Math-1060
7-1 (Part 1)

Vectors in the Plane

Vocabulary:

Vector

Position vector

Component form of a vector

Zero Vector

Magnitude of a Vector

Scalar Multiplication

Unit Vector

Standard Unit Vector

Direction Angle

Horizontal Component

Vertical Component

Resolving a Vector

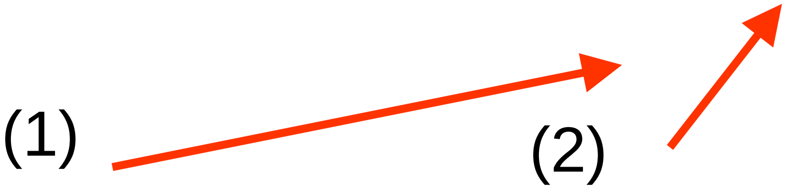
Vector: a single real number that represents both magnitude and direction.

We can show vectors as an arrow.

(1) length indicates magnitude

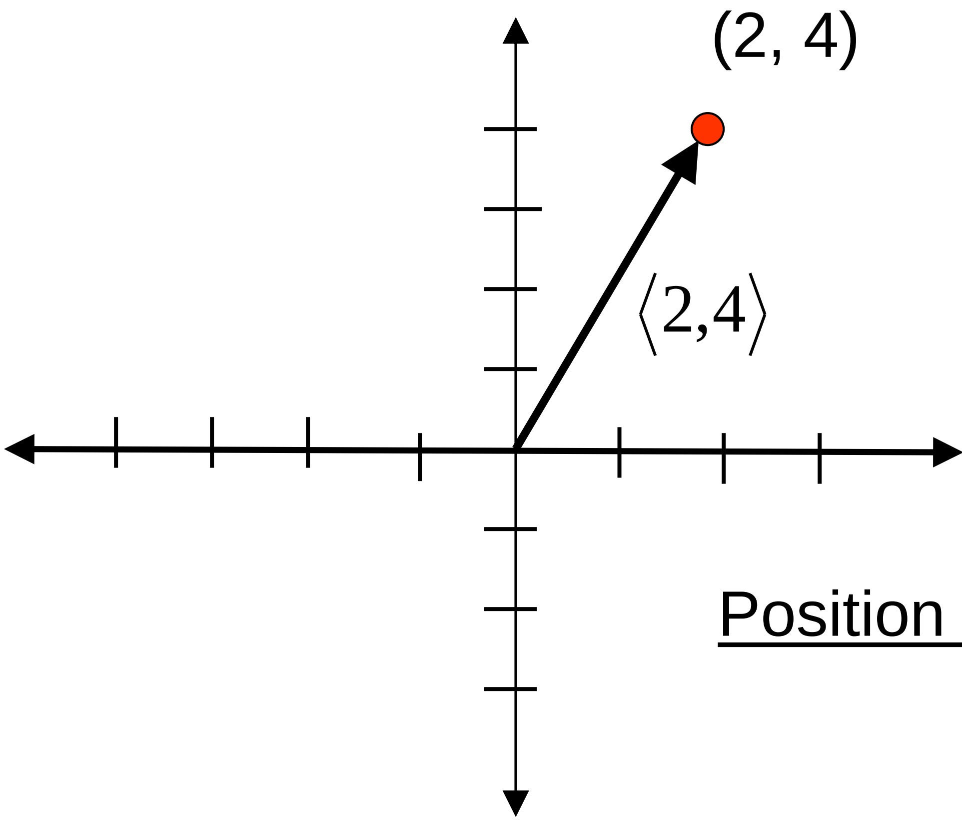
(2) arrow head indicates direction

Which vector is bigger?



Which vector has a *bigger component* in the up direction?

Position Vector: a vector whose "tail" is at the origin.



2 Meanings:

(1) point in the plane

(2) A vector

(tail at origin)
(arrow head at point)

