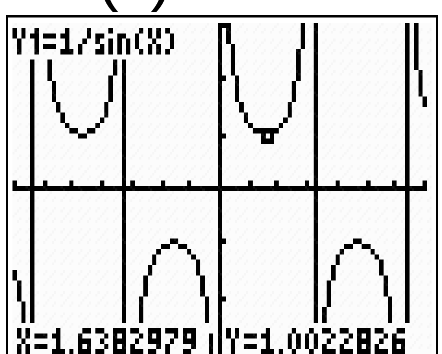
$$f(x) = \frac{a}{\sin(bx - c) + d} \qquad f(x) = \frac{2}{\sin(2x)}$$

1. Vertical stretch: factor of 2
2. Vertical asymptotes: even multiples of $\frac{\pi}{2} = \left(0 * \pi/2\right), \left(2 * \pi/2\right), etc.$
3. Horizontal shrink factor of ½, changes period to $\pi$

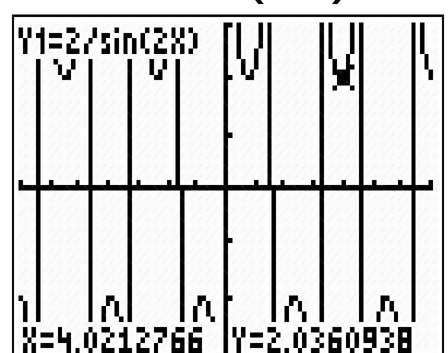
sin(x)



csc(x)

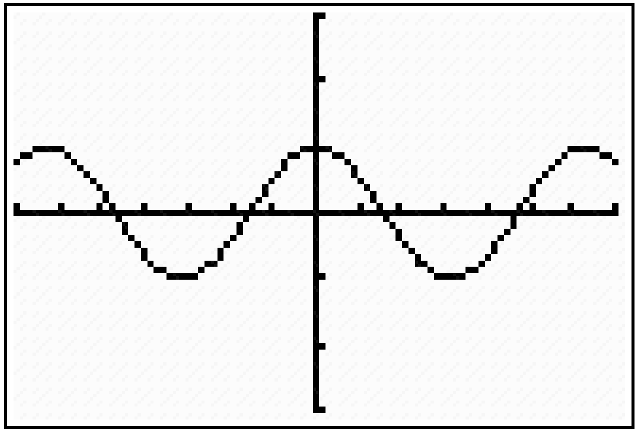



2csc(2x)



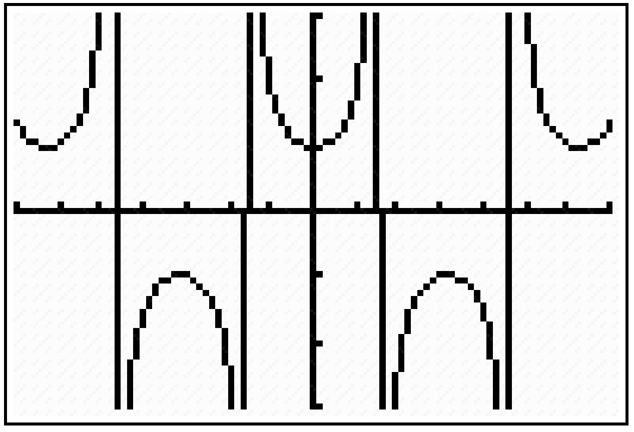
# The Cosine and Secant Functions

Cos x



$$\sec\theta = \frac{1}{\cos\theta}$$

Sec x



| Function | Period | Domain | Range |
|---|---|---|---|
| $\sin x$ | $2\pi$ | All reals | $[-1, 1]$ |
| $\cos x$ | $2\pi$ | All reals | $[-1, 1]$ |
| $\tan x$ | $\pi$ | $x \neq \pi/2 + n\pi$ | All reals |
| $\cot x$ | $\pi$ | $x \neq n\pi$ | All reals |
| $\sec x$ | $2\pi$ | $x \neq \pi/2 + n\pi$ | $(-\infty, -1] \cup [1, \infty)$ |
| $\csc x$ | $2\pi$ | $x \neq n\pi$ | $(-\infty, -1] \cup [1, \infty)$ |

| Function | Asymptotes | Zeros | Even/ Odd |
|---|---|---|---|
| $\sin x$ | None | $n\pi$ | Odd |
| $\cos x$ | None | $\pi/2 + n\pi$ | Even |
| $\tan x$ | $x = \pi/2 + n\pi$ | $n\pi$ | Odd |
| $\cot x$ | $x = n\pi$ | $\pi/2 + n\pi$ | Odd |
| $\sec x$ | $x = \pi/2 + n\pi$ | None | Even |
| $\csc x$ | $x = n\pi$ | None | Odd |