Math-1060
7-1 (Part 2)

Applications
Of Vectors

Vocabulary:

Vector

Position vector

Component form of a vector

Zero Vector

Magnitude of a Vector

Scalar Multiplication

Unit Vector

Standard Unit Vector

Direction Angle

Horizontal Component

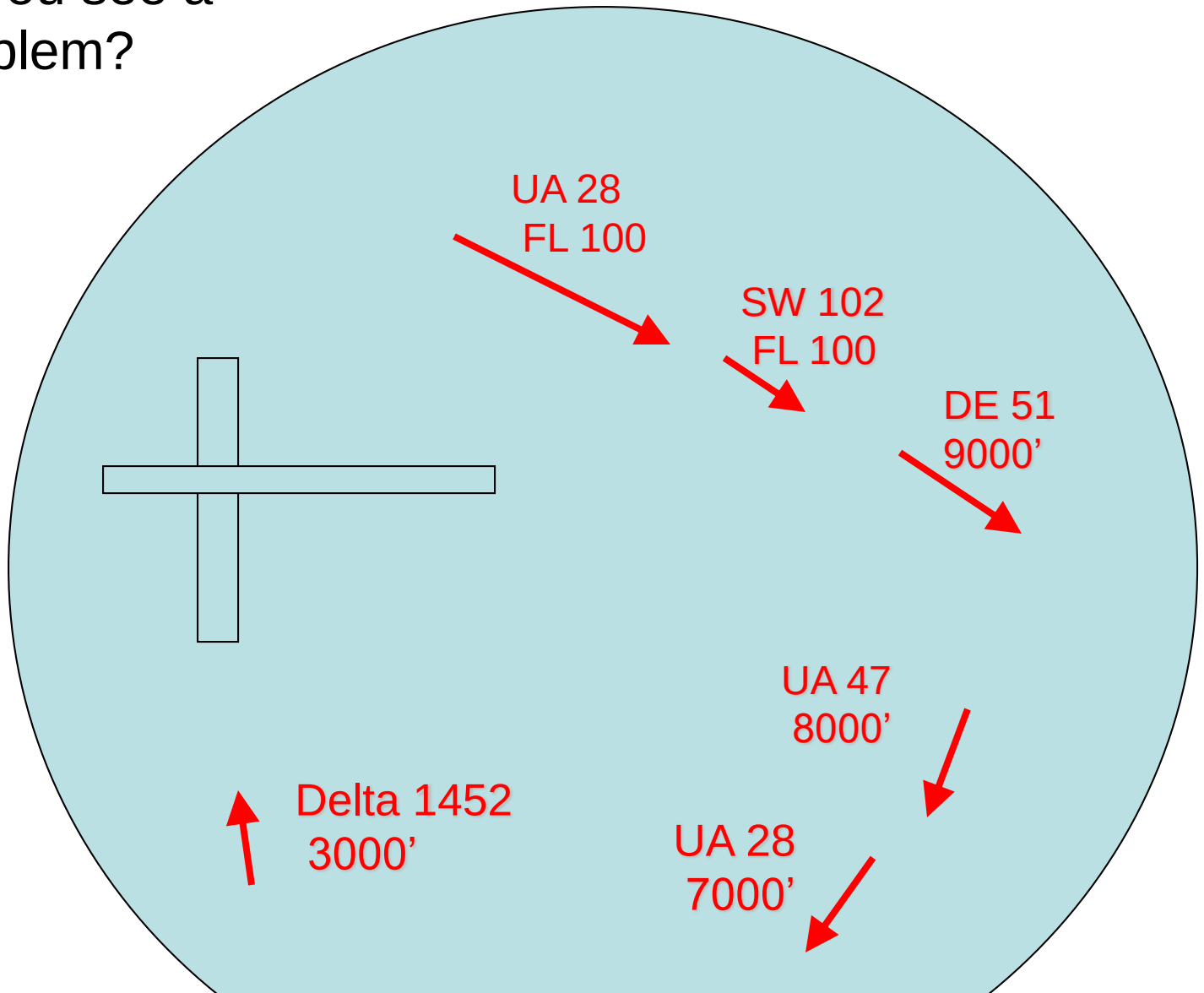
Vertical Component

Resolving a Vector

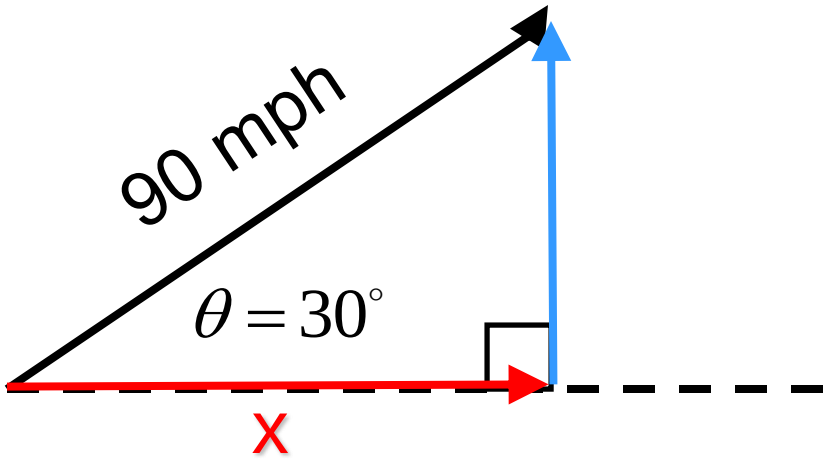
Uses of Vectors

Do you see a problem?

Radar Screen



A ball is thrown at an initial velocity of 90 mph (fastball) at a 30º upward from horizontal.



1. What is its <u>horizontal</u> velocity component?

$$\cos 30^\circ = \frac{x}{90mph} \qquad 90mph(\cos 30^\circ) = 90mph(0.8660) = 77.9mph$$

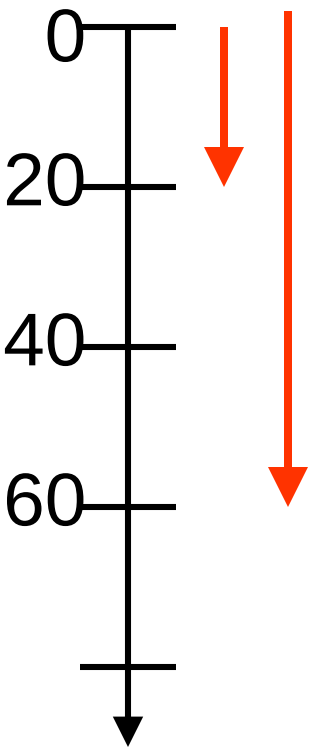
Scalar Multiplication.

A car is driving at a speed of 20 mph toward the south.

$$v = \langle 0, -20 \rangle$$

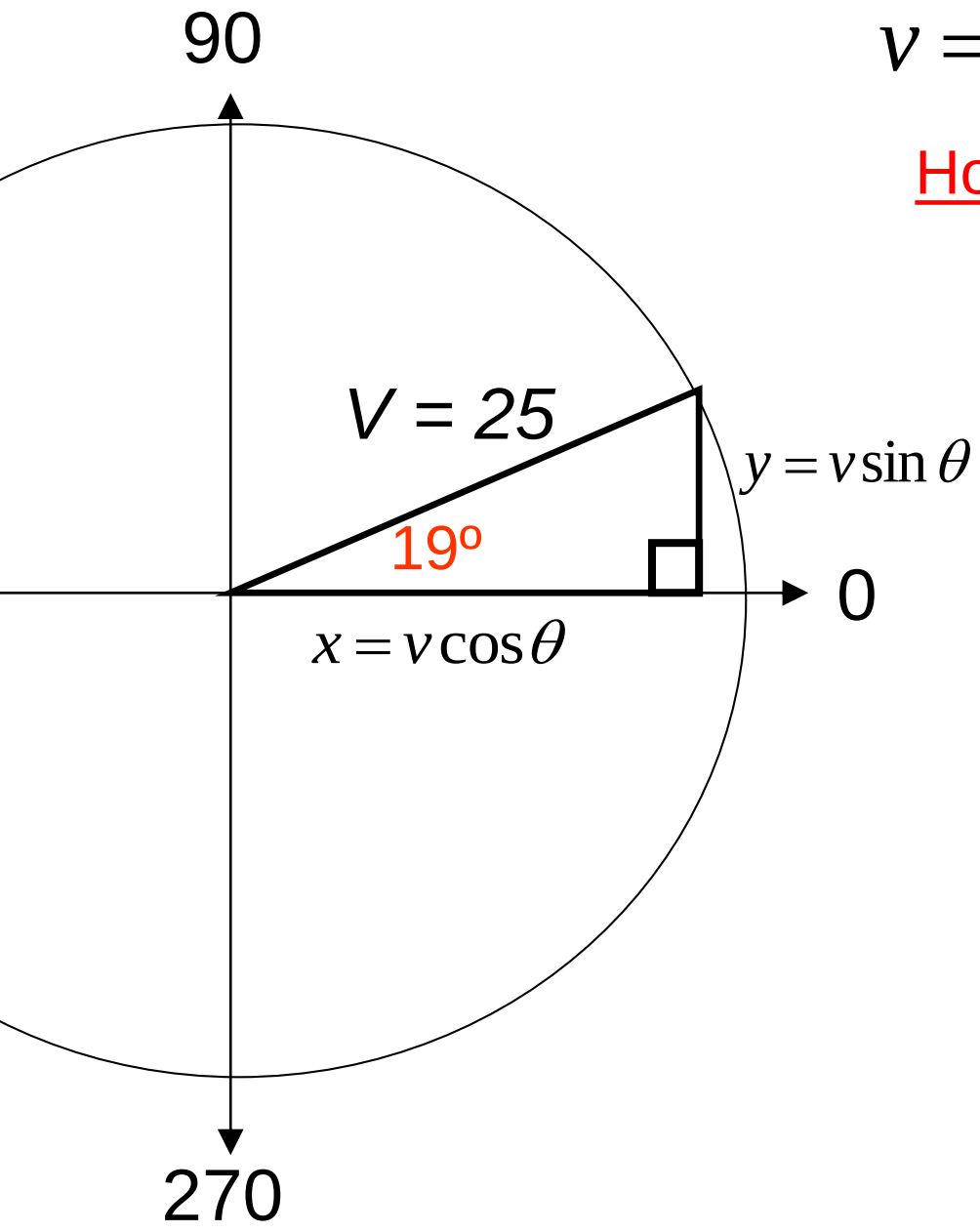
What would the vector look like if the car was traveling 3 times as fast?

$$3v = \langle 0, -60 \rangle$$



The new vector is 3 times as long (magnitude is 3 times as big—but in the same direction)

3 is a scalar



$$v = \langle |v|\cos\theta, |v|\sin\theta \rangle$$

Horizontal component:

$$f_i = 25\cos 19°$$

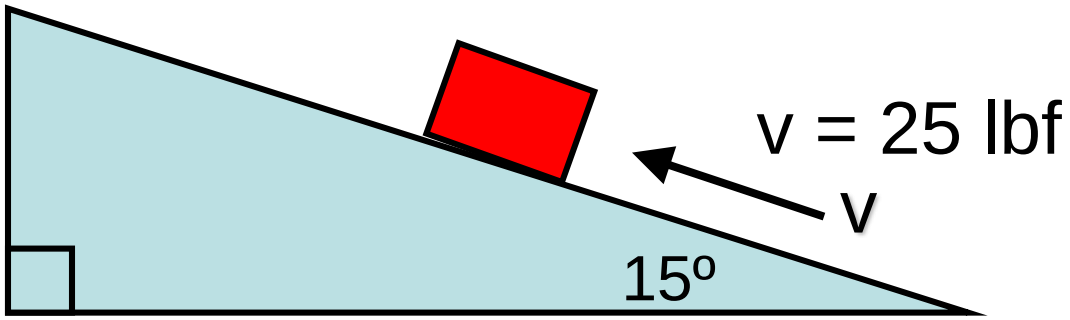
$$f_i = 23.6\ lbs_f$$

Vertical component:

$$f_j = 25\sin 19°$$

$$f_j = 8.1\ lbs_f$$

Force Vectors

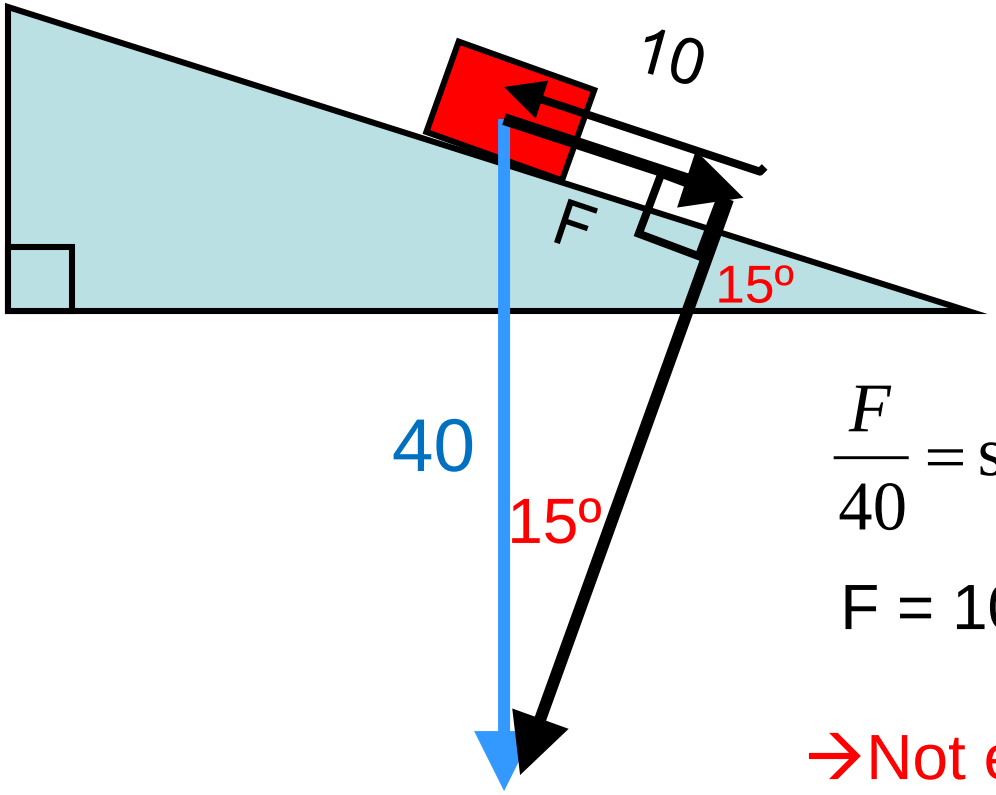


A box is being rolled (on a cart) up a 15 degree incline. The worker is pushing with a force of 25 lbf in a direction that is parallel to the incline.

Force vectors are always resolved to "*parallel the plane*" and "*normal to the plane*" (perpendicular to the plane)

A worker is pushing the box up the slope with 10 lb of force.

If the box weights 40 lbs, is the worker exerting enough force to move the box up the incline?



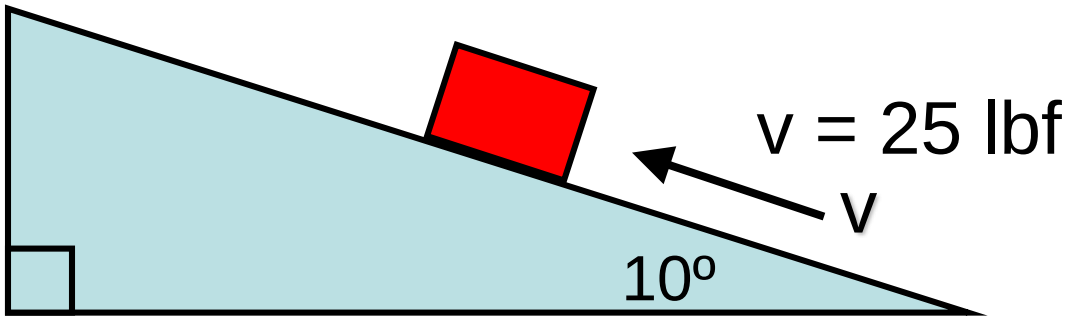
The component of the force of gravity in the direction of the slope must be no greater than 10 lb.

$$\frac{F}{40} = \sin 15^\circ \qquad F = 40 \sin 15^\circ$$

F = 10.4 lb

→Not enough force to move the box up the slope

Force Vectors



A box is being rolled (on a cart) up a 10 degree incline. The worker is pushing with a force of 25 lbf in a direction that is parallel to the incline.

What is the horizontal component of the force?

What is the vertical component of the force?

What is the difference between *velocity* and *speed*?

velocity is a vector.

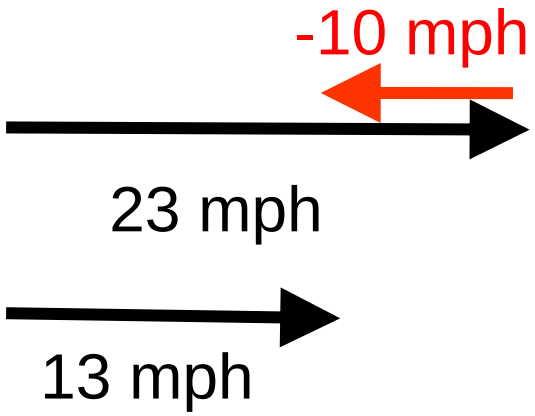
speed is a scalar magnitude

In calm water, a boat can travel at 23 mph. The boat is traveling upstream in a river that is flowing at 10 mph.

1) Draw the velocity vector of the boat.

2) Draw the velocity vector of the water.

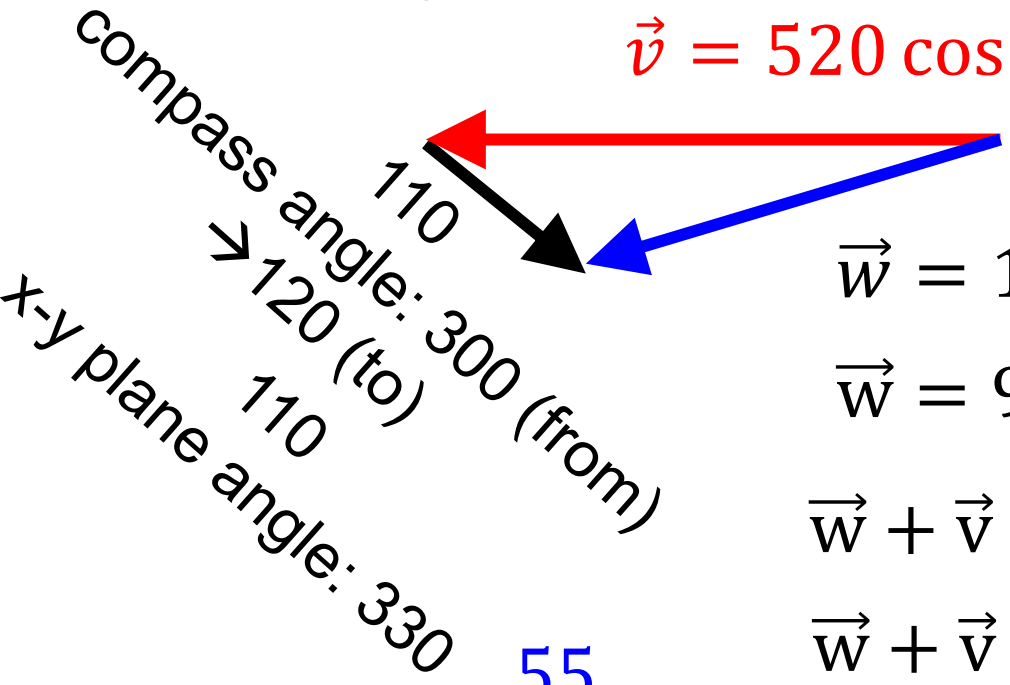
3) How fast will the boat move (relative to the river bank)?



A plane is flying on a compass heading of 270 (straight west) at an airspeed of 520 mph. The jet-stream is coming from a direction of 300 at a speed of 110 mph.

What is the speed of the airplane relative to the ground?

Challenging→ compass headings NOT the same as x-y plane direction angles.



$\vec{v} = 520 \cos 180\ i + 520 \sin 180\ j$

$\vec{v} = -520\ i + 0\ j$

$\vec{w} = 110 \cos 330\ i + 110 \sin 330\ j$

$\vec{w} = 95.3\ i - 55\ j$

$\vec{w} + \vec{v} = (-520 + 95.3)\ i - 55\ j$

$\vec{w} + \vec{v} = -424.7\ i - 55\ j$

$\theta_{ref} = \tan^{-1} \frac{55}{424.7}$

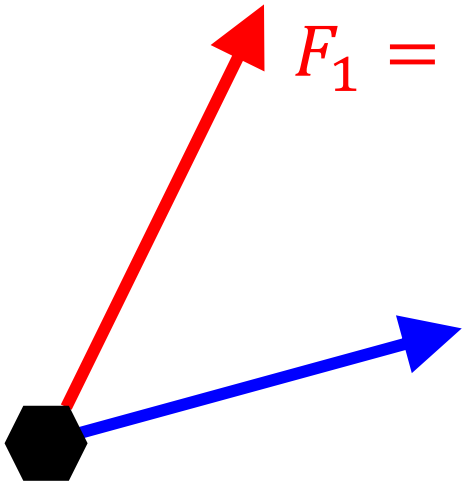
$\theta_{ref} = 7.4$

$|\overrightarrow{w + v}| = \sqrt{424.7^2 + 55^2} = 428.2$

$\theta_{dir} = 187.4$

$\theta_{compass} = 270 - 7.4$

A bolt secured to a bridge is being acted upon by forces being exerted by two cables attached to the bolt. The force vector for one of the cables is represented by $\boldsymbol{F}_1 = \langle 1.7, 9.8 \rangle$. The other force vector, $\boldsymbol{F}_2$, is known to have a magnitude 5 and a direction angle of 10˚. Determine the **angle** between the two cables that are attached to this bolt.



$F_1 = 1.7\ i + 9.8\ j$ $\theta_{dir} = \tan^{-1}(\frac{9.8}{1.7})$

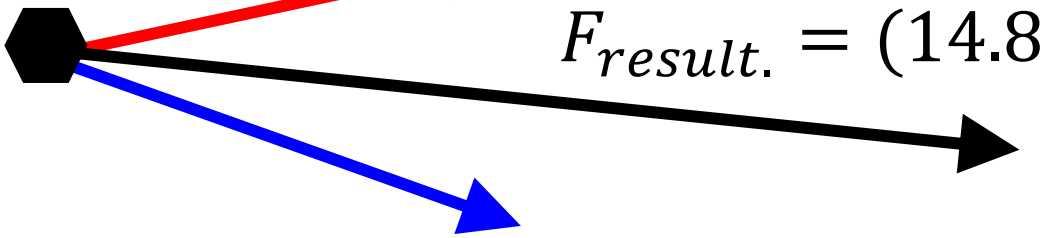
$\theta_{dir} = 80.2$

$F_2 = 5\cos 10\ i + 5\sin 10\ j$

$\theta_{dir} = 10$

$\theta_{difference} = 70.2$

Two forces are acting on an object. The vectors representing the forces are given by $\boldsymbol{F}_1 = \langle 14.8, 2.6 \rangle$ and $\boldsymbol{F}_2 = \langle 16.9, -6.1 \rangle$. Find the **direction angle** of the **resultant force**.



$F_1 = 14.8\ i + 2.6\ j$

$F_{result.} = (14.8 + 16.9)\ i + (2.6 - 6.1)\ j$

$F_{result.} = 31.7\ i - 3.5\ j$

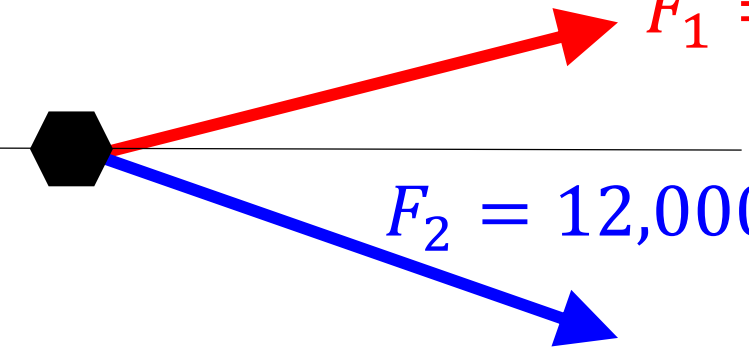
$F_2 = 16.9\ i - 6.1\ j$

$$\theta_{ref} = \tan^{-1}\left(\frac{3.5}{31.7}\right)$$

$\theta_{ref} = 6.3$ $\quad \theta_{dir} = 360 - 6.3$ $\quad \boxed{\theta_{dir} = 353.7}$

Two tugboats are connected to a disabled cruise ship and are attempting to tow the ship in an easterly direction. The 1st tug is exerting a force on the ship that has a magnitude of 10,000 kilograms at an angle of 10° from horizontal; this force is represented by the vector $\boldsymbol{F}_1$. The 2nd tug is exerting a force of 12,000 kg. at an angle of $-15°$ from horizontal; this force is represented by the vector $\boldsymbol{F}_2$. (Round your answers to 2 decimal places.)

$F_1 = 10{,}000 \cos 10\ i + 10{,}000 \sin 10\ j$

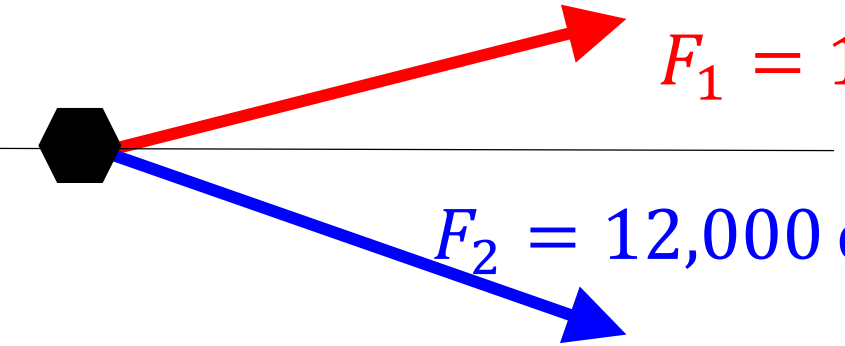


$F_2 = 12{,}000 \cos(-15)\ i + 12{,}000 \sin(-15)\ j$

(a) What is the <u>horizontal</u> component of the vector representing the force for the 2nd tugboat?

$F_{2i} = 12{,}000 \cos(-15)\ i$

$F_{2i} = 11{,}591.11$ kg



$F_1 = 10{,}000\cos 10\ i + 10{,}000\sin 10\ j$

$F_2 = 12{,}000\cos(-15)\ i + 12{,}000\sin(-15)\ j$

(b) In order to move the ship at all, the resultant force exerted by the two tugboats needs to be 20,000 kg. Will these tugboats be able to move the ship? yes

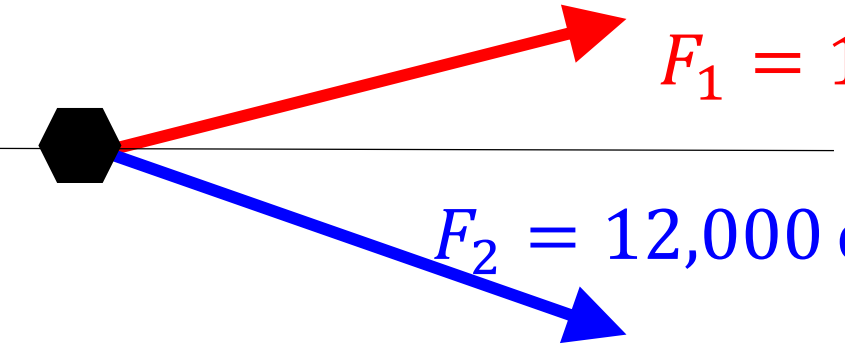
$$F_1 + F_2 = (10K\cos 10 + 12K\cos(-15)i$$
$$+(10Ksin10 + 12Ksin(-15)j$$

$$F_1 + F_2 = 21{,}439.19\ i - 1{,}369.35\ j$$

$$|F_1 + F_2| = \sqrt{(21{,}439.19)^2 + (-1{,}369.35)^2}$$

$$|F_1 + F_2| = \sqrt{(21{,}439.19)^2 + (-1{,}369.35)^2}$$

$$|F_1 + F_2| = 21{,}482.87\ kg$$



$F_1 = 10{,}000\cos 10\ i + 10{,}000\sin 10\ j$

$F_2 = 12{,}000\cos(-15)\ i + 12{,}000\sin(-15)\ j$

(c) Using the direction angle for the resultant force, determine whether or not the ship will move due east (direction angle of zero).

$$F_1 + F_2 = (10K\cos 10 + 12K\cos(-15)i + (10Ksin10 + 12Ksin(-15)j$$

$$F_1 + F_2 = 21{,}439.19\ i - 1{,}369.35\ j$$

$$\theta_{ref} = \tan^{-1}\left(\frac{1369.35}{21439.19}\right)$$

$$\theta_{ref} = 3.65$$

$$\theta_{dir} = 360 - 3.65 \qquad \boxed{\theta_{dir} = 356.35} \qquad \boxed{\text{no}}$$

Which of the following statement is TRUE concerning the vector $\boldsymbol{v} = -5\boldsymbol{i} + 12\boldsymbol{j}$?

Quadrant II

(a) The vector $\boldsymbol{u} = -512\boldsymbol{i} + \boldsymbol{j}$ is a **unit vector** in the **same direction** as $\boldsymbol{v}$.

(b) $|\boldsymbol{v}| = 169$

(c) $1/13\boldsymbol{v}$ is a **unit vector** in the **same direction** as $\boldsymbol{v}$.

(d) The **direction angle** of $\boldsymbol{v}$ is 292.62°.

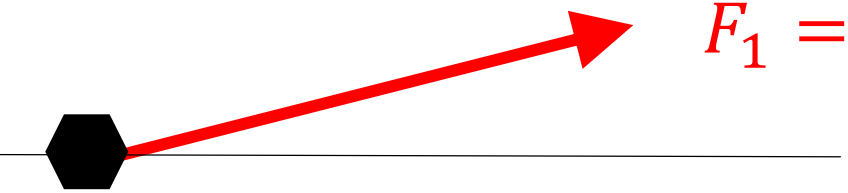
(e) Both (d) and (e) are TRUE.

$$u = \frac{v}{|v|} \qquad |v| = \sqrt{(-5)^2 + (12)^2} \qquad |v| = 13$$

$$\theta_{ref} = \tan^{-1}\left(\frac{12}{5}\right) \qquad \theta_{ref} = 67.4 \qquad \theta_{dir} = 180 - 67.4$$

$$\theta_{dir} = 112.6$$

A 400-pound force is applied by a logger to a cable attached to a tree that has been cut down. The cable from the end of the tree to the logger is at an angle of 30° from horizontal. How much work is done if the logger drags the tree **horizontally along the ground** for 40 feet? Use the work equation $W = \boldsymbol{F} \cdot \boldsymbol{d}$. (Round your answer to one decimal place.)



$$F_1 = 400 \cos 30\ i + 400 \sin 30\ j$$

$$F_{1i} = 364.41\ lbf$$

$$w = F_{1i} * d = 364.41\ lbf * 40\ ft$$

$$13{,}856.41\ \ ft * lbf$$