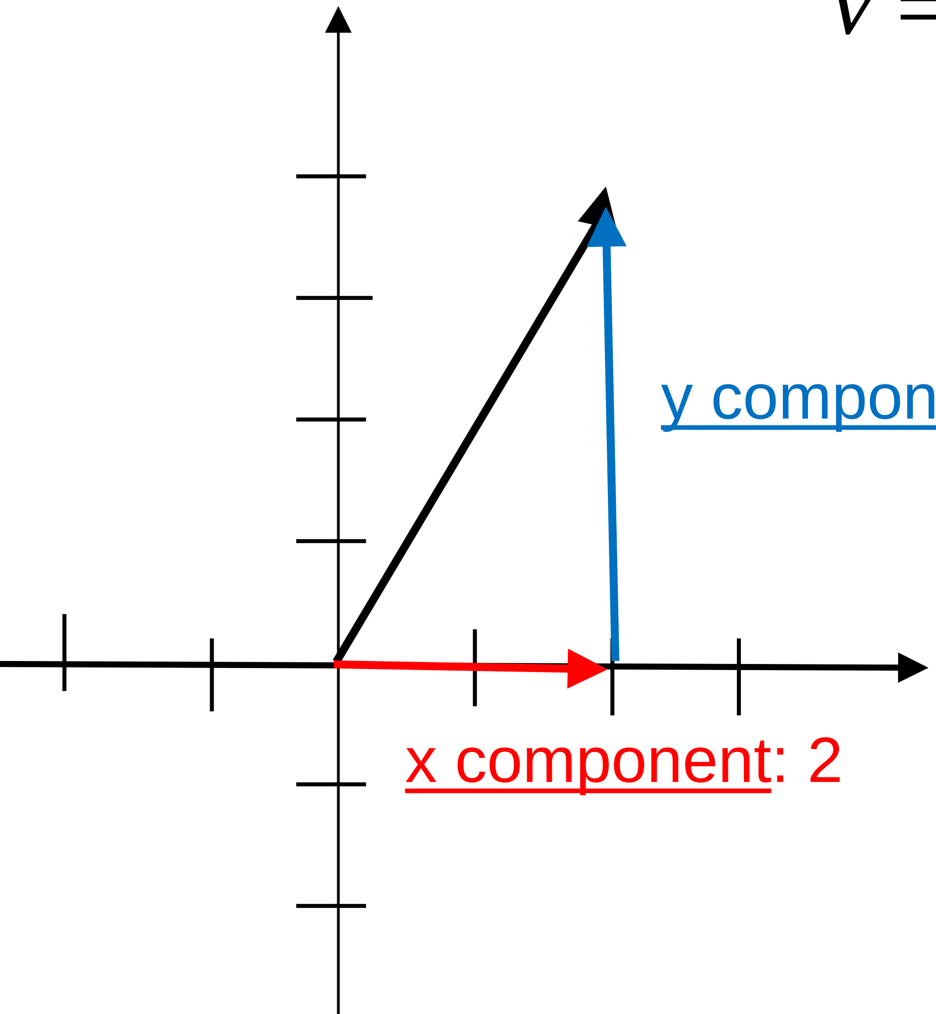
Position Vector of the point (2, 4)

<u>2 Dimensional Vectors</u>

An ordered pair of real numbers in <u>component form</u>

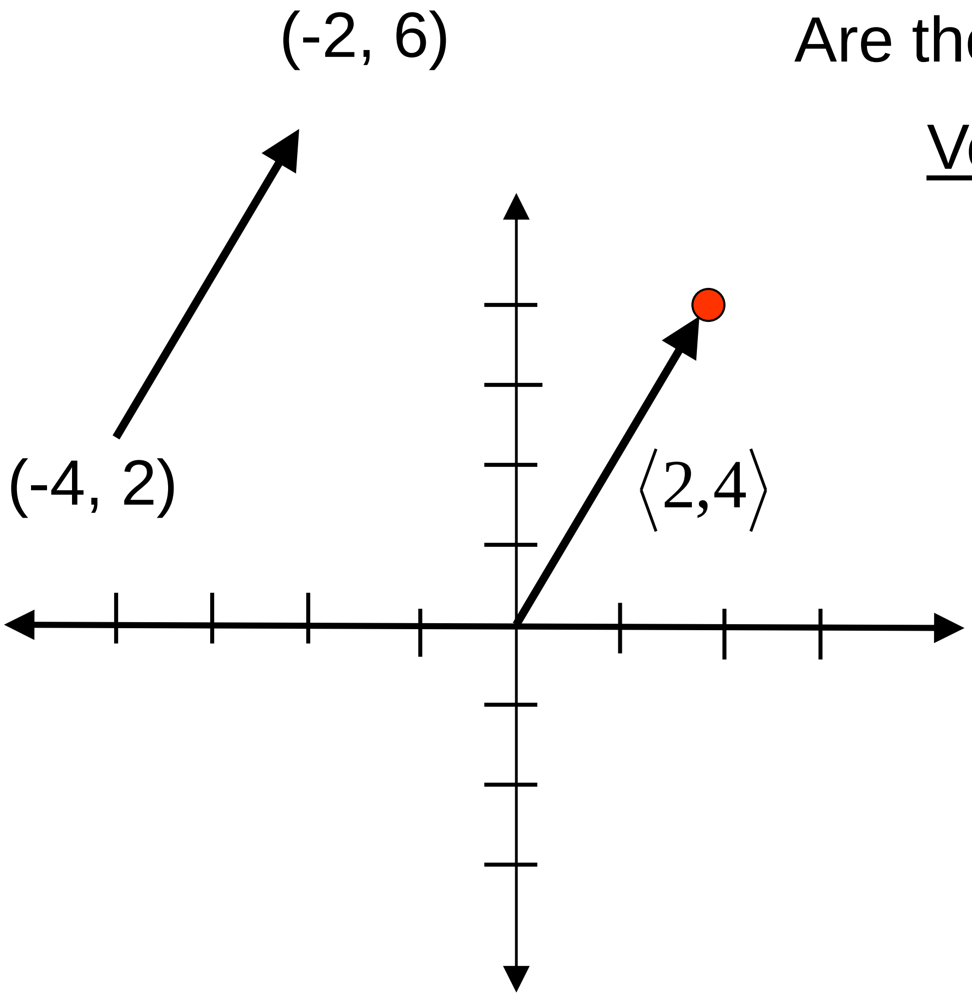
$$v = \langle 2,4 \rangle$$

"walk across somebody's yard or go to the corner then up the next street"



Other Vectors

Vector with tail not at the origin.



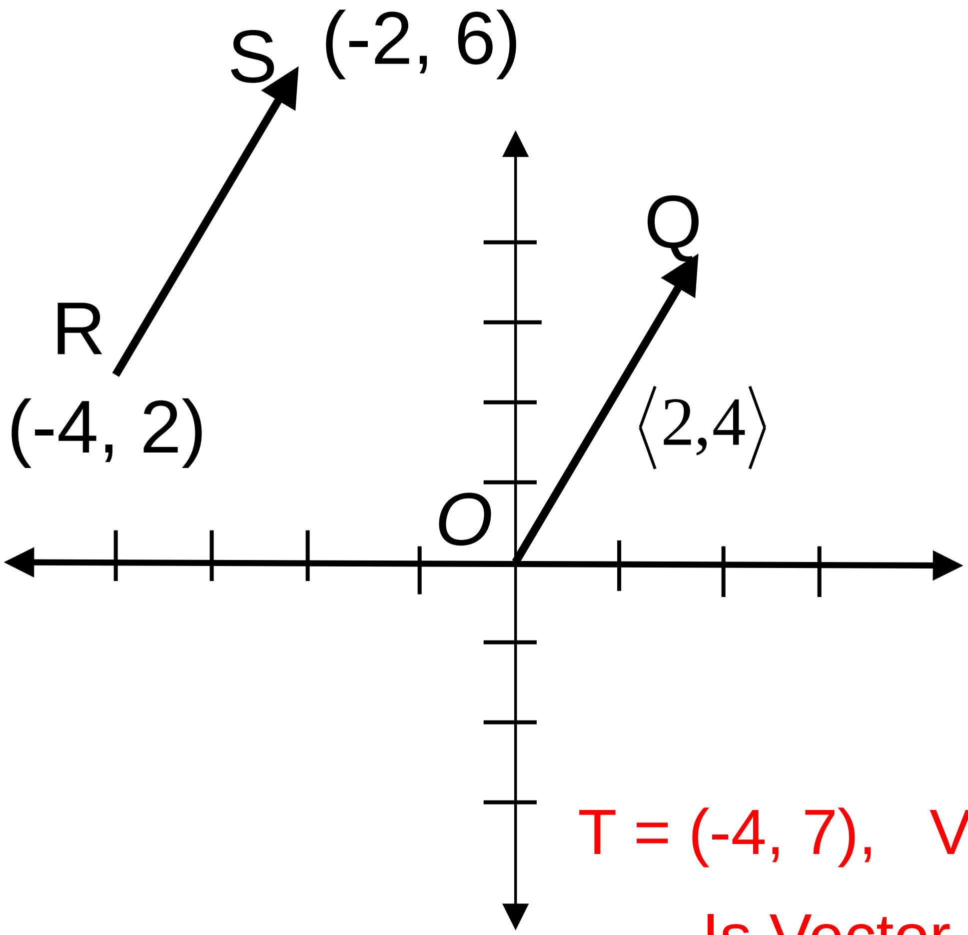
Are these equivalent vectors?

Vector: magnitude & direction

(1) Do they have the same magnitude (length)?

(2) Do they have the same direction (horizontal and vertical components)?

Head Minus Tail (HMT) