Math-1060

Lesson 5-2

Graphs of the sine and cosine functions; and Periodic Behavior

What is a function? $f(x)$

Function: a rule that matches each input to exactly one output.

What is the domain of a function?

Domain: the set of all allowable input values of a function (the ‘x’ values that have corresponding ‘y’ values).

What is the range of a function?

Range: the set of all output values of a function (the ‘y’ values).

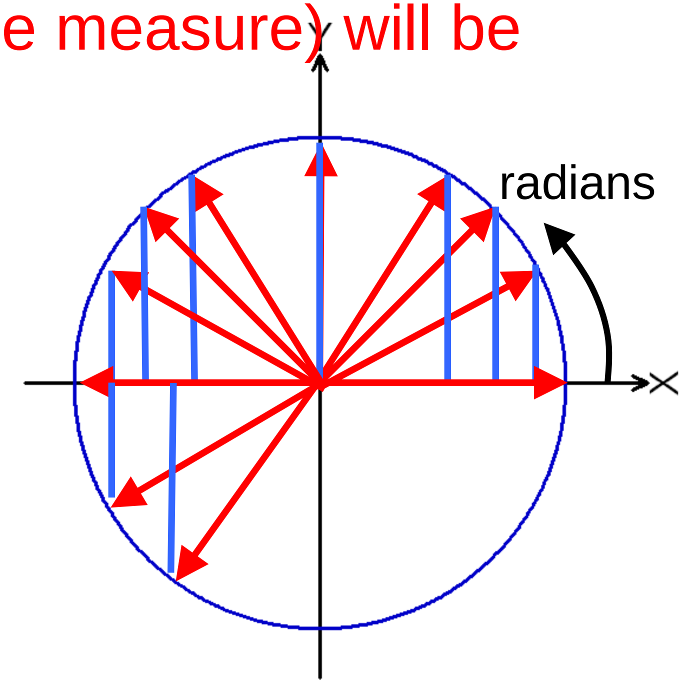
Graph of the Sine Function

$$f(\theta) = \sin\theta$$

Think of a point moving around the unit circle.

Input value: the angle

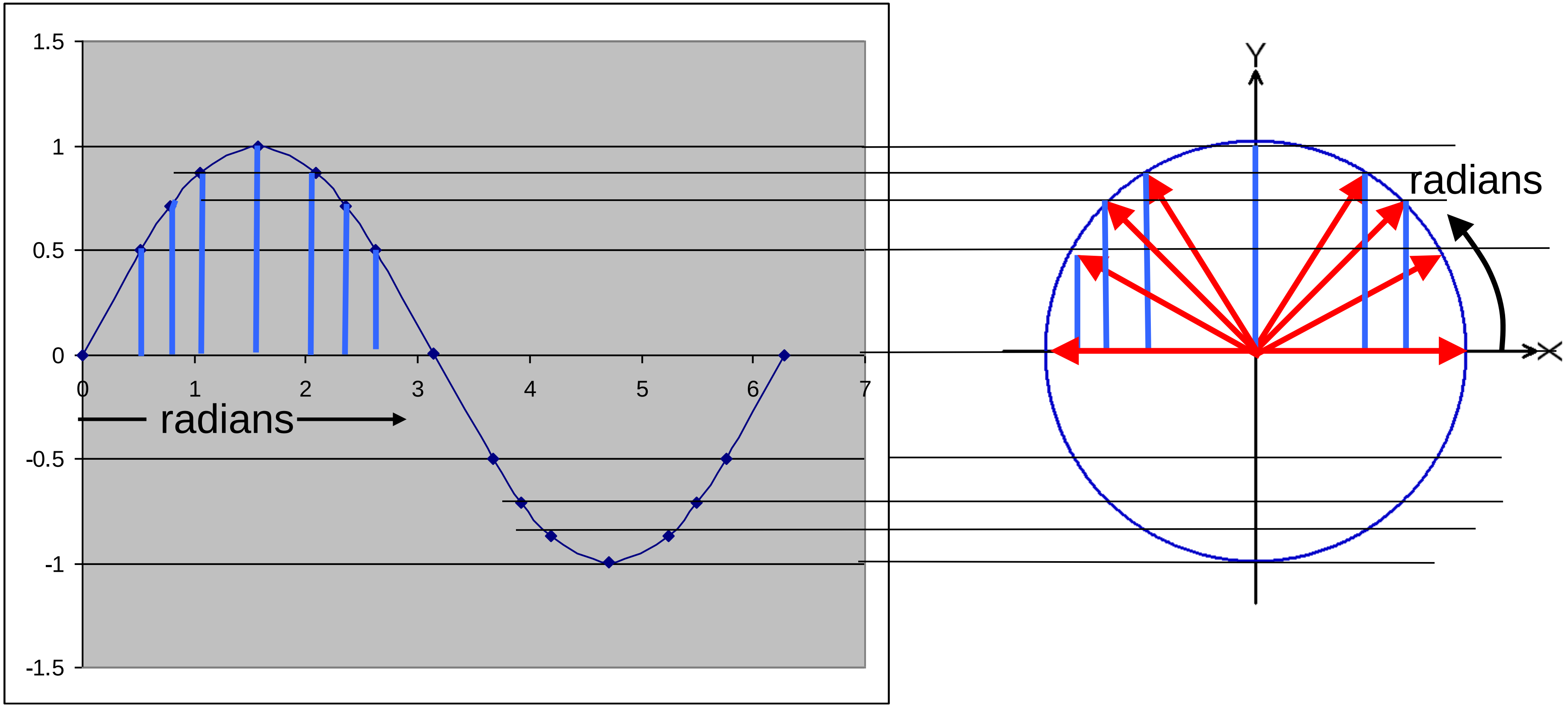
The input value (standard position angle measure) will be graphed on the x-axis of another plot



Output value: the y-value of the point as it goes around the circle.

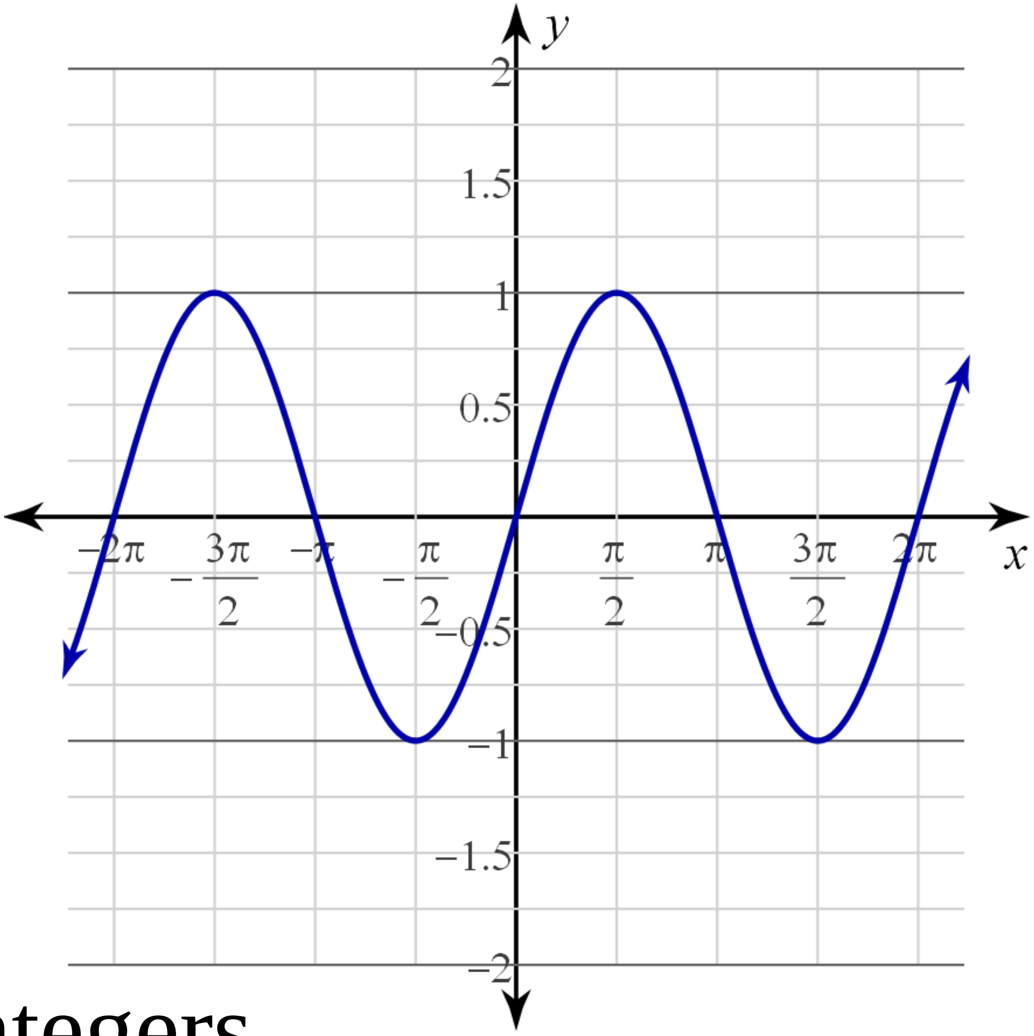
The output value (the y-value of the point) will be graph on the y-axis of the other plot

Graph of the Sine Function:



Think of sin θ as the distance that the point is above (or below) the x-axis, determined by 'θ' (the standard position angle passing through the point).

