Math-1060
Session #10
(Textbook 5-1)
Exact Trigonometric Ratios for “Nice” Angles

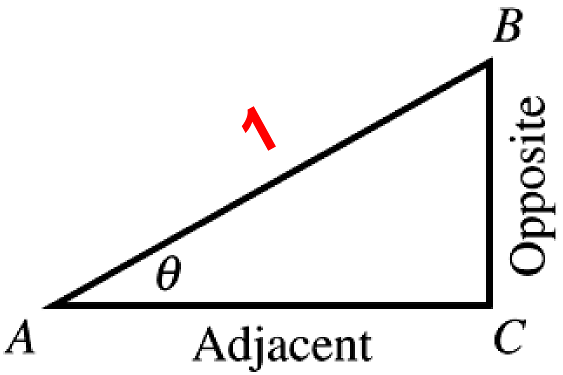
hypotenuse = 1

Why is it "nice" to have a hypotenuse whose length is '1'?

Sin ϴ= opposite side

Cos ϴ= adjacent side

Tan ϴ= opp/adj



The length of the hypotenuse is no longer in the ratio!

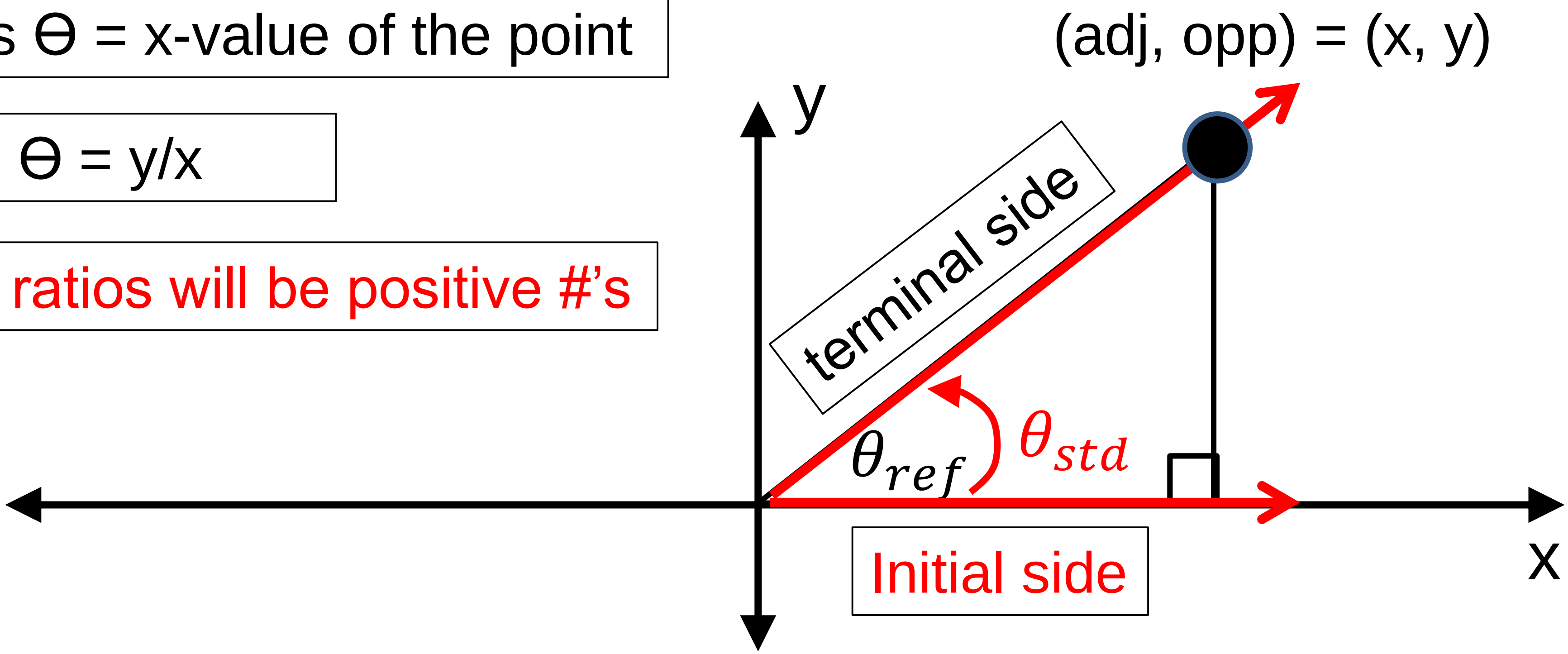
For Quadrant I $\theta_{ref} = \theta_{std}$

Sin Ө = y-value of the point

Cos Ө = x-value of the point

Tan Ө = y/x

Trig ratios will be positive #'s



Reference angle: The acute angle with the x-axis.

For Quadrant II $\theta_{ref} = 180 - \theta_{std}$

Sine ratio is a positive number

Cosine ratio is a negative number

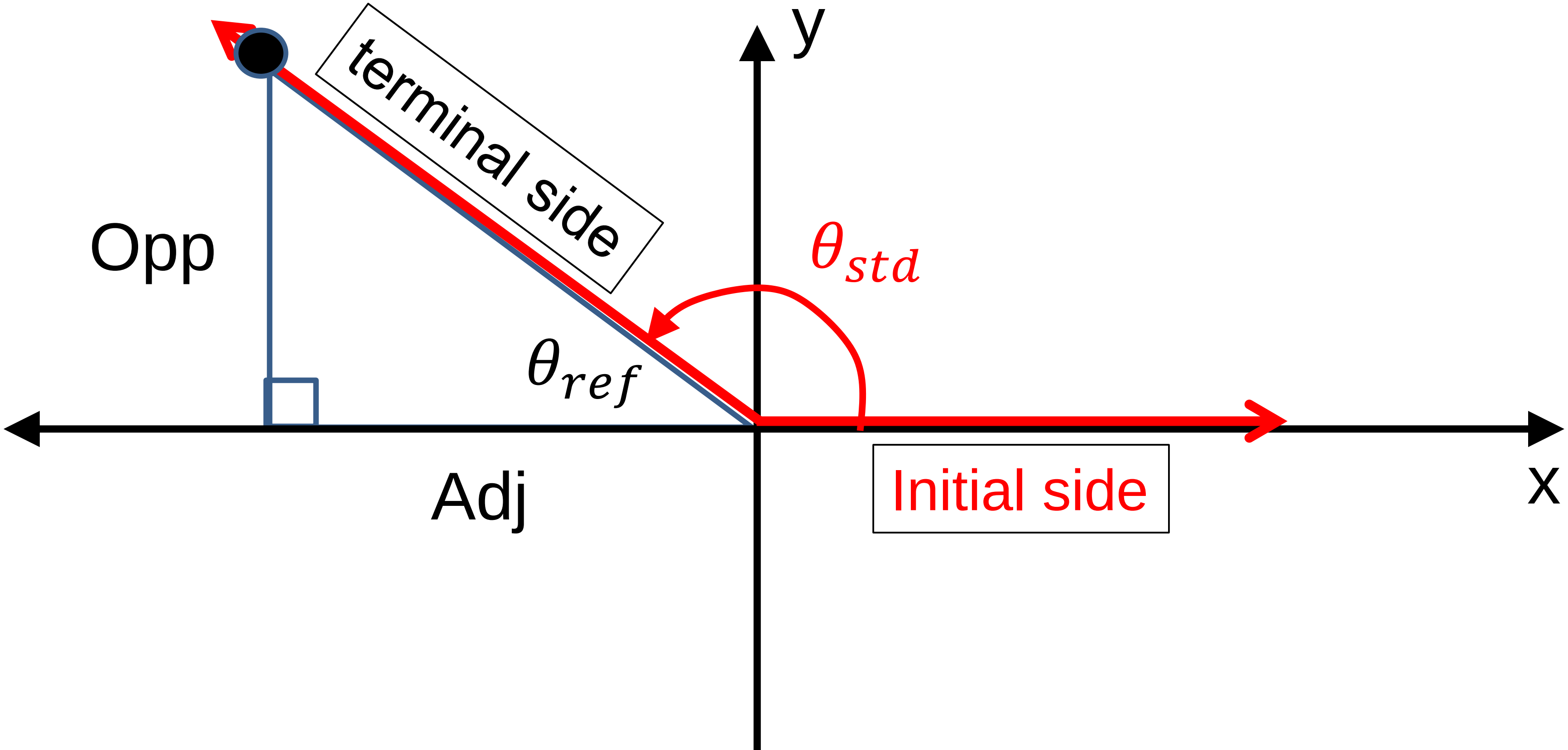
Tangent ratio is a negative number

Sin Ө = y

Cos Ө = x

Tan Ө = y/x

(adj, opp) = (x, y)



For Quadrant III

$$\theta_{ref} = \theta_{std} - 180$$

Sin ϴ = y

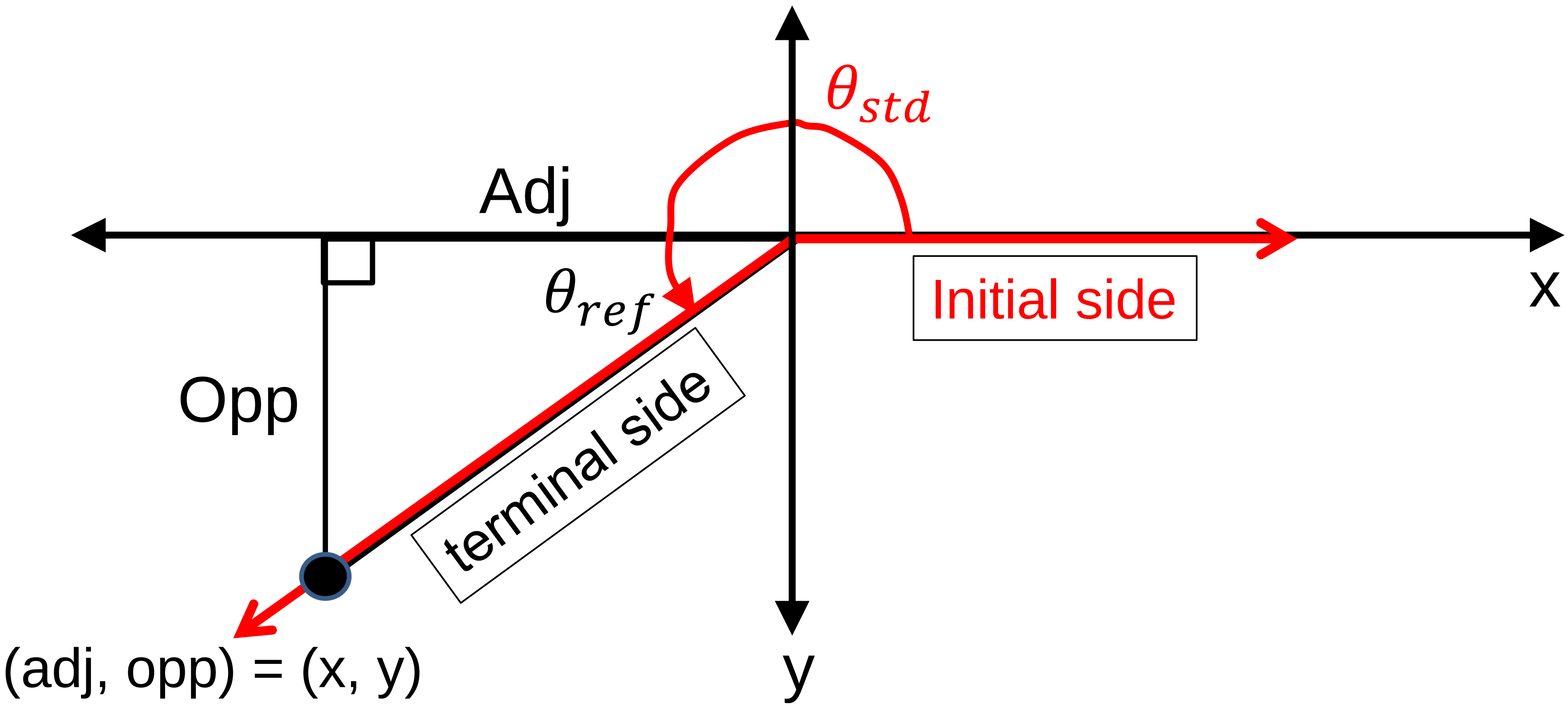
Cos ϴ = x

Tan ϴ = y/x

Sine ratio is a negative number

Cosine ratio is a negative number

Tangent ratio is a positive number



For Quadrant IV

$\theta_{ref} = 360 - \theta_{std}$

Sin ϴ = y

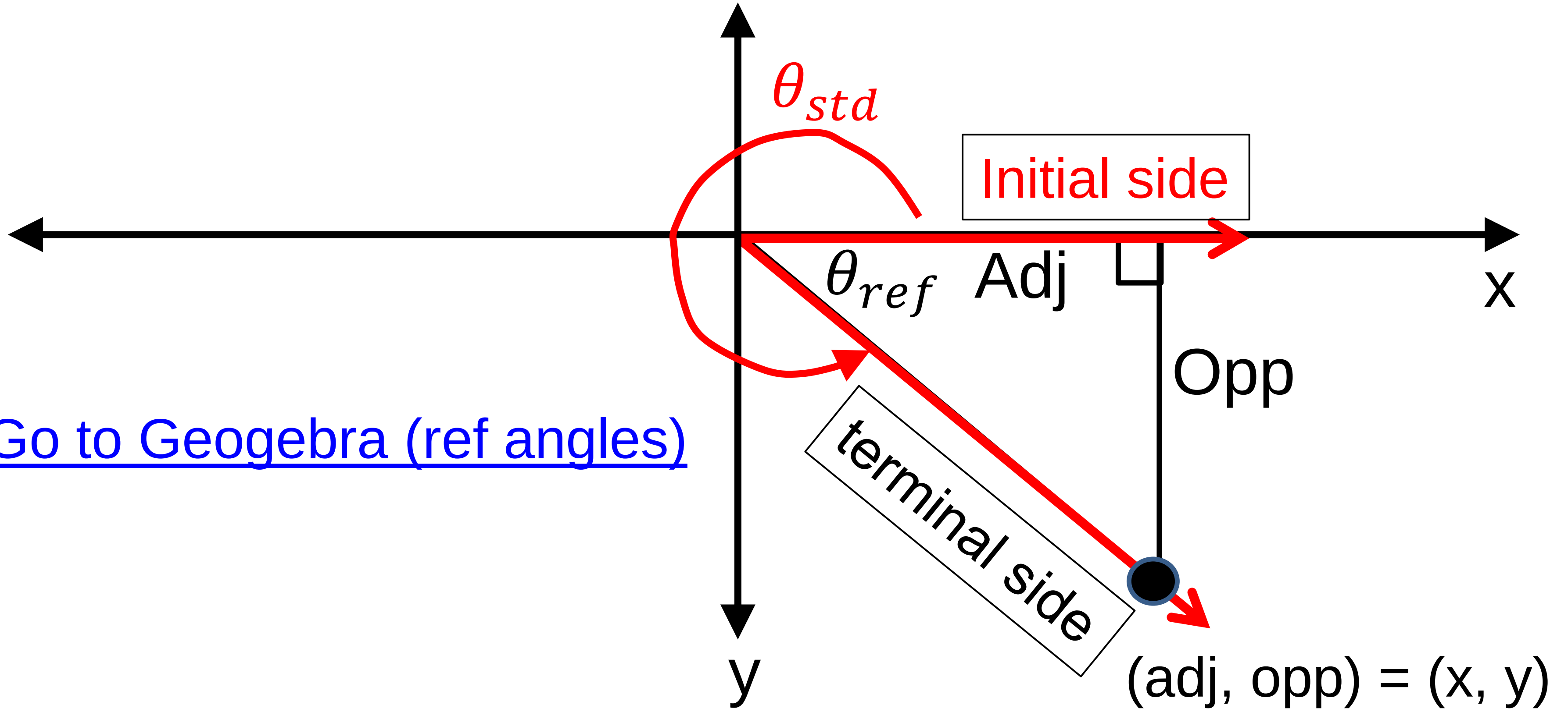
Cos ϴ = x

Tan ϴ = y/x

Sine ratio is a negative number

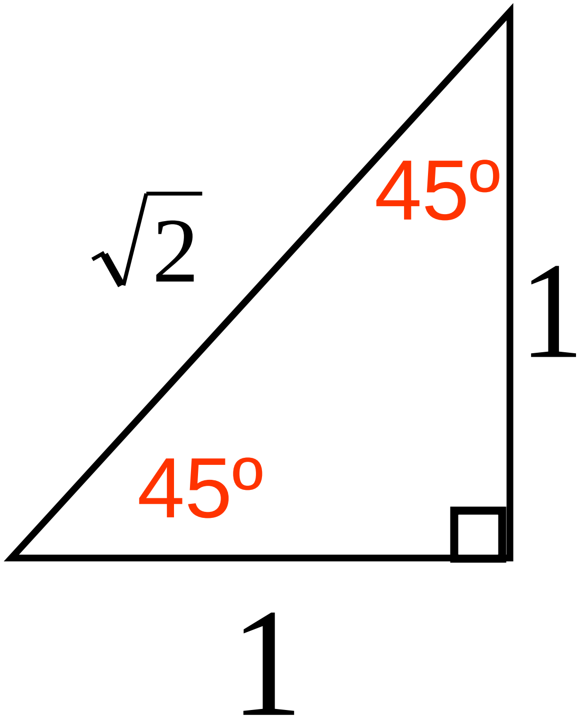
Cosine ratio is a positive number

Tangent ratio is a negative number

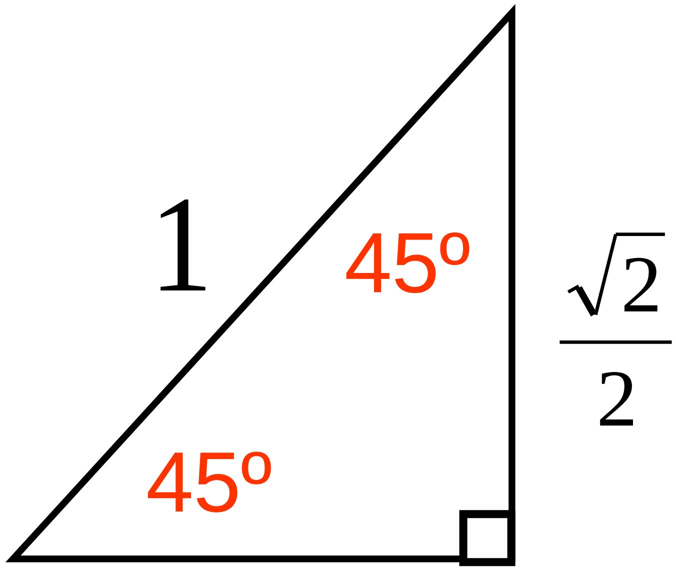


Go to Geogebra (ref angles)

Do you remember the side lengths for a 45-45-90 triangle?

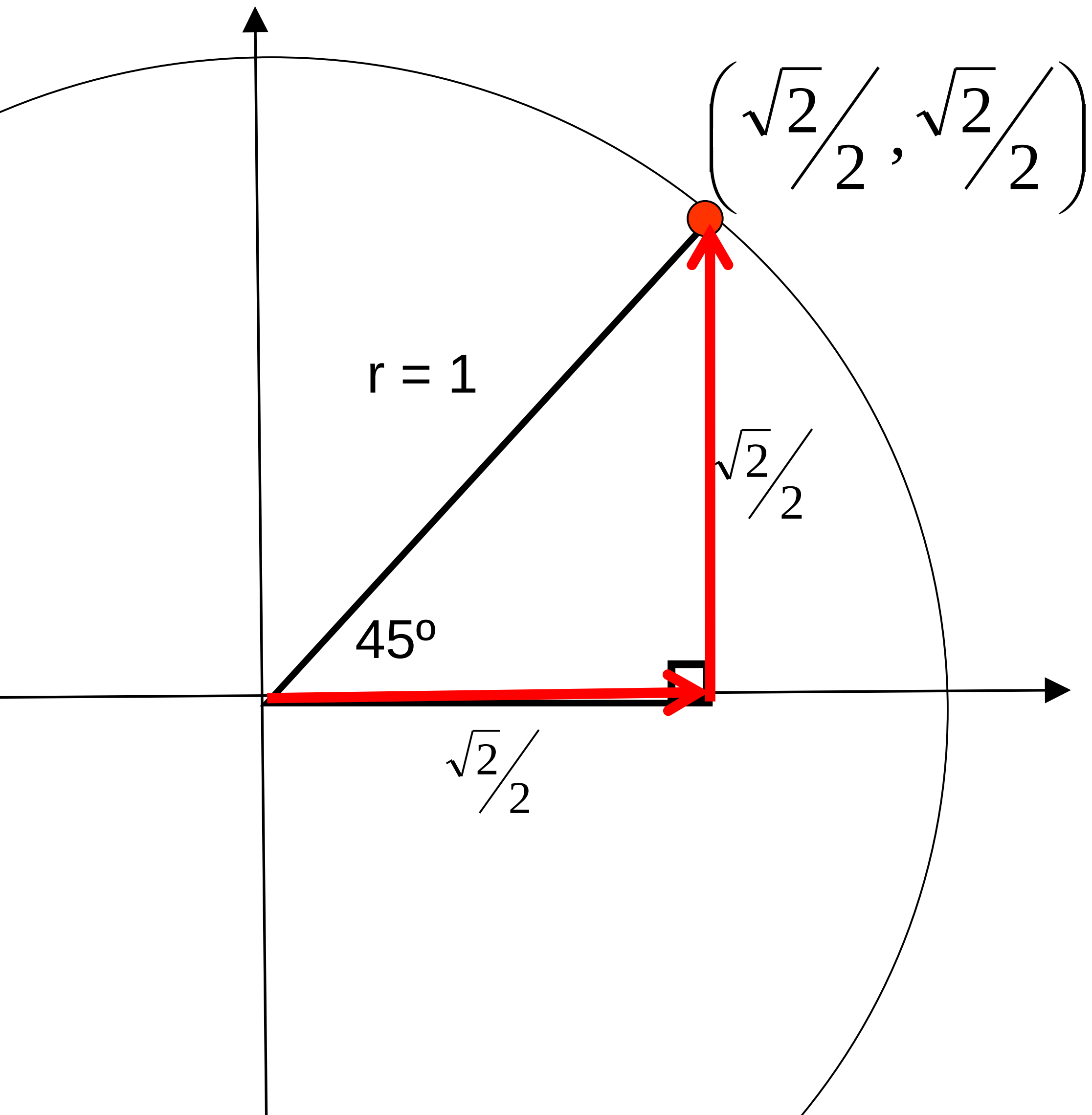


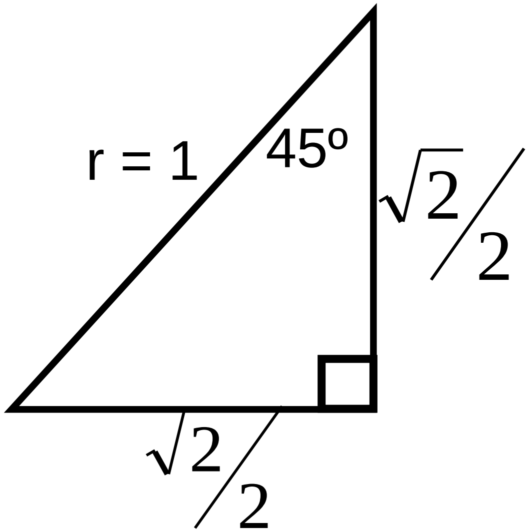
What are the leg lengths if the hypotenuse = 1?



$$\frac{1}{\sqrt{2}} * \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

Let's put the triangle on top of a circle with radius = 1.



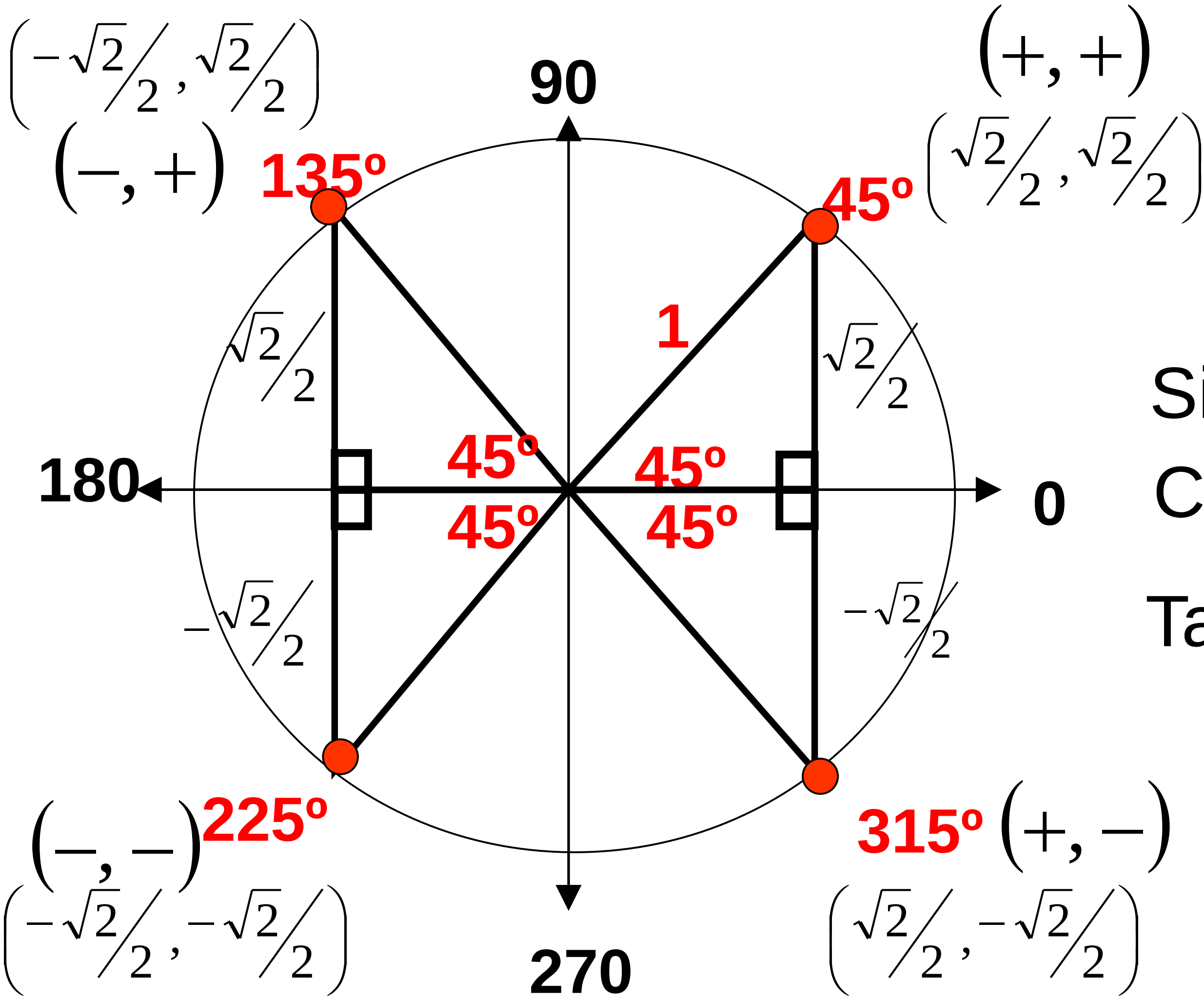


Sin ϴ = y

Cos ϴ = x

Tan ϴ = y/x

We can use a 45° reference angle 4 times

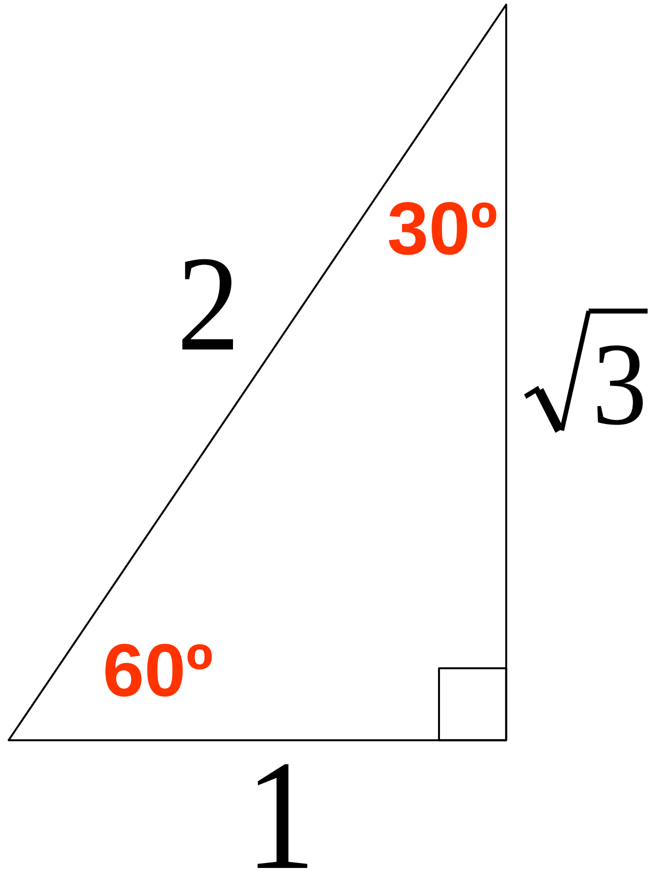


Sin Θ = y

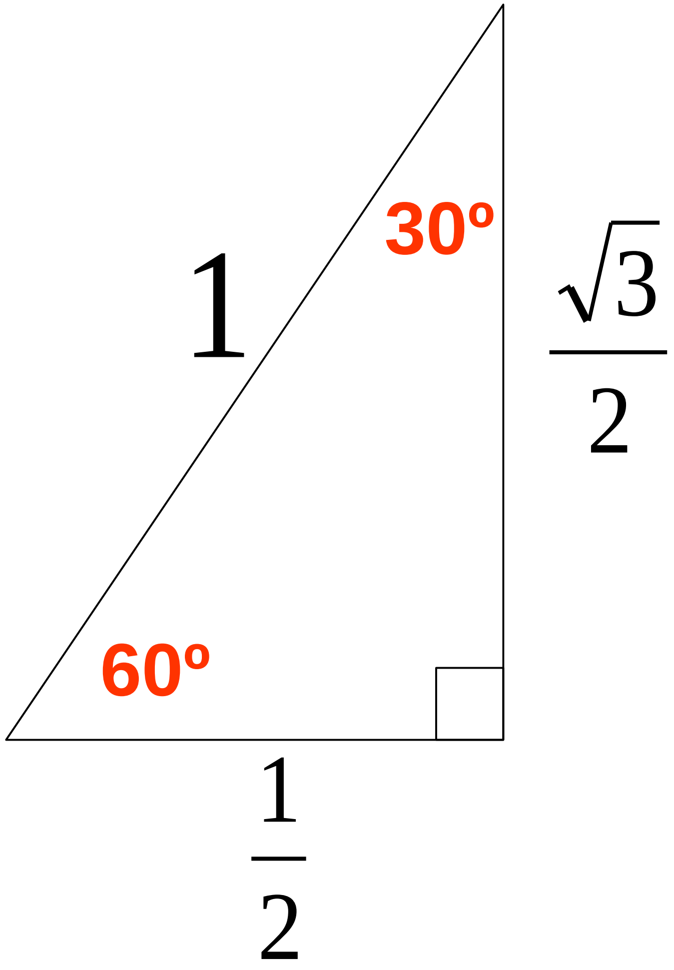
Cos Θ = x

Tan Θ = y/x

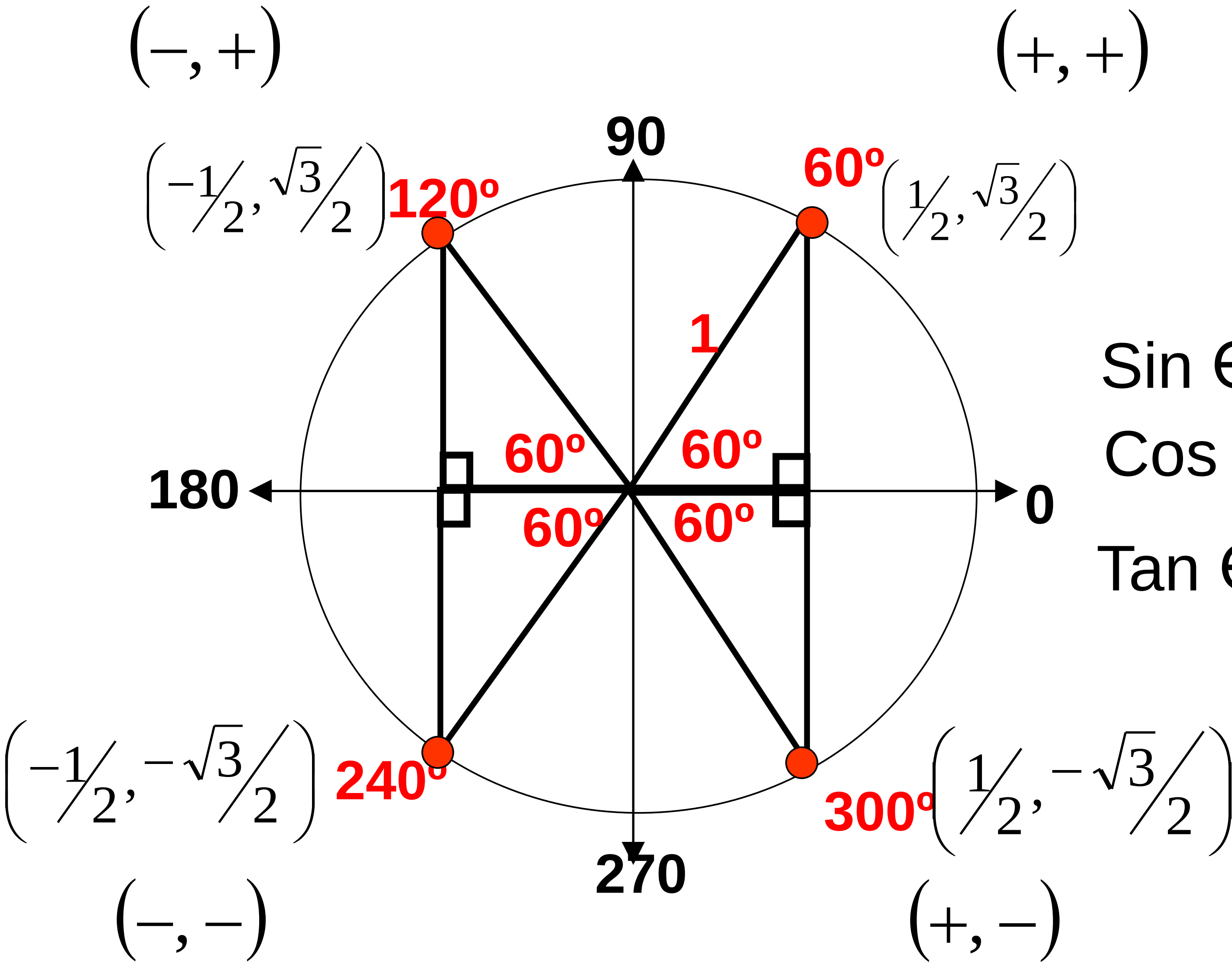
Do you remember the side lengths for the 30-60-90 triangle?



What are the leg lengths if the hypotenuse = 1?



We can use a 60° reference angle 4 times

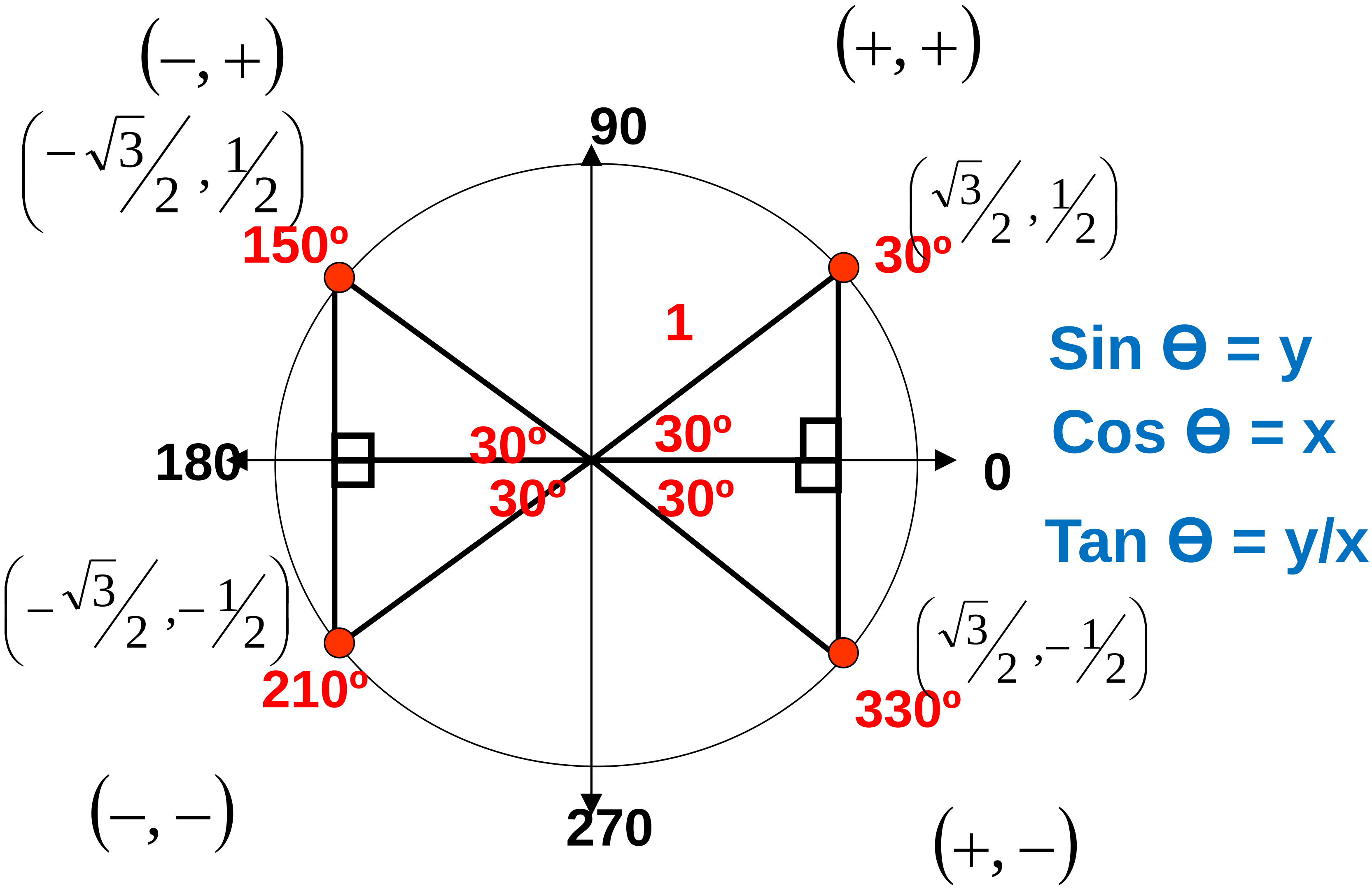


Sin Θ = y

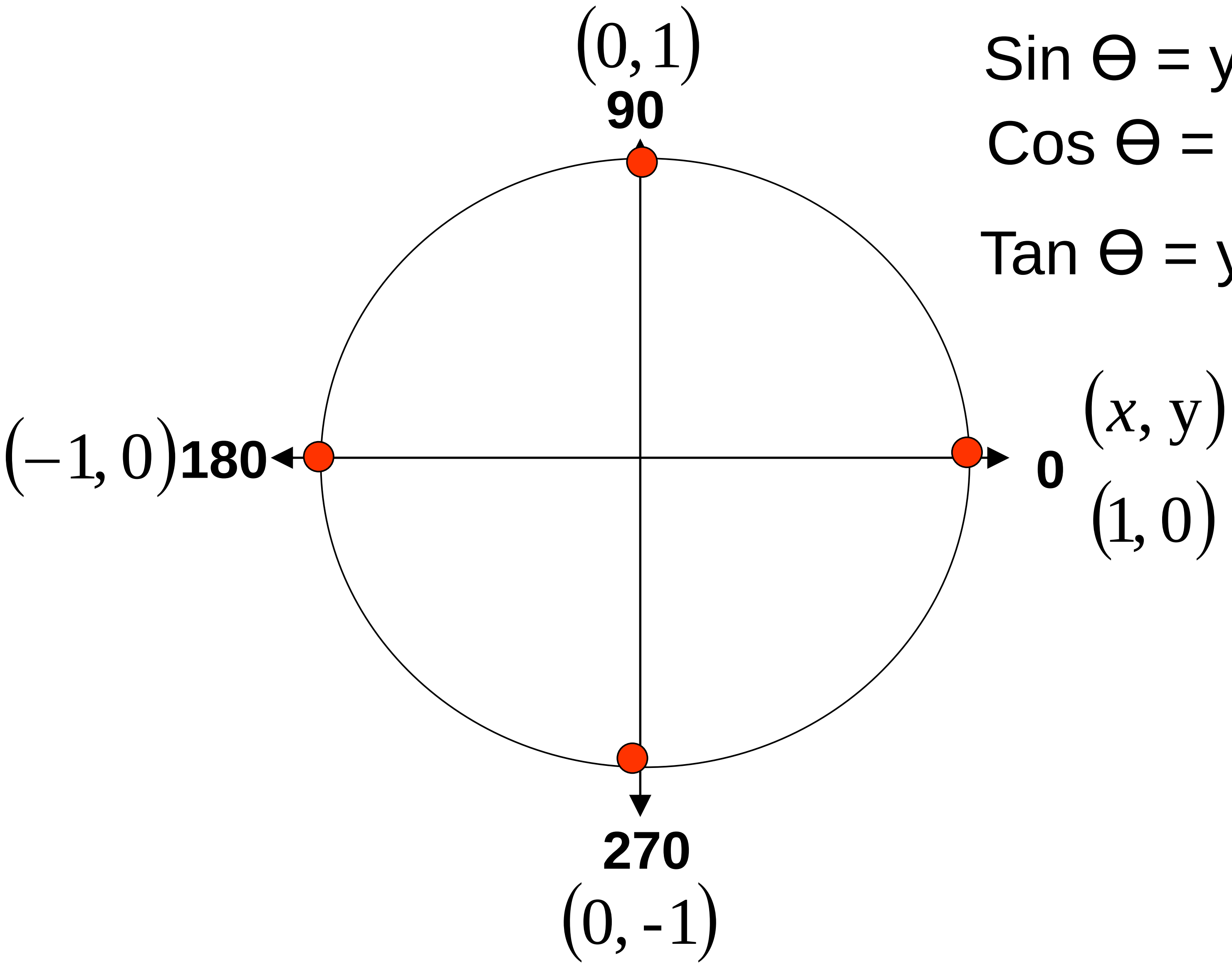
Cos Θ = x

Tan Θ = y/x

We can use a 30° reference angle 4 times



What about the "cardinal angles"?



Sin Θ = y

Cos Θ = x

Tan Θ = y/x

We know the exact ratios for the following angles.

Angle	Sine	Cosine	Tangent
0	0	1	0
90	1	0	$undef$
180	0	-1	0
270	-1	0	$undef$

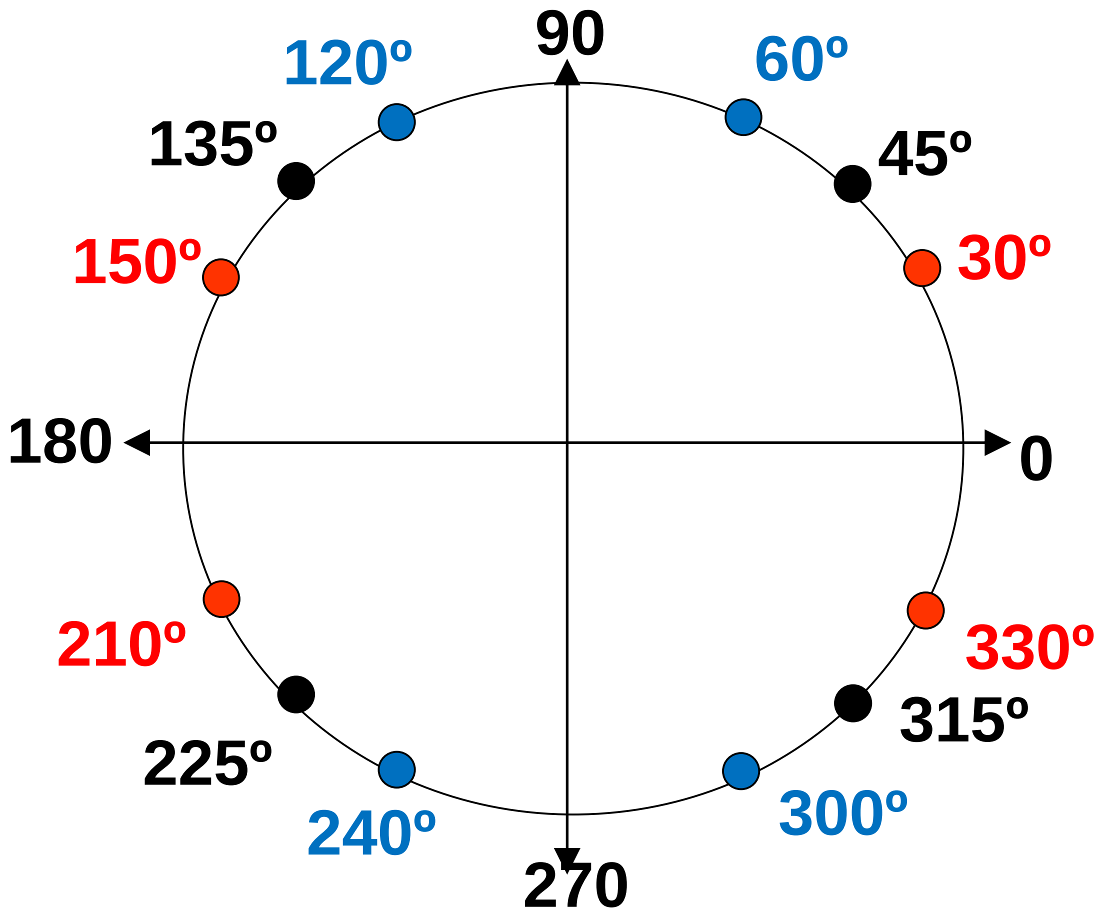
Tan 0° $= {}^{y}\!/\!{}_{x} = \frac{0}{1} = 0$

Tan 90° $= {}^{y}\!/\!{}_{x} = \frac{1}{0}$

$= undefined$

The tangent function does **NOT** have a domain of "all real numbers".

Can you quickly come up with the exact ratio?



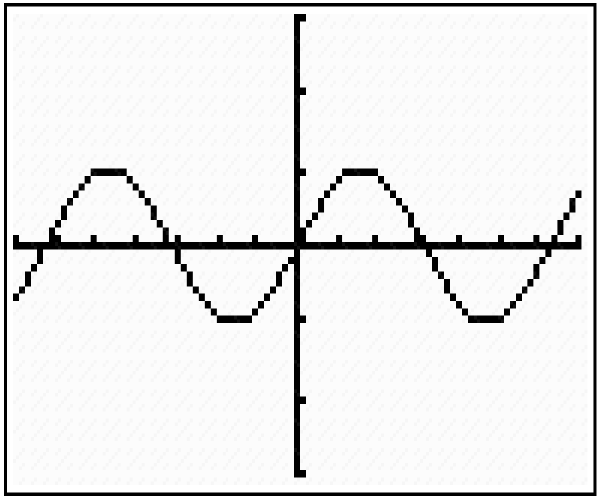
Sin ϴ = y

Cos ϴ = x

Tan ϴ = y/x

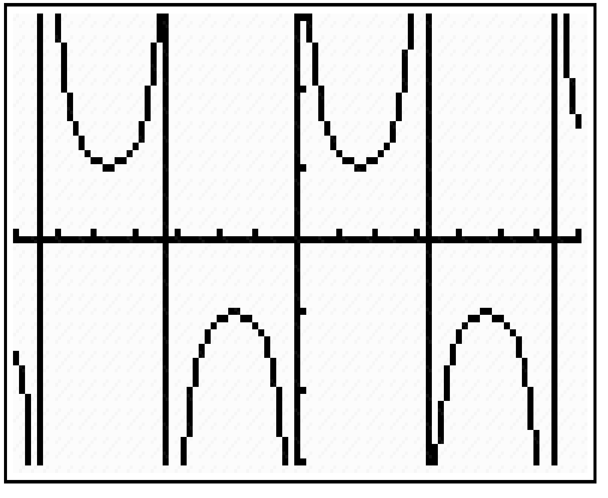
The Sine and Cosecant Functions

Sine (x)



$$\csc\theta = \frac{1}{\sin\theta}$$

Cosecant (x)

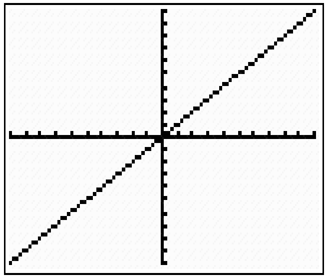


Function	Period	Domain	Range
$\sin x$	2π	All reals	$[-1, 1]$
$\cos x$	2π	All reals	$[-1, 1]$
$\tan x$	π	$x \neq \pi/2 + n\pi$	All reals
$\cot x$	π	$x \neq n\pi$	All reals
$\sec x$	2π	$x \neq \pi/2 + n\pi$	$(-\infty, -1] \cup [1, \infty)$
$\csc x$	2π	$x \neq n\pi$	$(-\infty, -1] \cup [1, \infty)$

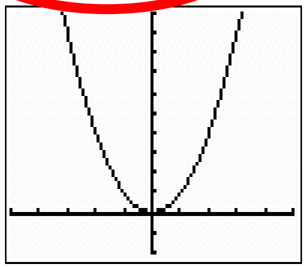
	Asymptotes	Zeros	Even/ Odd
$\sin x$	None	$n\pi$	Odd
$\cos x$	None	$\pi/2 + n\pi$	Even
$\tan x$	$x = \pi/2 + n\pi$	$n\pi$	Odd
$\cot x$	$x = n\pi$	$\pi/2 + n\pi$	Odd
$\sec x$	$x = \pi/2 + n\pi$	None	Even
$\csc x$	$x = n\pi$	None	Odd

Which functions are symmetric about the y-axis?

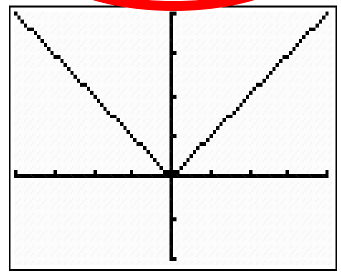
$f(x) = x$



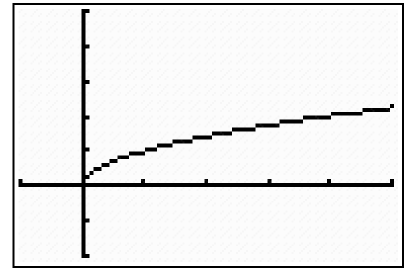
$f(x) = x^2$

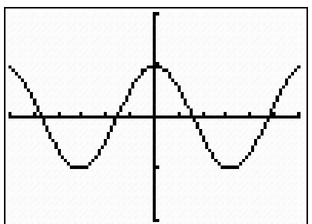


$f(x) = |x|$

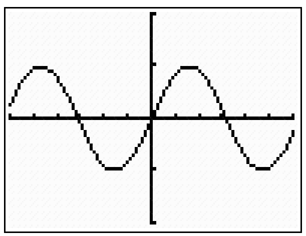


$f(x) = \sqrt{x}$

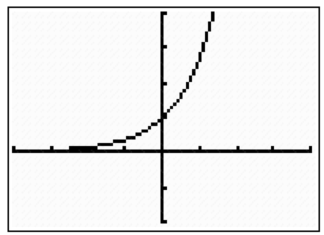




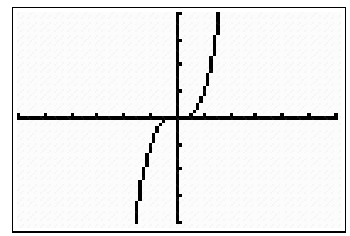
$f(x) = \cos x$



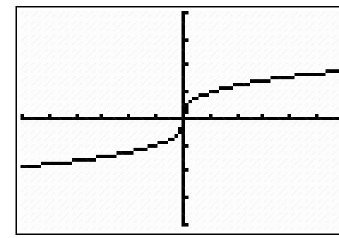
$f(x) = \sin x$



$f(x) = 2^x$



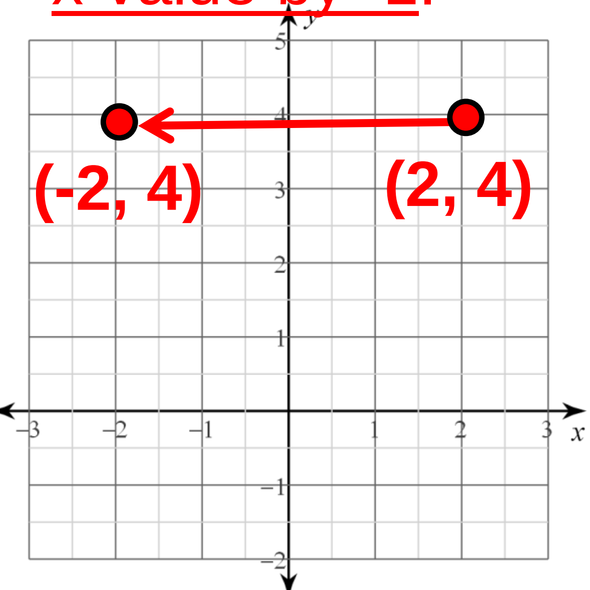
$f(x) = x^3$



$f(x) = \sqrt[3]{x}$

We call functions that are
symmetric about the 'y'-axis,
even functions.

graphically: To reflect a point across the y-axis, we multiply the x-value by -1.

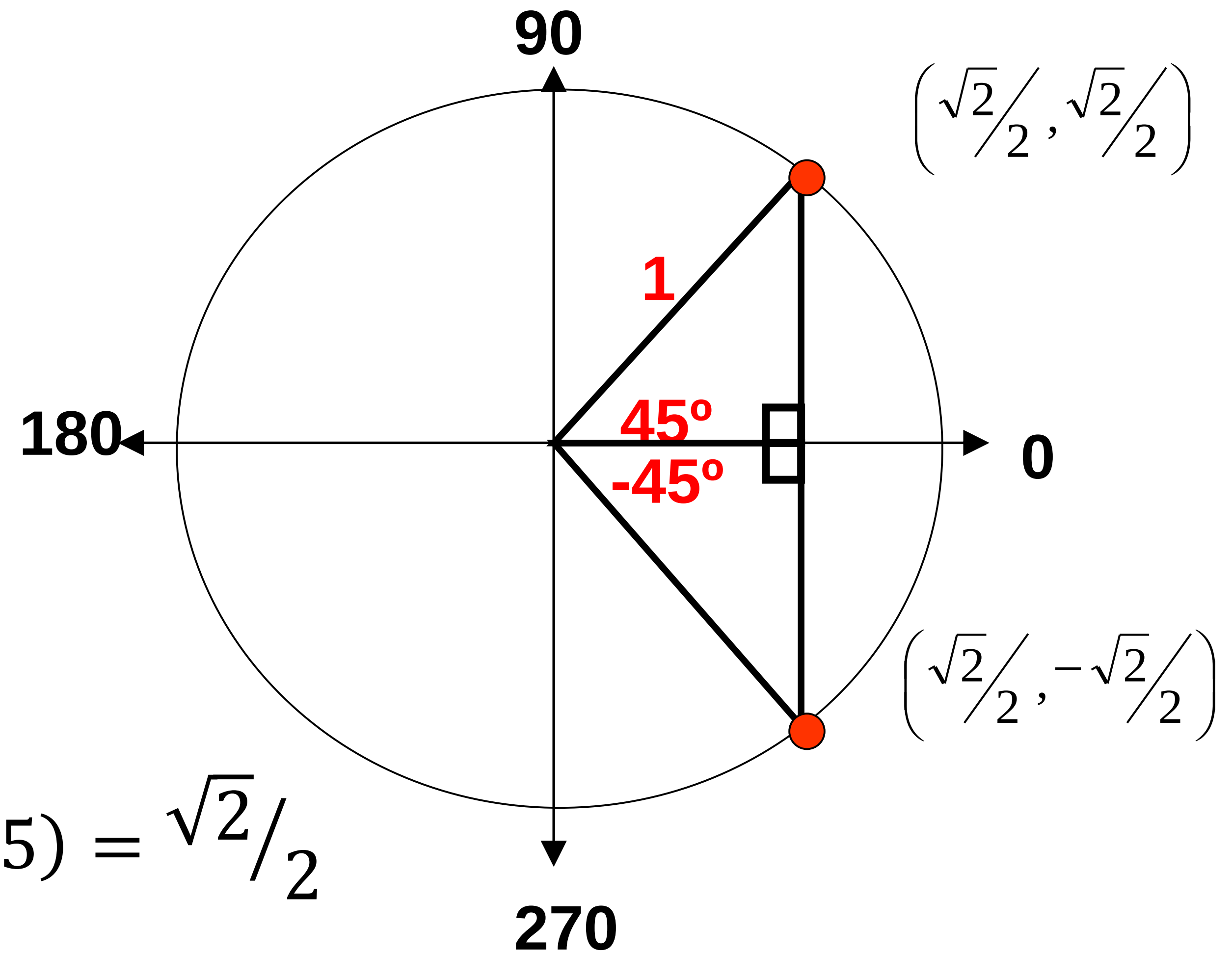


$$f(x) = f(-x)$$

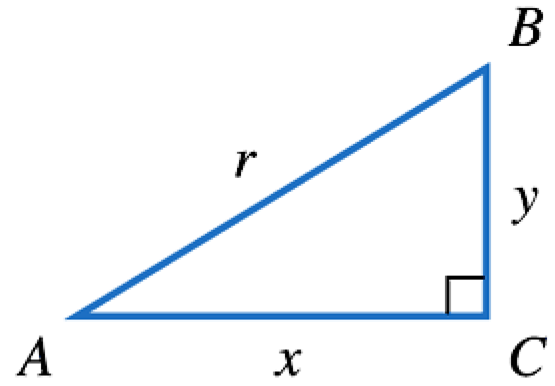
The mathematical definition of an even function.

$$\cos(45) = \cos(-45) = \sqrt{2}/_2$$

Cos Ɵ = x



function of angle A = “cofunction” of angle B.

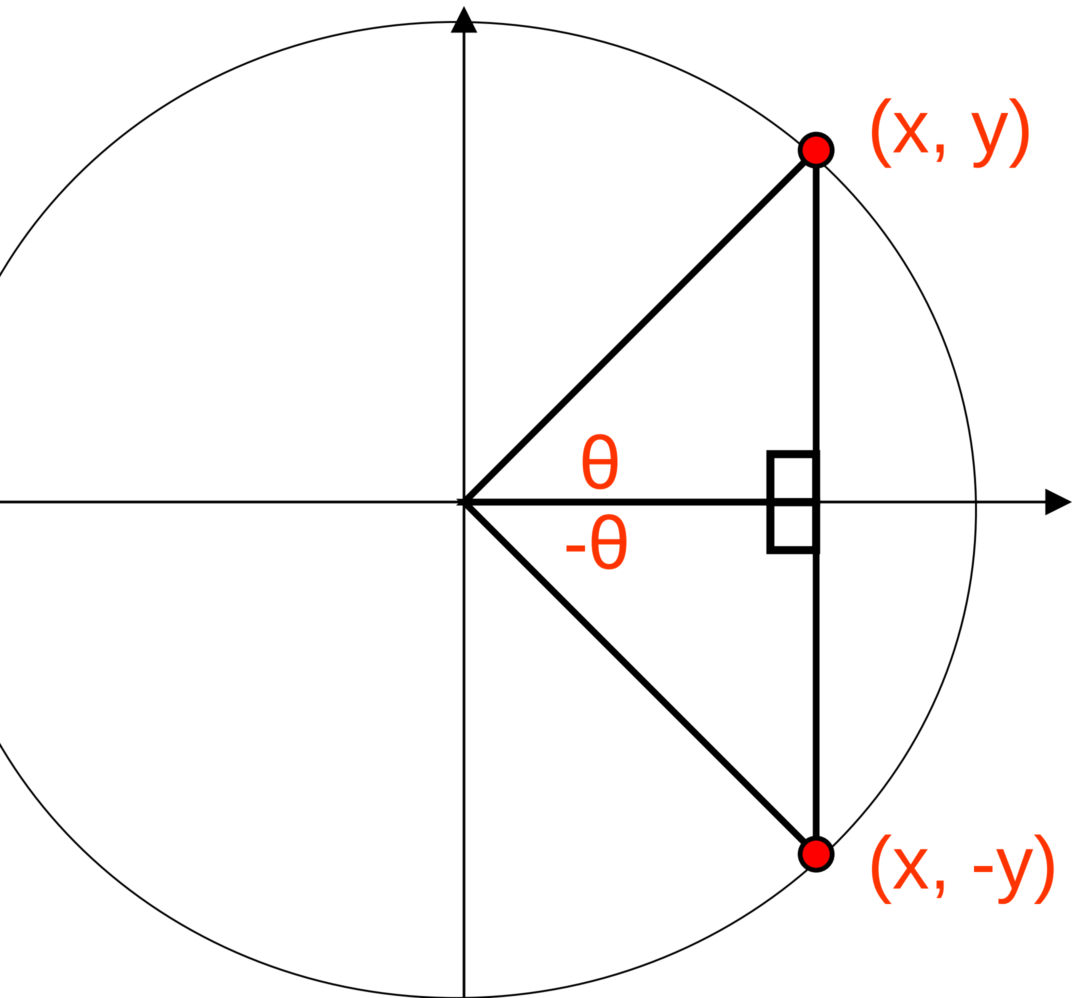


Angle A:	$\sin A = \dfrac{y}{r}$	$\tan A = \dfrac{y}{x}$	$\sec A = \dfrac{r}{x}$
	$\cos A = \dfrac{x}{r}$	$\cot A = \dfrac{x}{y}$	$\csc A = \dfrac{r}{y}$
Angle B:	$\sin B = \dfrac{x}{r}$	$\tan B = \dfrac{x}{y}$	$\sec B = \dfrac{r}{y}$
	$\cos B = \dfrac{y}{r}$	$\cot B = \dfrac{y}{x}$	$\csc B = \dfrac{r}{x}$

Even-Odd Identities

Sin θ = y coordinate of the point on the circle.

$\sin(-\theta) = -\sin(\theta)$



"the y coord. of point through which (-θ) passes is the <u>negative</u> of the y-coord of the point through which (θ) passes.

Cos Θ = x

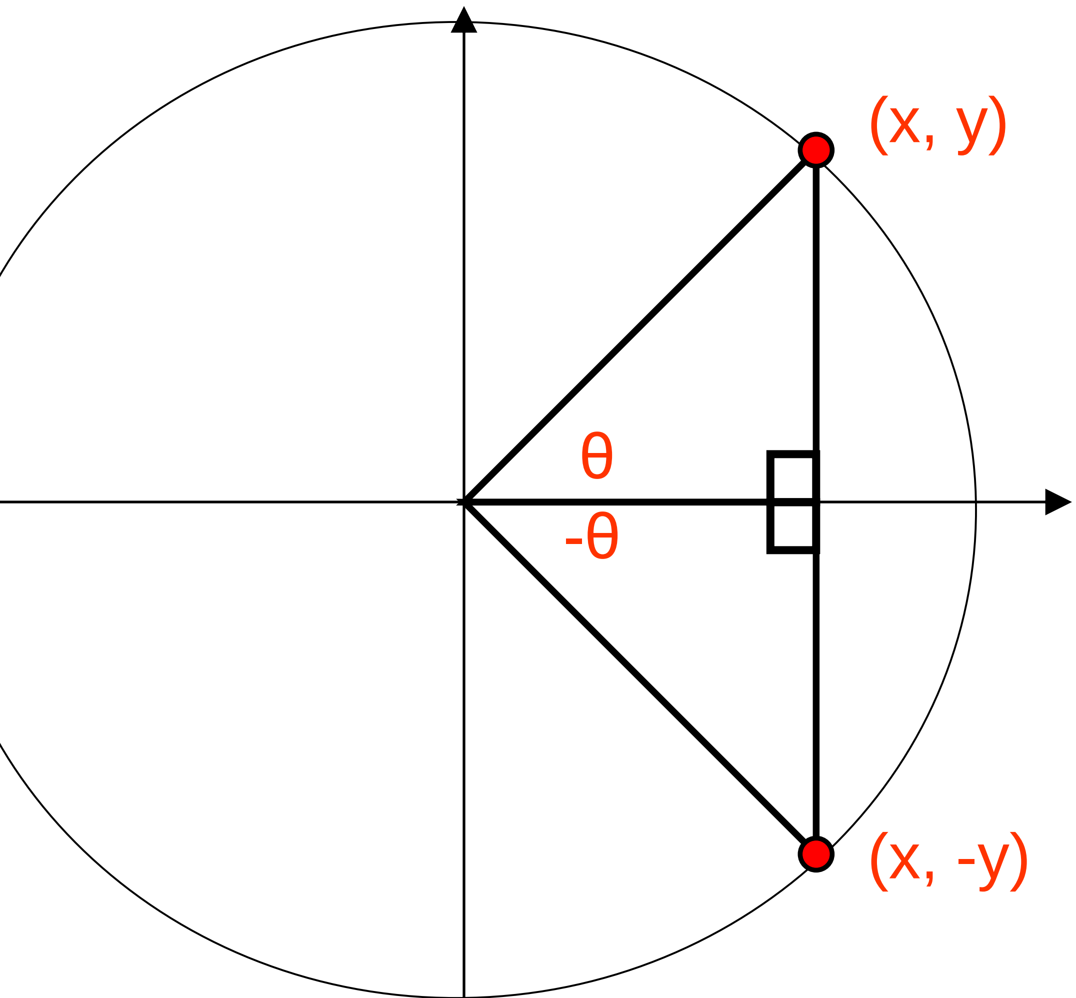
Sin Θ = y

Tan Θ = y/x

Even-Odd Identities

cos θ = x coordinate of the point on the circle.

Cos (-θ) = cos (θ)



"the x coord. of point through which (-θ) passes is the same as the x-coord of the point through which (θ) passes.