Sinusoid $f(\theta) = \sin\theta$



Domain = ?

$$-\infty < \theta < \infty$$

Range = ?

$$-1 \leq y \leq 1$$

x-intercepts?

$$x = n * \pi \quad \text{n} \in \text{integers}$$

The cosine function
(as opposed to the cosine ratio)

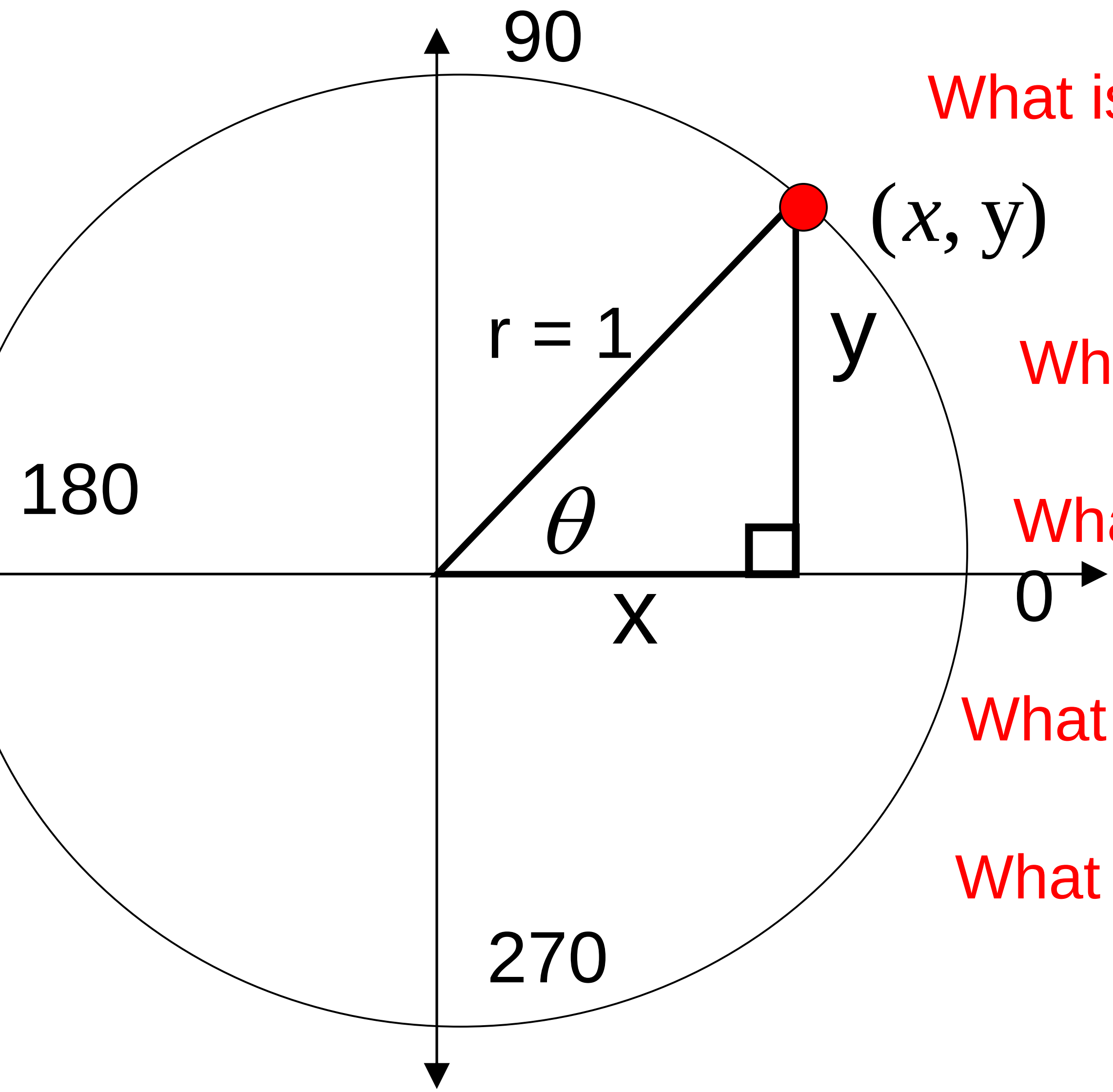
$$f(\theta) = \cos\theta$$

What is the input to the function?

angle

What is the output of the function?

x-value of the point



What is the maximum ϴ-value?

∞

What is the minimum ϴ-value?

-∞

What is the maximum y-value?

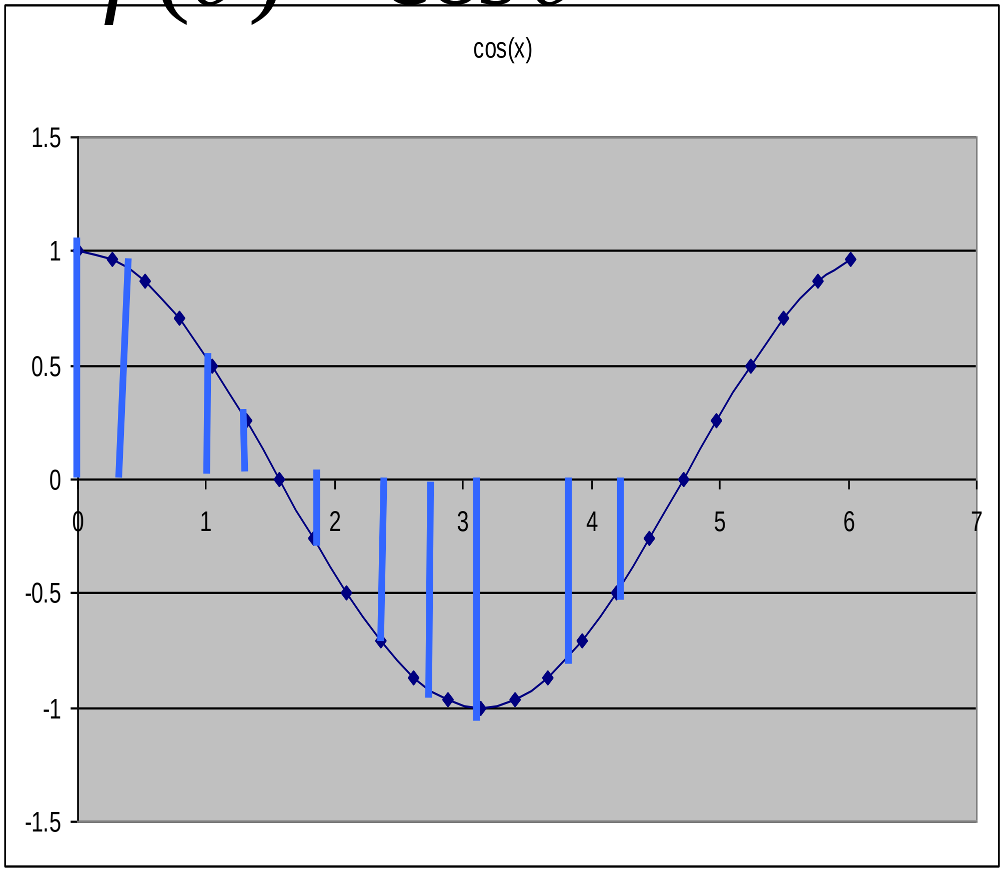
1

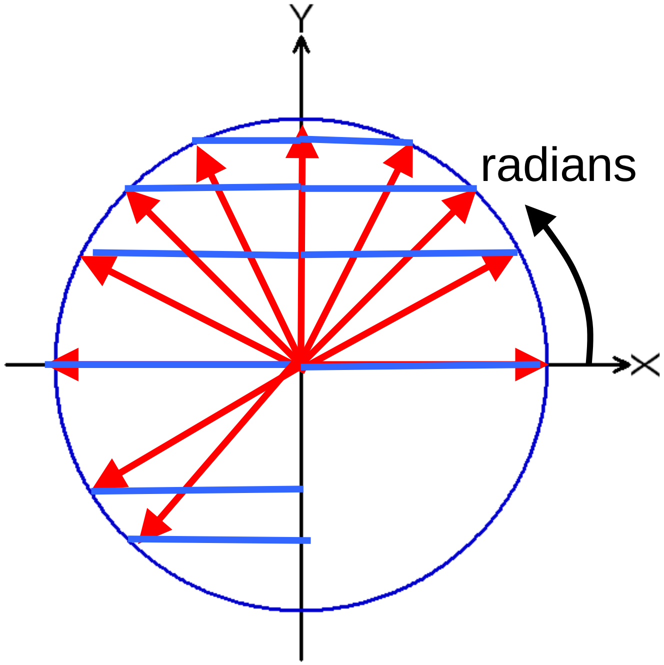
What is the minimum y-value?

-1

Graph of the Cosine Function

$$f(\theta) = \cos\theta$$





Think of a dot traveling around the circle to the right.

Think of <u>cos θ</u> as the distance <u>to the right</u> (or left) of the y-axis as determined by '<u>θ</u>' (the angle).

Degrees

$$f(\theta) = \cos\theta$$

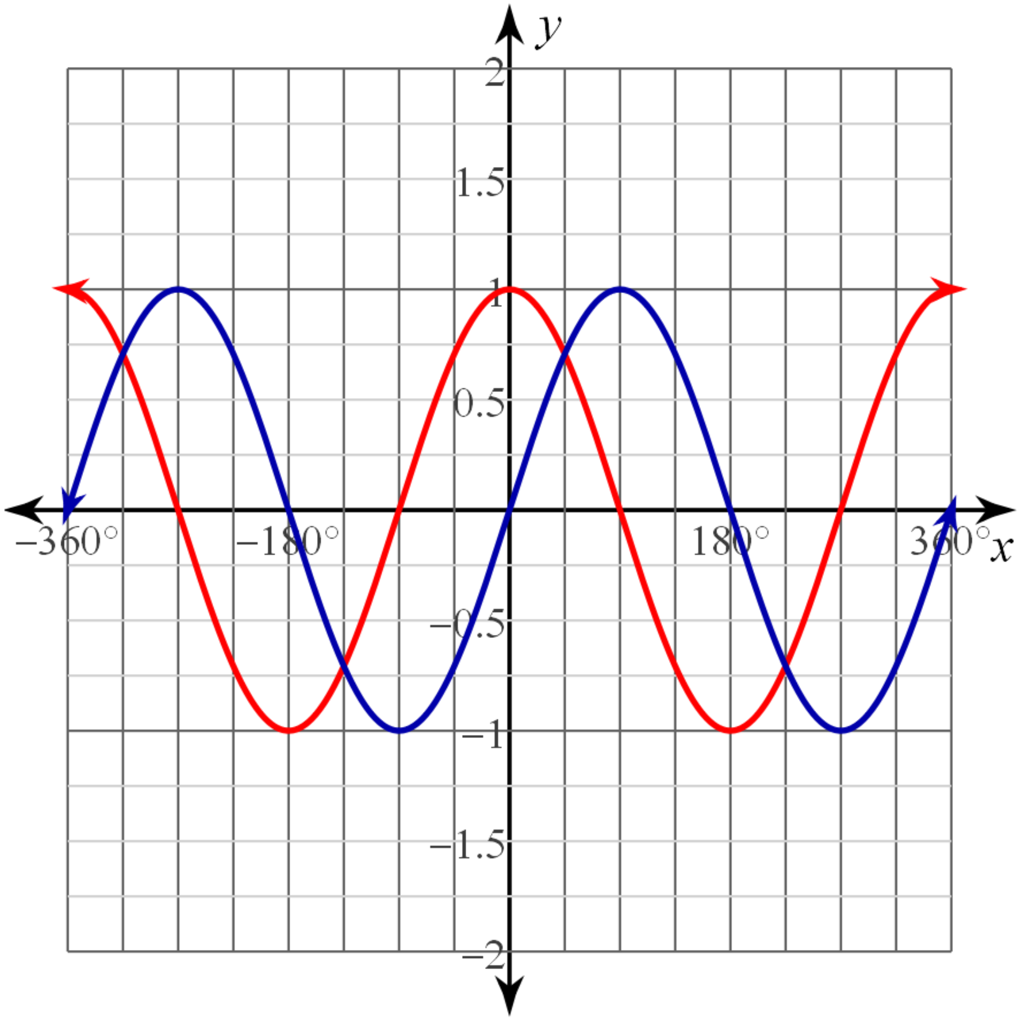
Domain = ?

$$-\infty < \theta < \infty$$

Range = ?

$$-1 \le y \le 1$$

x-intercepts?

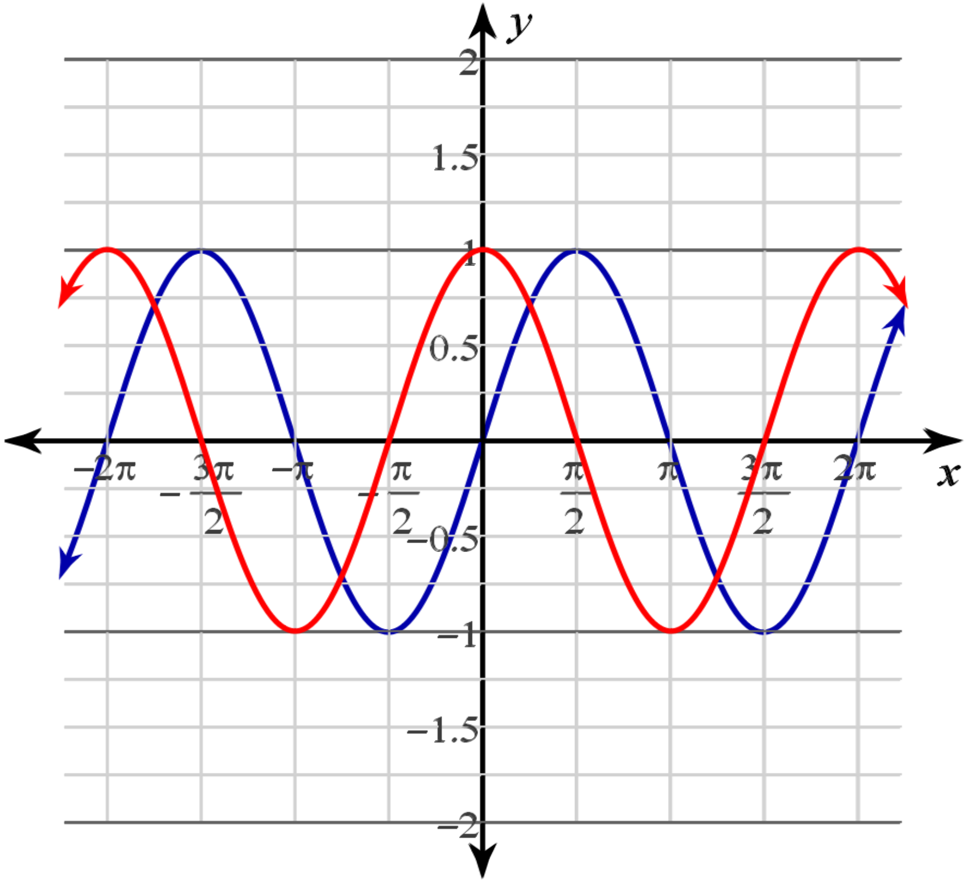


$$x = 90^0 + 180 * n \quad \text{n} \in \text{integers}$$

$$\cos(\theta - 90) = \sin(\theta)$$

Radians

$$f(\theta) = \cos\theta$$



x-intercepts?

$$x = \frac{\pi}{2} + n\pi \quad \text{n} \in \text{integers} \qquad \cos(\theta - \frac{\pi}{2}) = \sin(\theta)$$

The “Transformation Equation”

reflection · Vertical stretch · shift · shift

$$y = (-1) * a * f(x - c) + d$$

$$f(x) = -x^2$$ Reflection across x-axis

$$f(x) = a|x|$$ Vertical stretch

$$f(x) = |x - c|$$ Horizontal shift

$$f(x) = |x| + d$$ Vertical shift

Describe the transformations these function make on their parent functions.

$$f(x) = |x|$$

$$g(x) = 2|x-4| + 5$$

Vertically stretched by a factor of 2, shifted right 4 and up 5

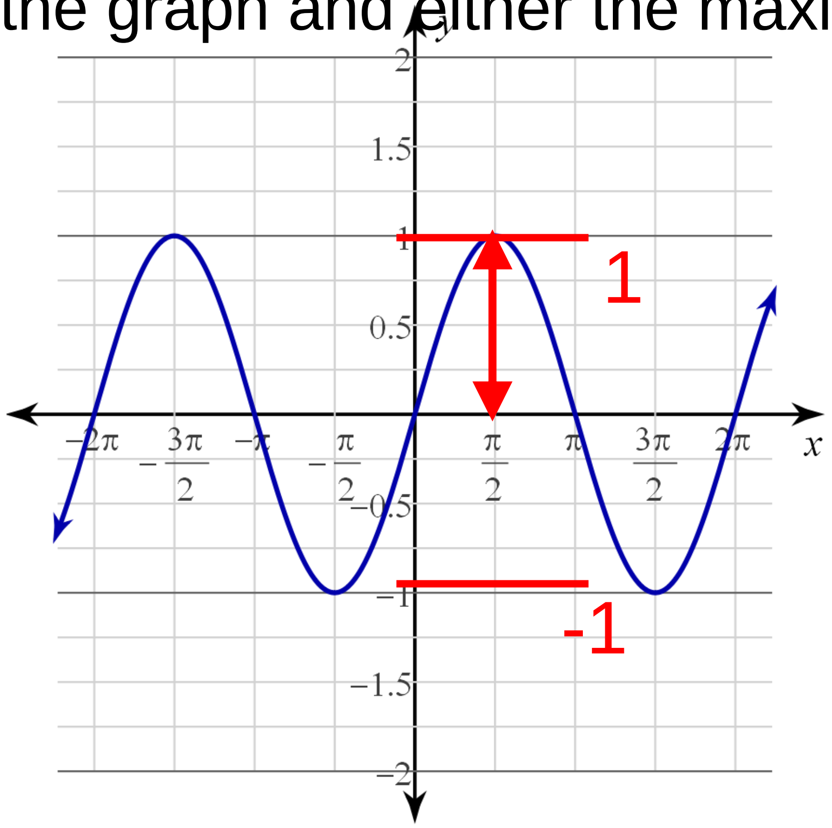
$$h(x) = x^2$$

$$k(x) = 0.25(x+1)^2 - 6$$

Vertically stretched by a factor of 0.25, shifted left 1 and down 6

$$f(x) = a \sin x$$

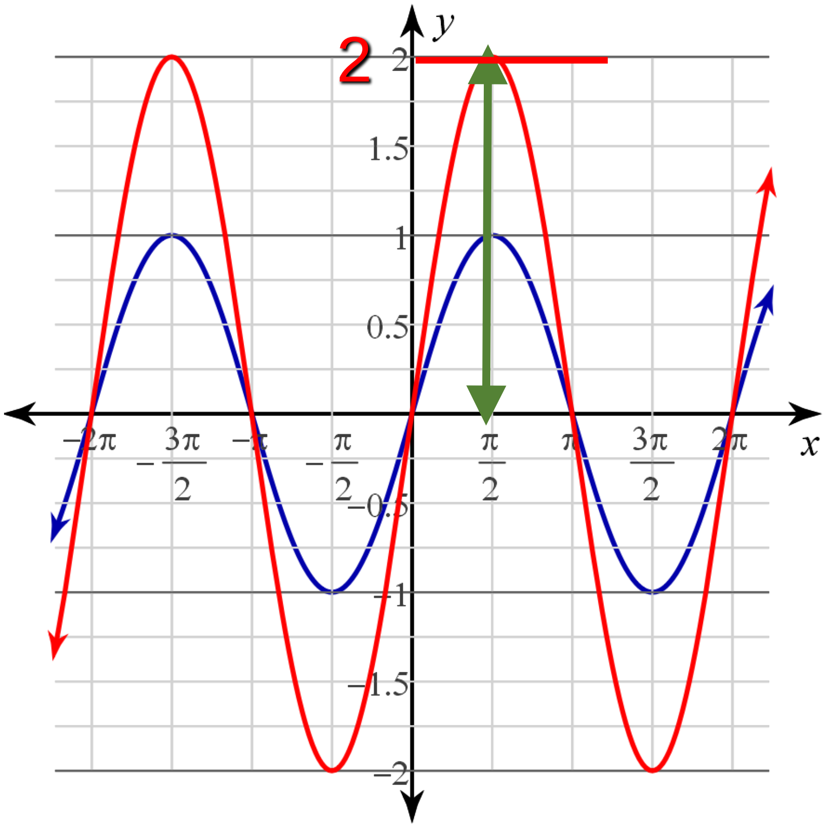
Amplitude: The vertical distance between the centerline of the graph and either the maximum or minimum output value.



For the sine and cosine functions, we call the coefficient ‘a’ the amplitude of the function (which in general refers to the vertical stretch factor).

Compare:

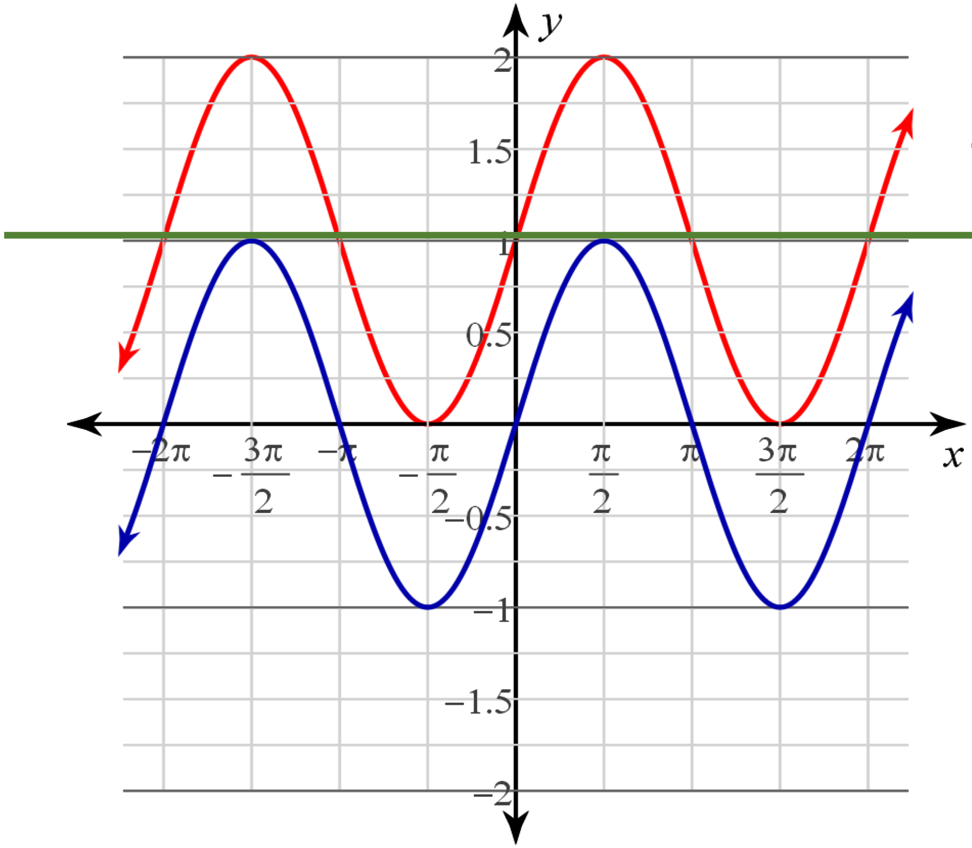
$f(x) = \sin x \qquad g(x) = 2\sin x$



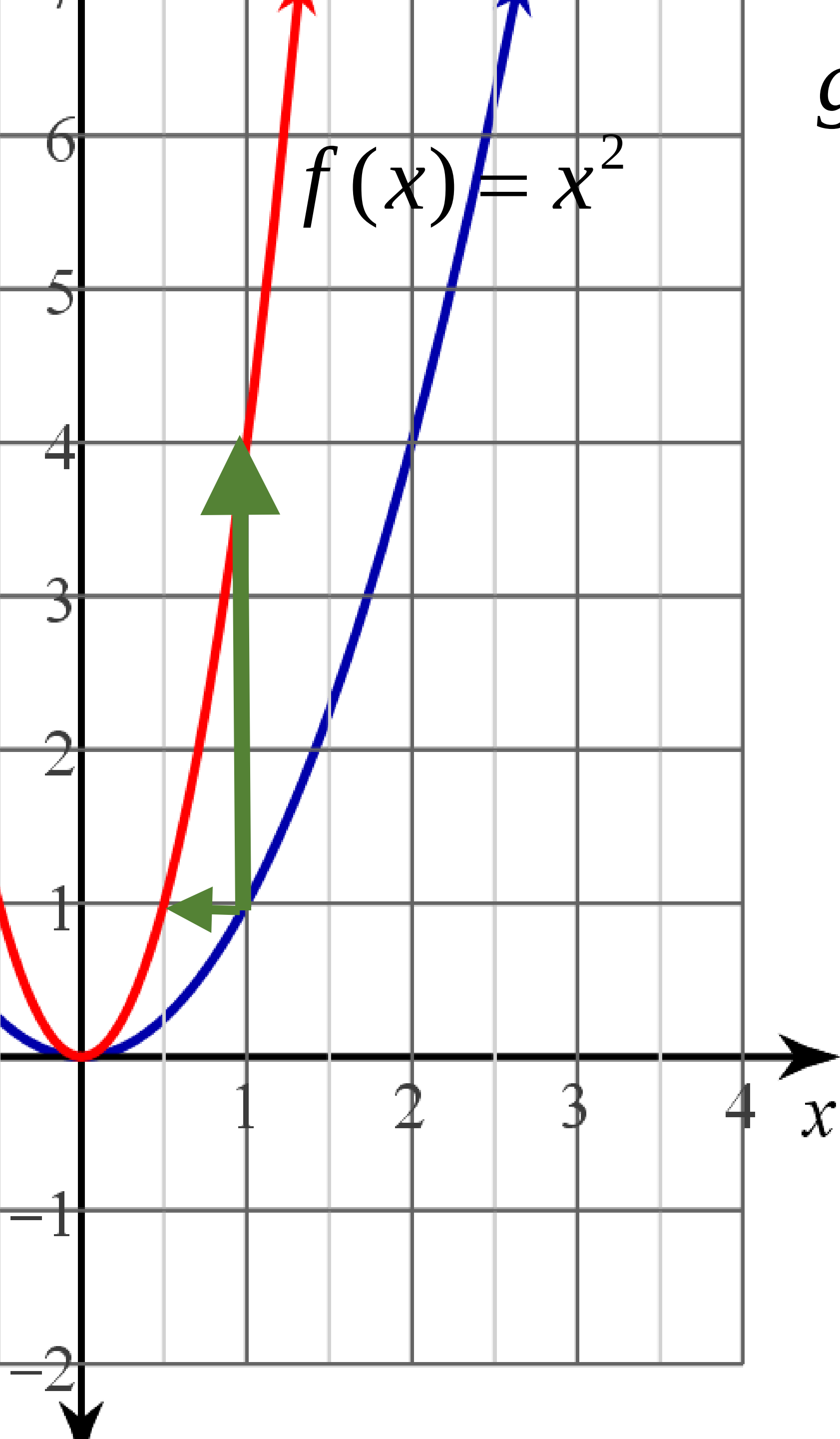
$$g(x) = \sin x$$

$$f(x) = 1 + \sin x$$

shift



Centerline of the Oscillation: corresponds to the up/down translation.



$$g(x) = 4f(x) \qquad g(x) = 4x^2$$

<u>Vertical Stretch</u>: multiplying the original function by 4, "vertically stretches" it by a factor of 4.

<u>Horizontal Stretch</u>: replacing 'x' with a number multiplied by 'x',

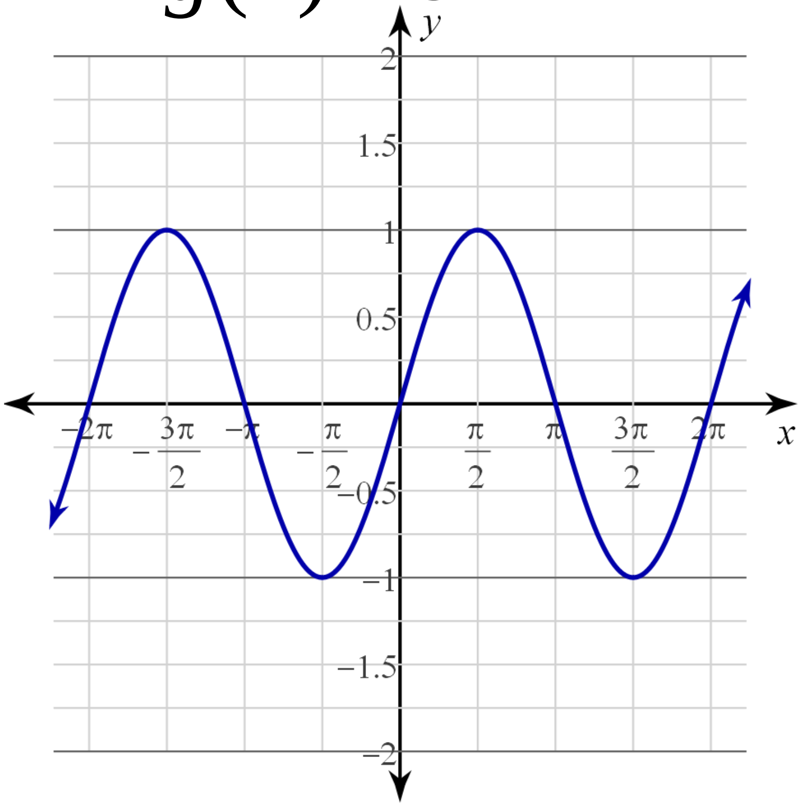
$$h(x) = (2x)^2 \quad = 4x^2$$

For the <u>square function</u> a <u>vertical Stretch</u> is the same as a <u>horizontal shrink</u> (so you really can't tell the difference between them).

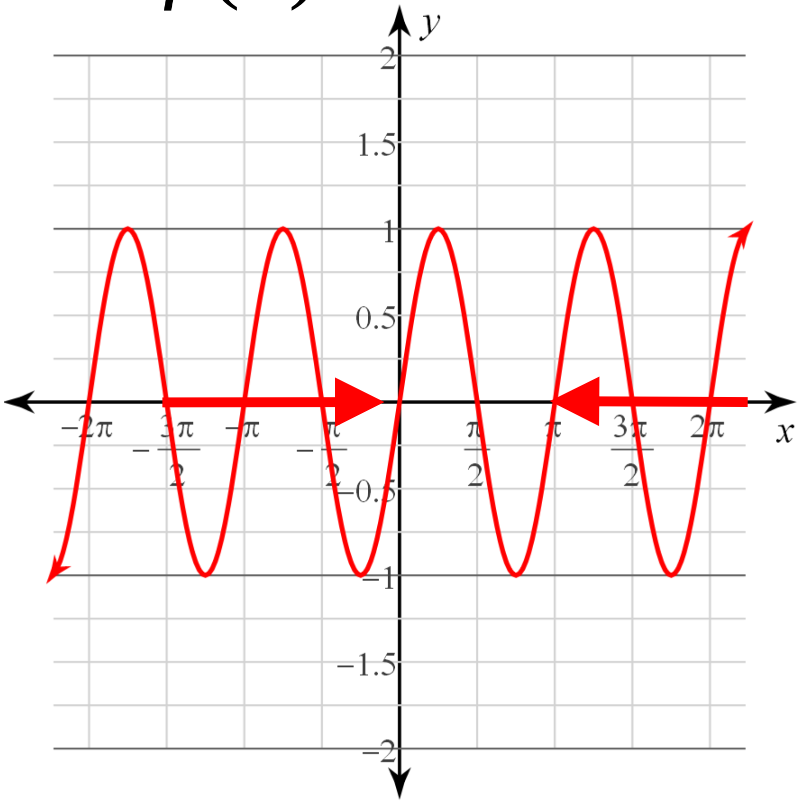
Predict what you think happens

Horizontal stretch or shrink?

$$g(x) = \sin x$$



$$f(x) = \sin 2x$$



horizontal shrink

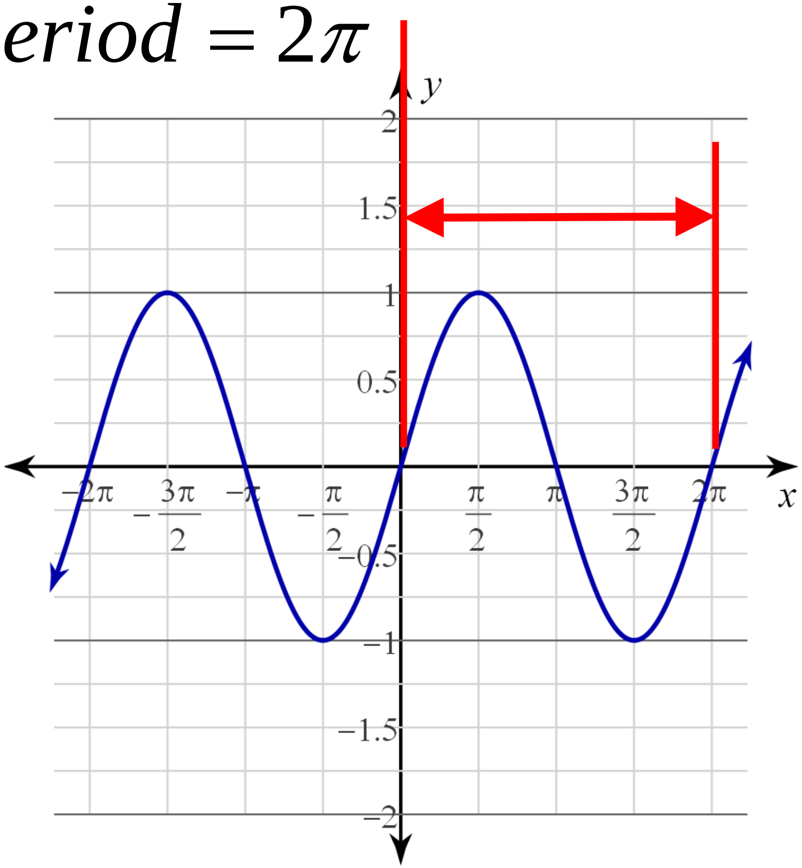
Stretched by a factor of ½ (we just use the word stretch)

$$f(x) = a\sin bx$$

Period: the horizontal distance along the x-axis needed to complete one full cycle of the oscillation.

$$g(x) = \sin x$$

$$Period = 2\pi$$



Period = $\dfrac{2\pi}{|b|}$

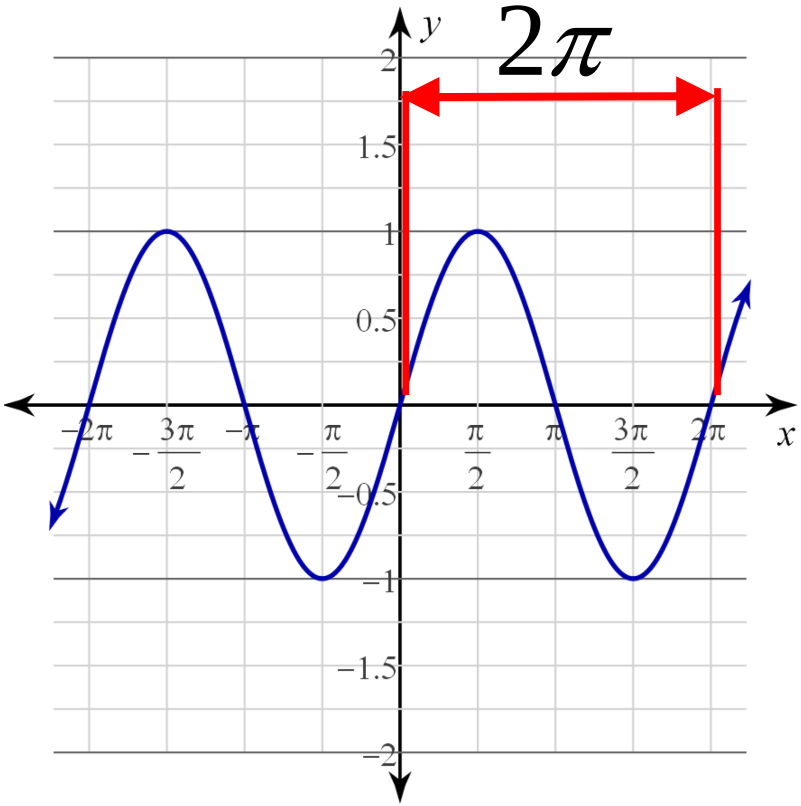
Frequency = $\dfrac{|b|}{2\pi}$

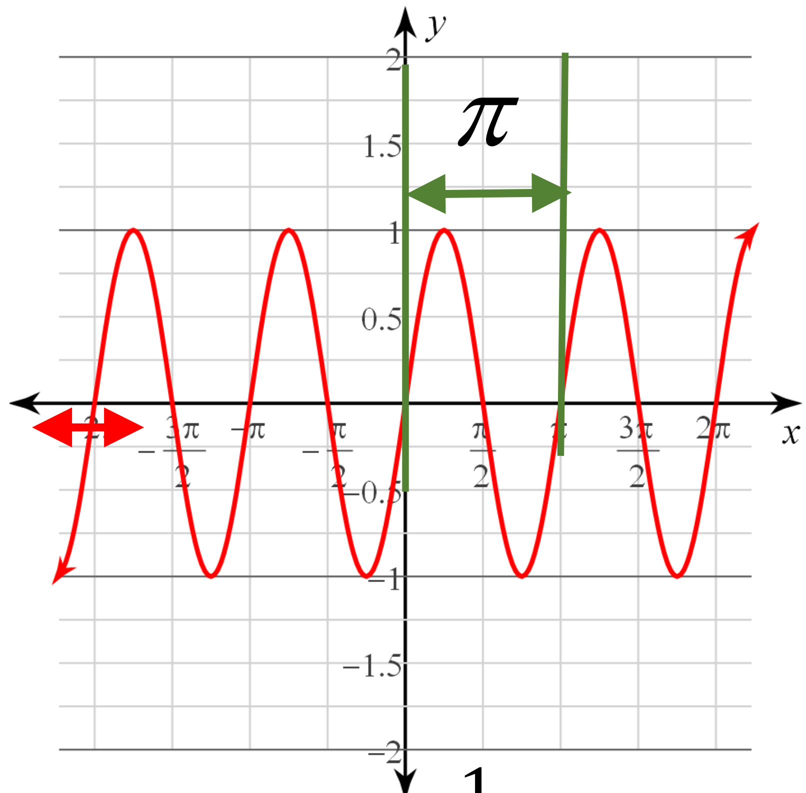
Frequency = 1/period

$$g(x) = \sin x$$

$$f(x) = \sin 2x$$

$$\text{horizontal stretch factor} = \frac{1}{2}$$





Frequency = 1 cycle every 2pi radians.

$$Period = \frac{1}{2} * 2\pi = \pi$$

Frequency = 1 cycle every pi radians.

Compare: $f(x) = a \sin bx$

$g(x) = \sin x$ $\quad f(x) = \sin 3x$

horizontal stretch factor = $\frac{1}{b}$ $\quad = \frac{1}{3}$

What is the period of g(x) ? $= 2\pi$

What is the period of f(x) ? $= \frac{1}{b} * 2\pi \quad = \frac{1}{3} * \frac{2\pi}{b} \quad = \frac{2\pi}{3}$

What is the frequency of f(x) ?

Frequency = 3 cycles every 2pi radians.

Your turn: $f(x) = a \sin bx$

$g(x) = \cos x$ $f(x) = 4\cos 5x$

What is the horizontal stretch factor ?. $= \frac{1}{5}$

What is the period of g(x) ? $= 2\pi$

What is the period of f(x) ? $= \frac{2\pi}{b} = \frac{2\pi}{5}$

What is the amplitude of f(x) ? $= 4$

What is the frequency of f(x) ? $= \frac{5}{2\pi}$

5 cycles every 2pi radians.

Vertical and now horizontal stretch factors

$$f(x) = a\sin bx$$

$$f(x) = -2\sin\left(\frac{\pi}{2}x\right)$$

a: Vertical stretch factor $= a$

Reflected across x-axis.

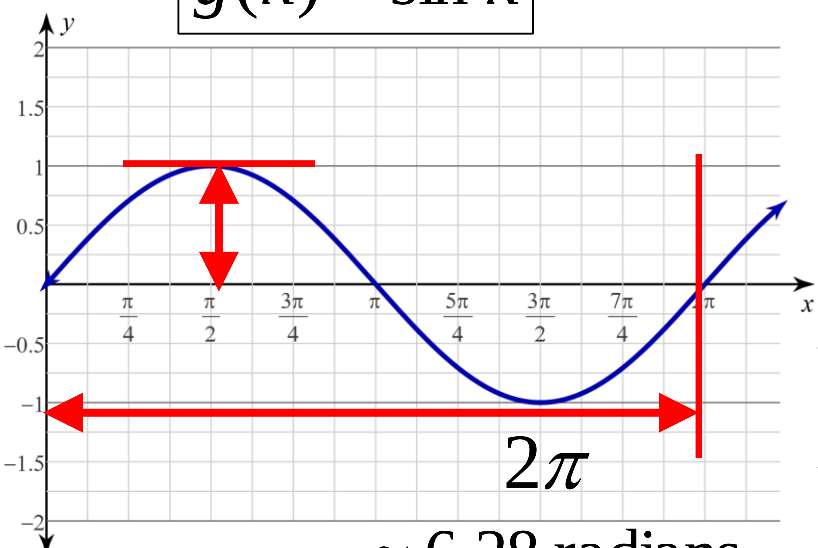
Vertically stretched by a factor of 2.

b: horizontal stretch factor. $= \frac{1}{b}$

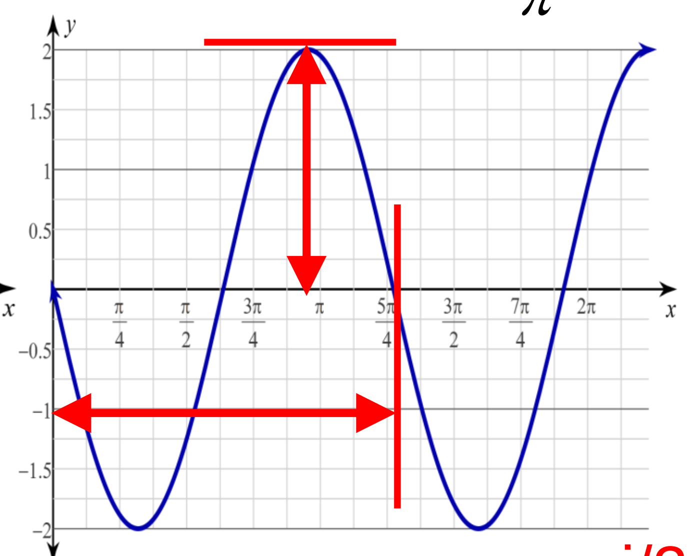
Horizontally stretched by a factor of $\frac{2}{\pi}$

Period = HSF*2π $\frac{2}{\pi} * 2\pi = 4\text{ radians}$

$$g(x) = \sin x$$



$\approx 6.28\text{ radians}$



Frequency = 1/period

pi/2 cycles every 2pi radians.

$g(x) = \sin x$ $f(x) = -5\sin 3x$

Describe how f(x) is a transformation of the parent function g(x).

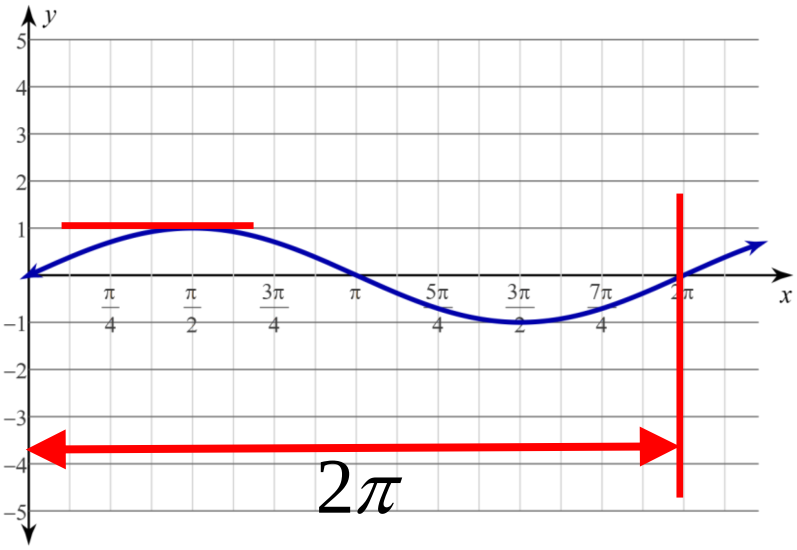
$f(x) = -5\sin 3x$ Reflected across x-axis.

Vertically stretched by a factor of 5 (amplitude).

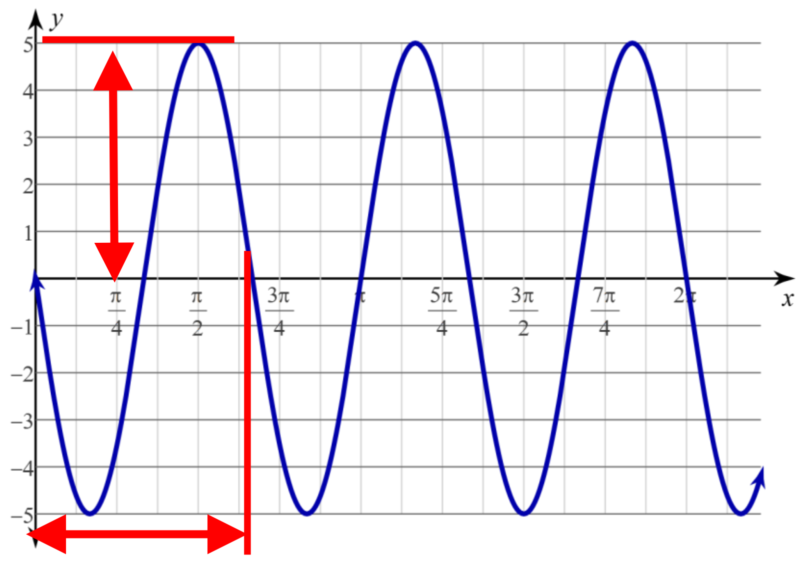
Horizontally shrunk by a factor of $\frac{1}{3}$

Period = HSF * 2π = $\frac{2\pi}{3}$ radians

Frequency = 3 cycles every 2pi radians.



$g(x) = \sin x$



$\frac{2\pi}{3}$ radians

Equivalent Equations
(input variable is ***theta***)

$$f(x) = \mathrm{a}\sin(b\theta - c) + k$$

In this version, the left/right shift is “mixed together” with the horizontal stretch factor.

$$f(x) = a\sin b\,(\theta - c/b) + k$$

By factoring out the coefficient of theta, we have separated the HSF from the phase shift.

$$f(x) = 4sin\,(3\theta - \pi) + 2$$

$$f(x) = 4\sin 3\,(\theta - \pi/3) + 2$$

$$f(x) = -5sin\,(\frac{\theta}{3} - \frac{\pi}{2}) + 2$$

$$f(x) = 4\sin\frac{1}{3}(\theta - \frac{3\pi}{2}) + 2$$

$$f(x) = 3sin\left(2\theta + \frac{\pi}{2}\right)$$

$$f(x) = a\sin(bx - c) + d$$

$$f(x) = a\sin b(x - c/b) + d$$

$$f(x) = + 3\sin 2\left(\theta - \frac{\pi}{4}\right)$$

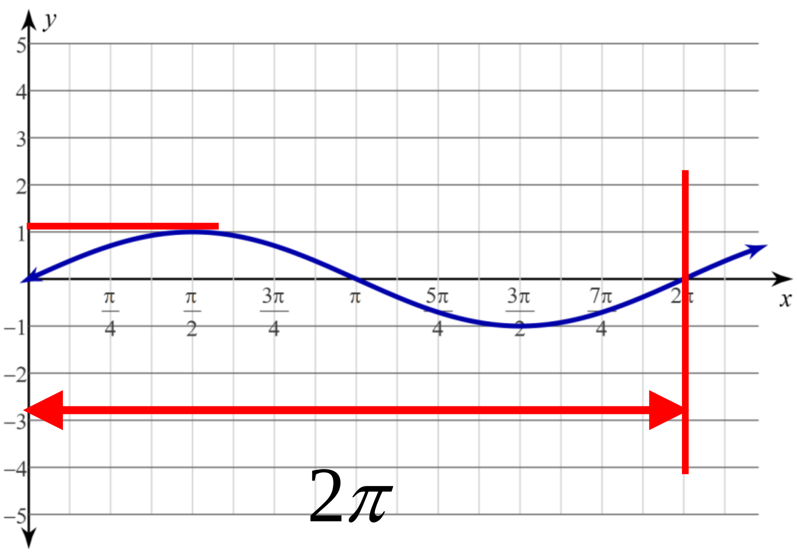
VSF = 3   HSF = 1/2

Not Reflected across x-axis.

Shifted right by π/4 radians

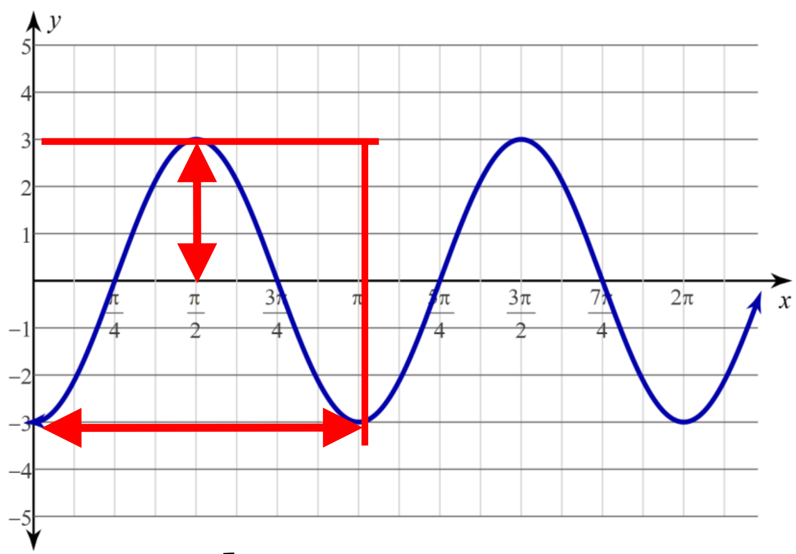
Frequency = 2 cycles every 2pi radians → 1/pi

$$g(x) = \sin x$$



2π

≈ 6.28 radians



π radians

$$f(x) = -0.5\sin 3\left(x + \frac{\pi}{4}\right) - 2$$

Amplitude = ? 0.5 units

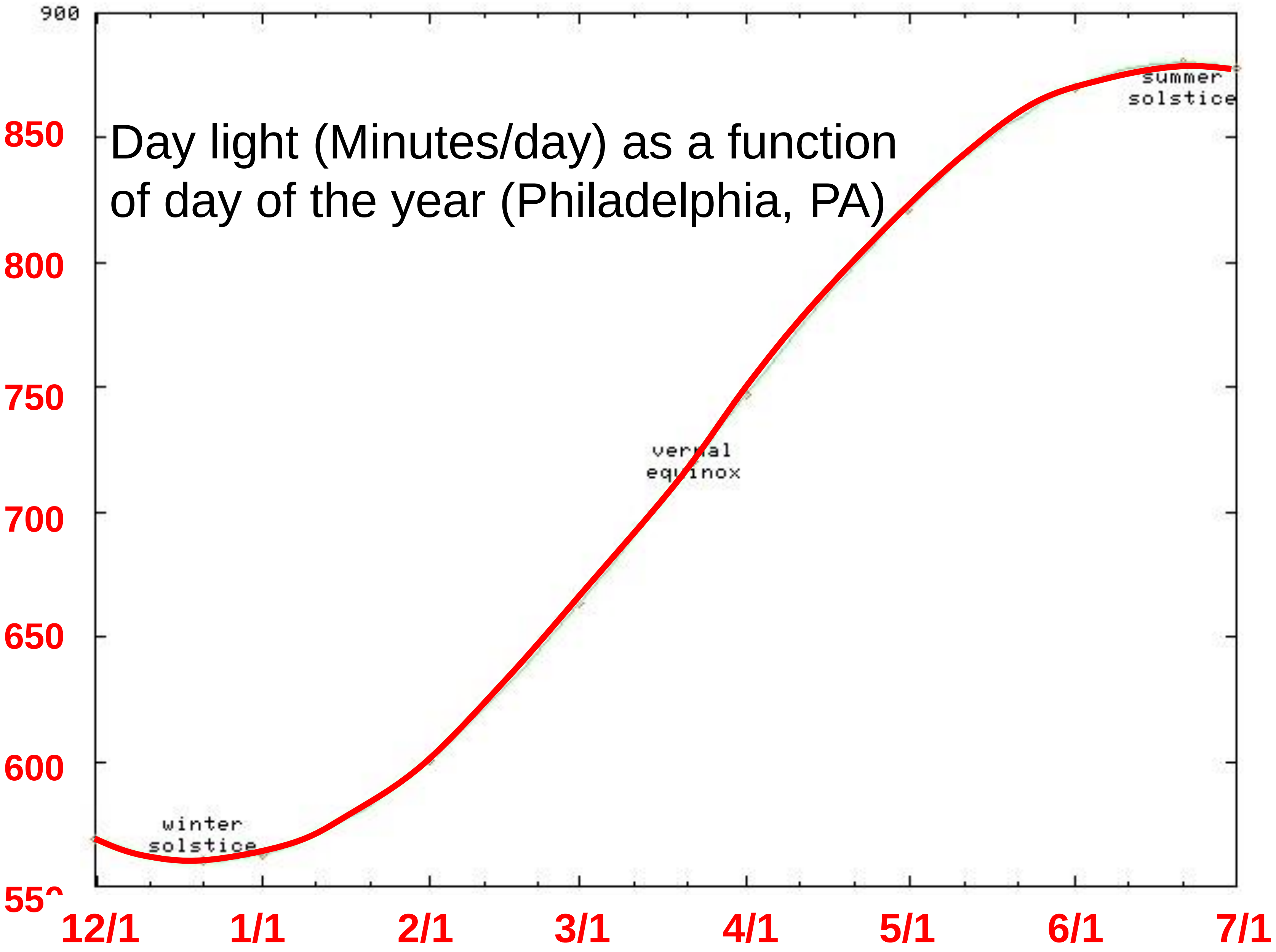
Phase shift = ? Left pi/4

Period = ? 2pi/3 radian per cycle.

Frequency = ? 3 cycles every 2pi radians.

Center line: y = -2

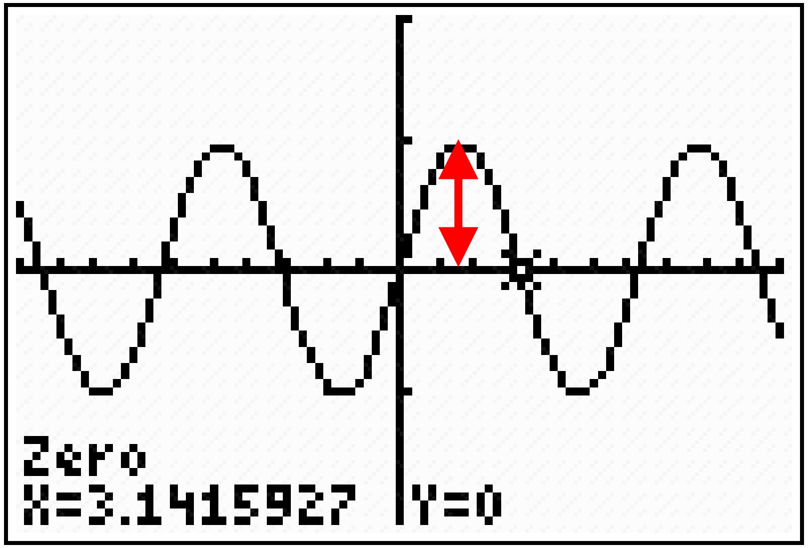
Periodic Behavior: a pattern that repeats itself over a fixed period of time.

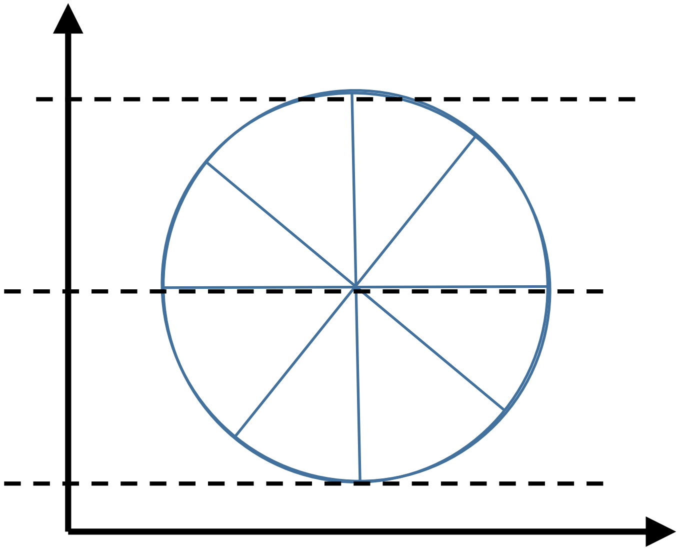


Sinusoid: $h(t) = a\sin(bt) + k$

The Radius of the Lagoon Ferris Wheel is 21.8 m. The bottom of the Ferris Wheel is 1.2 meter above ground level (you have to go up onto a platform to get on the Ferris Wheel). Once the Ferris Wheel is spinning it takes 40 seconds to complete one revolution.

Write an equation that models the height of a point on the Ferris wheel as a function of time.

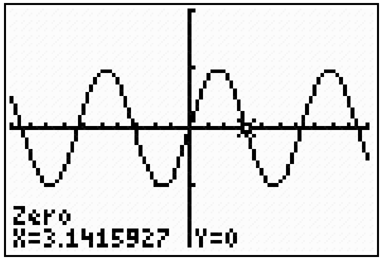




Sinusoid: $h(t) = a\sin(bt) + k$

The Radius of the Lagoon Ferris Wheel is 21.8 m. The bottom of the Ferris Wheel is 1.2 m above ground level (you have to go up onto a platform to get on the Ferris Wheel). Once the Ferris Wheel is spinning it takes 40 seconds to complete one revolution.

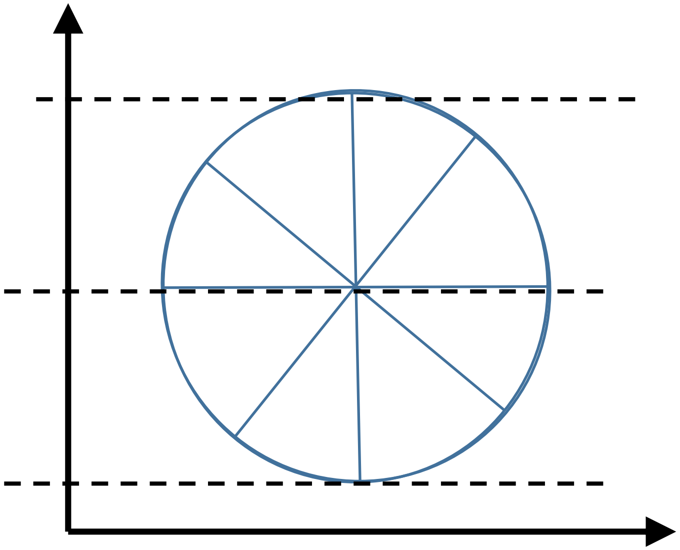
Period = $\frac{2\pi}{|b|}, \frac{360^\circ}{|b|}$ (time required to complete one cycle)



$$40 = \frac{360}{|b|}$$

$$b = \frac{360}{40} = 9$$

$$h(t) = a\sin(9t) + k$$



Sinusoid: $h(t) = a\sin(bt) + k$ $\quad h(t) = a\sin(9t) + k$

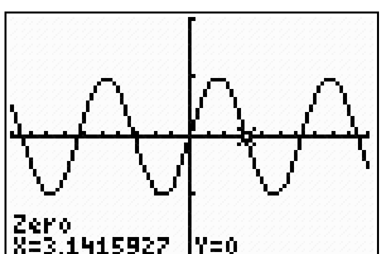
The Radius of the Lagoon Ferris Wheel is 21.8 m. The bottom of the Ferris Wheel is 1.2 m above ground level (you have to go up onto a platform to get on the Ferris Wheel). Once the Ferris Wheel is spinning it takes 40 seconds to complete one revolution.

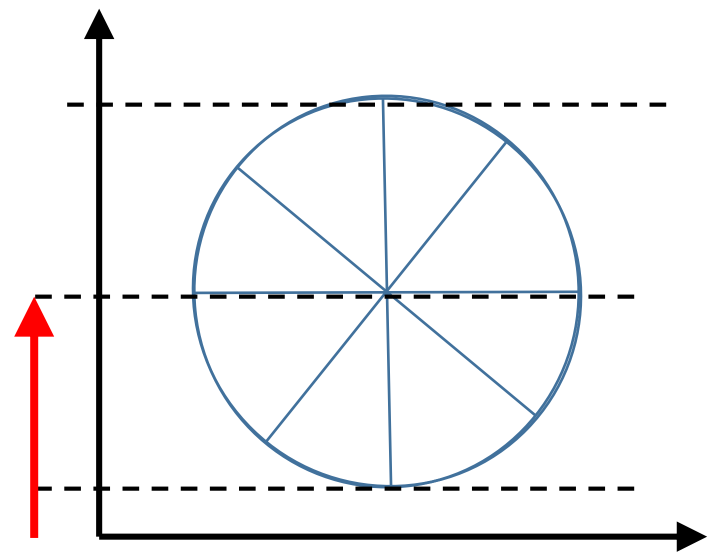
k = Vertical shift = centerline of the oscillation.

$$k = 1.2\ \text{m} + 21.8\ \text{m}$$

$$k = 23\ \text{m}$$

$$h(t) = a\sin(9t) + 23$$





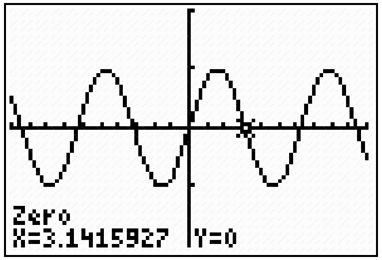
Sinusoid: $h(t) = a\sin(bt) + k$ $h(t) = a\sin(9t) + 23$

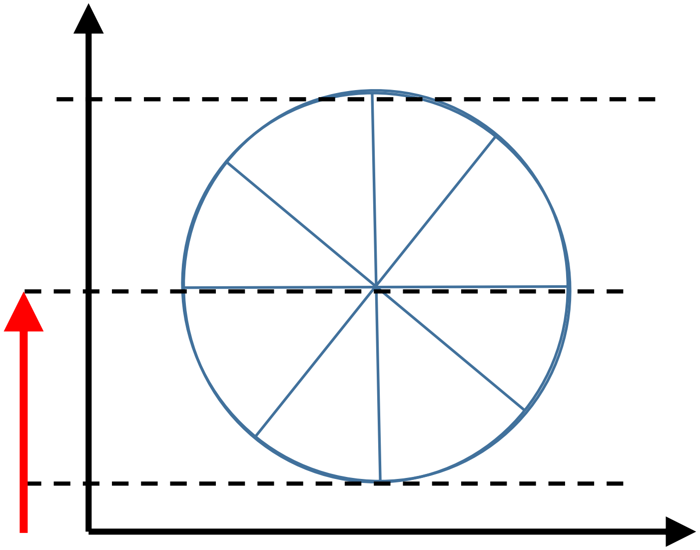
The Radius of the Lagoon Ferris Wheel is 21.8 m. The bottom of the Ferris Wheel is 1.2 m above ground level (you have to go up onto a platform to get on the Ferris Wheel). Once the Ferris Wheel is spinning it takes 40 seconds to complete one revolution.

amplitude = one half the "peak to peak" distance = circle radius

$a = 21.8\text{ m}$

$h(t) = 21.8\sin(9t) + 23$





Harmonic Motion

$$d(t) = a\sin(bt) + k$$

A mass oscillating up and down on the bottom of a spring can be modeled as harmonic motion (assuming perfect elasticity and no friction or air resistance).

If the weight is displaced a maximum of 4 cm, find the modeling equation if it takes 3 seconds to complete one cycle.

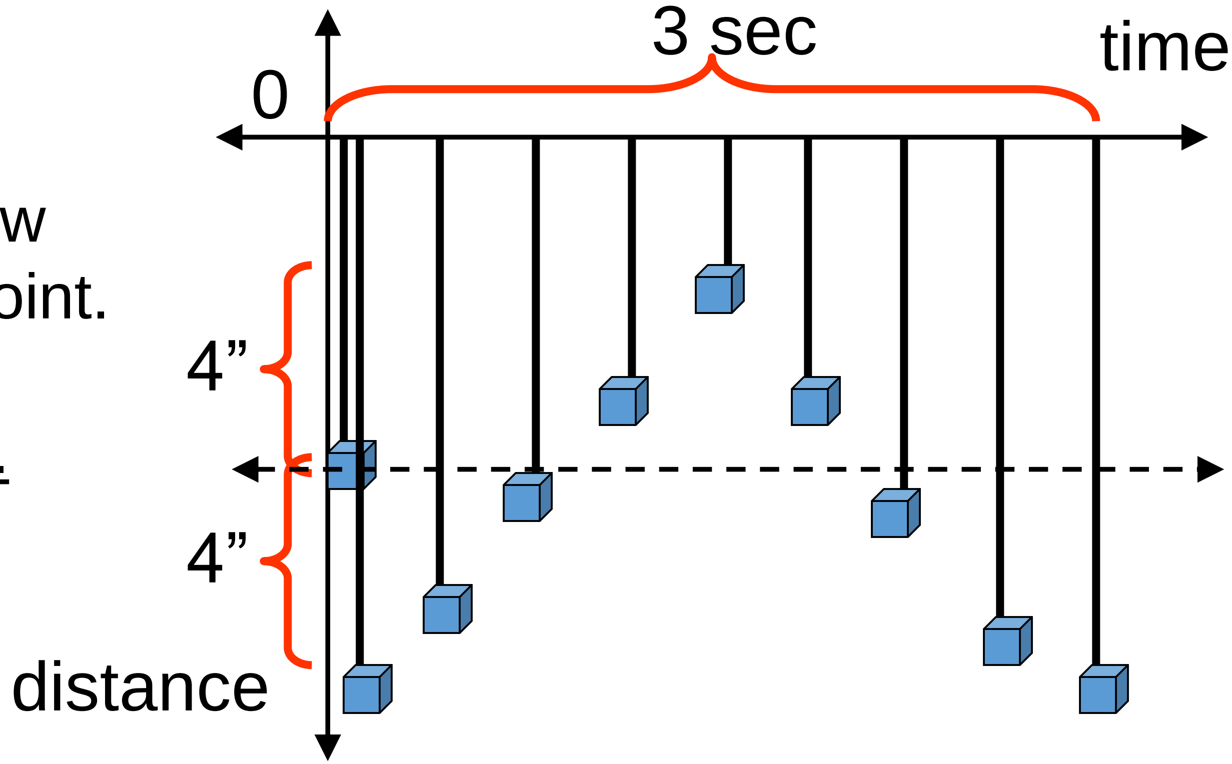
1. Draw the picture.

x-axis: time

y-axis: Distance below spring attachment point.

2. Write the equation.

$$d = a\sin(\omega t)$$



Calculating Harmonic Motion

1. Draw the picture.

x-axis: time

y-axis: Distance below spring attachment point.

2. Write the equation. $d = a\sin(\omega t)$

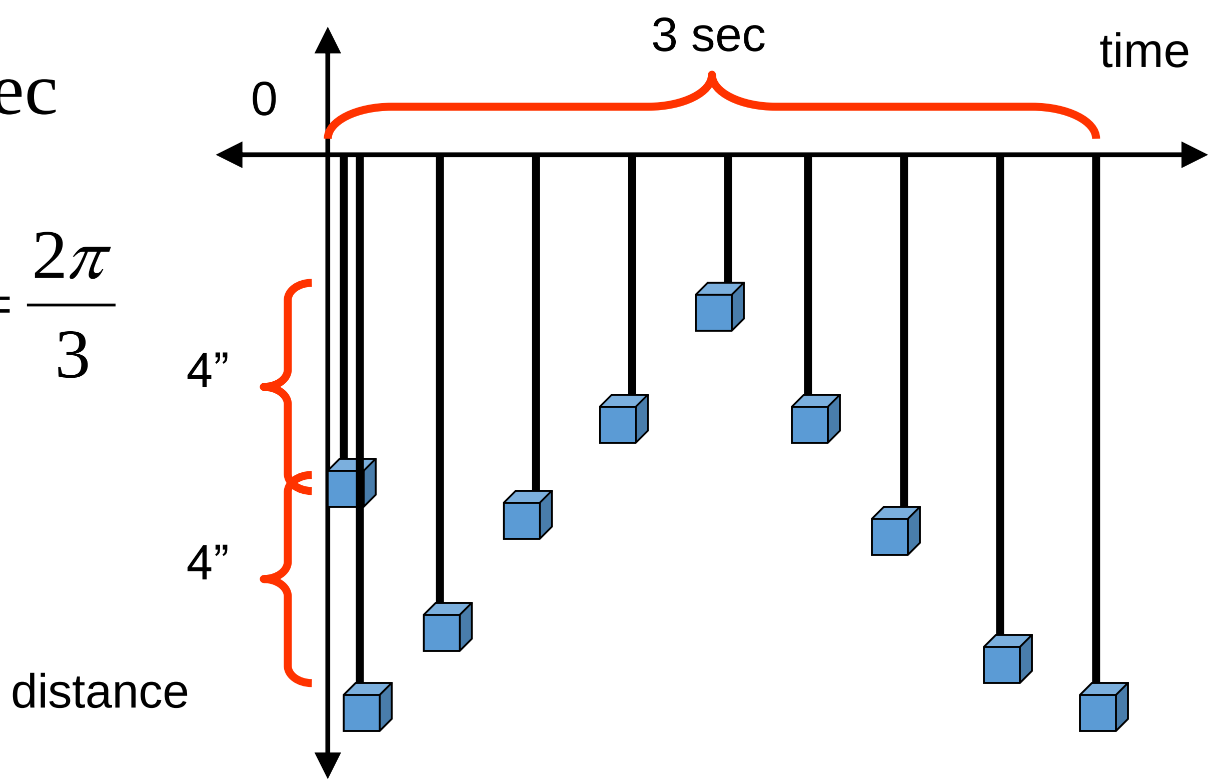
3. Solve the equation. Amplitude = 4”

$$period = \frac{2\pi}{\omega} = 3\sec$$

$$\omega = 2\pi * \frac{1}{3} hz$$

$$\omega = \frac{2\pi}{3}$$

$$d = 4\sin\left(\frac{2\pi}{3}t\right)$$



A tuning fork vibrates at a frequency of 6000 Hz (6000 cycles per second) The amplitude of motion of the tuning fork is 0.05 cm. Find the equation for harmonic motion for this situation.

$$d = a\sin(\omega t)$$

$$\text{frequency} = \frac{\omega}{2\pi}$$

$$6000 = \frac{\omega}{2\pi}$$

$$12000\pi = \omega$$

$$d = 0.05\sin(12000\pi * t)$$