Cos Ө = x

Sin Ө = y

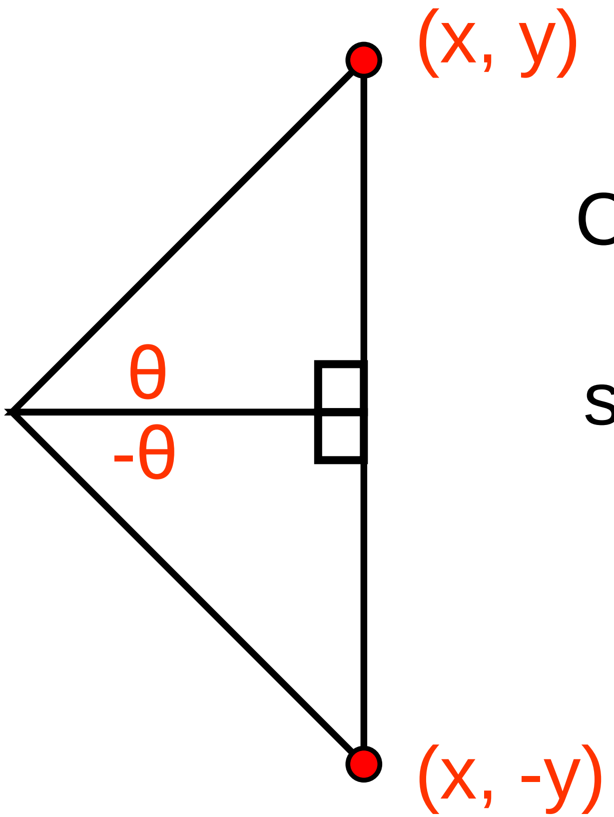
Tan Ө = y/x

Even-Odd Identities

Sin (-θ) = -sin (θ)

$$\csc\theta = \frac{1}{\sin\theta}$$

csc (-θ) = - csc (θ)



Cos (-θ) = cos (θ)

$$\sec\theta = \frac{1}{\cos\theta}$$

sec (-θ) = sec (θ)

Even-Odd Identities

Tan Ө = y/x

Sin Ө = y

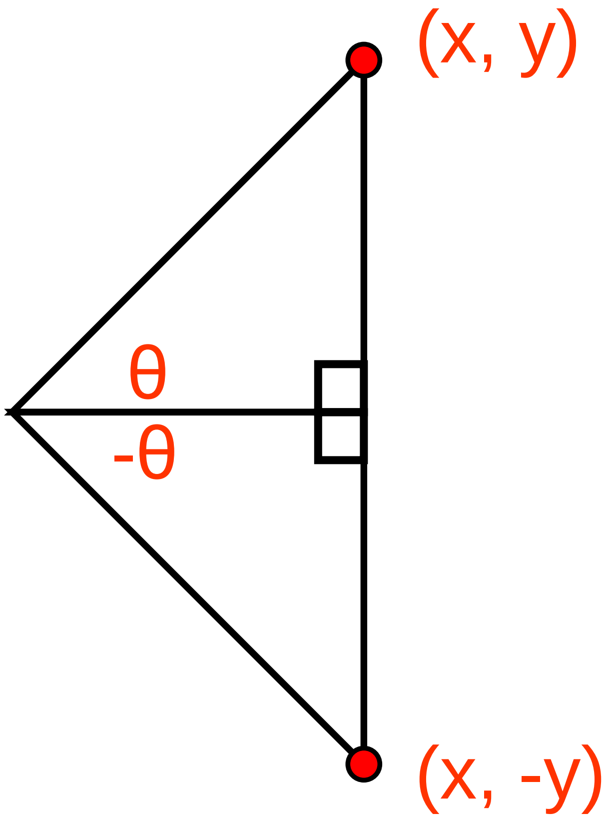
Cos Ө = x

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\tan(-\theta) = \frac{\sin(-\theta)}{\cos(-\theta)}$$

Sin (-θ) = -sin (θ)

Cos (-θ) = cos (θ)



$$\tan(-\theta) = \frac{-\sin(\theta)}{\cos(\theta)}$$

tan (-θ) = - tan (θ)

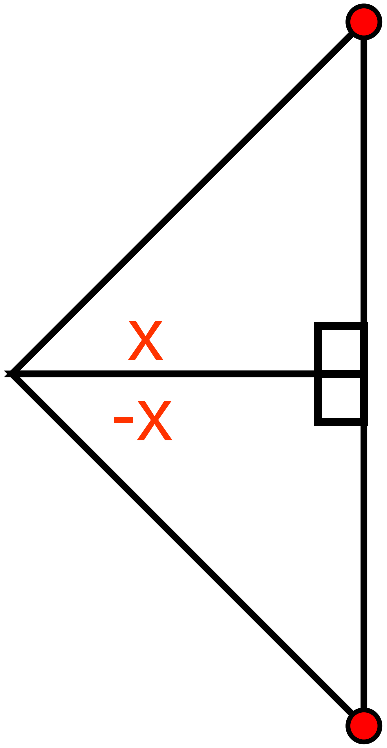
cot (-θ) = - cot (θ)

Even-Odd Identities

$\sin(-x) = -\sin x$ $\cos(-x) = \cos x$ $\tan(-x) = -\tan x$

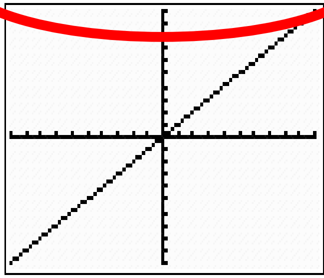
$\csc(-x) = -\csc x$ $\sec(-x) = \sec x$ $\cot(-x) = -\cot x$

The book uses ‘x’ instead of ‘θ’ for the angle variable.

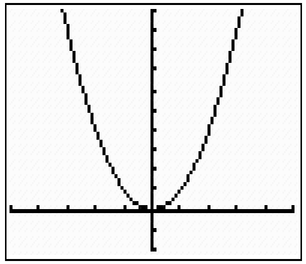


Which functions are symmetrical across the origin?

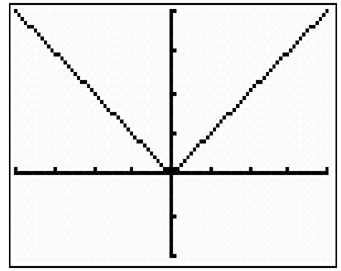
$f(x) = x$



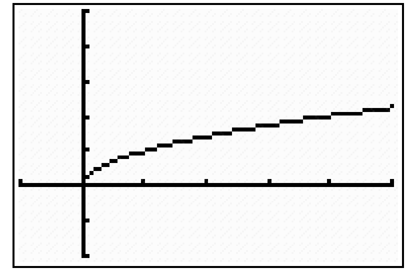
$f(x) = x^2$

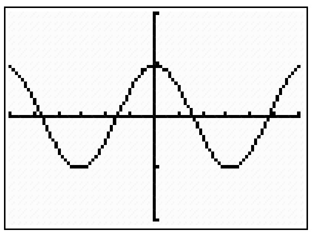


$f(x) = |x|$

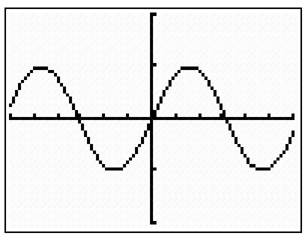


$f(x) = \sqrt{x}$

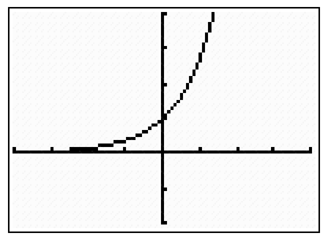




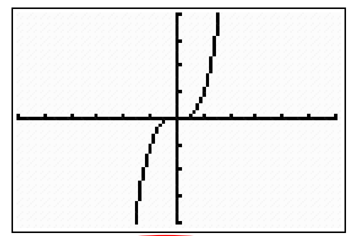
$f(x) = \cos x$



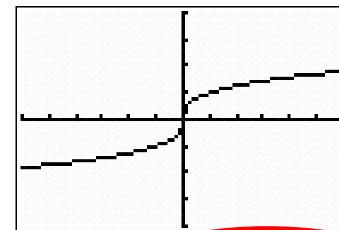
$f(x) = \sin x$



$f(x) = 2^x$



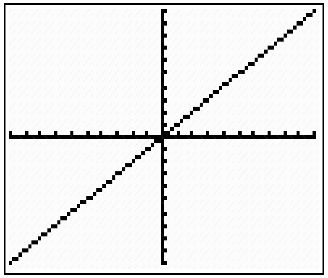
$f(x) = x^3$



$f(x) = \sqrt[3]{x}$

We call these functions “odd” functions.

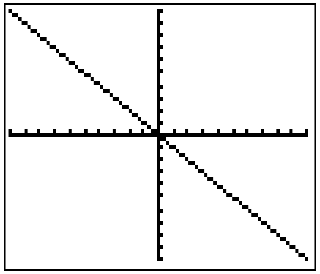
<u>“Odd” functions</u> If a reflection of the function across the x-axis looks exactly like the same function reflected across the y-axis. $f(-x) = -f(x)$



$f(x) = x$

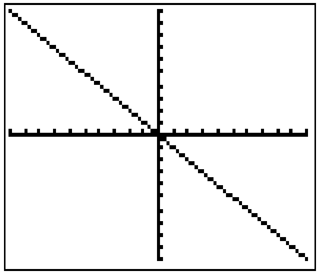
$k(x) = f(-x)$

y-axis reflection.



$g(x) = -x$

x-axis reflection.



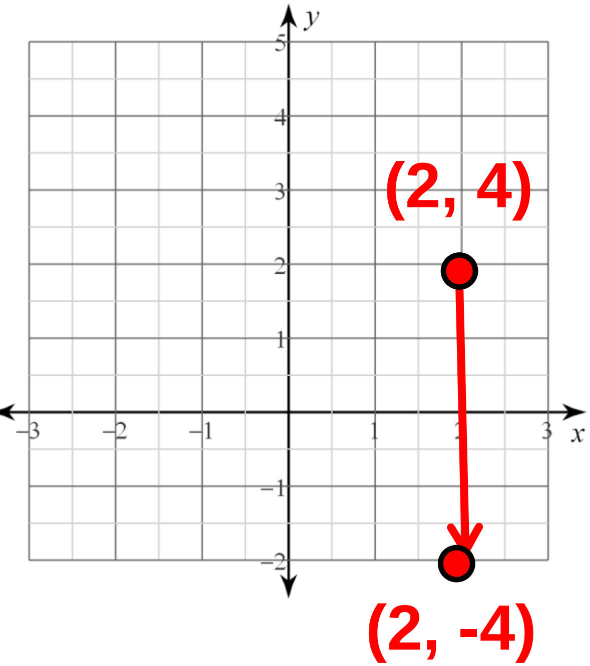
graphically: To reflect a point across the x-axis, we multiply the y-value by -1.

Sin ϴ = y

$$\sin(45) = \sqrt{2}/2$$

$$-\sin(45) = -\sqrt{2}/2$$

$$\sin(-45) = -\sqrt{2}/2$$



$$-f(x) = f(-x)$$

Mathematical definition of an odd function.

$$-\sin(45) = \sin(-45) = -\sqrt{2}/2$$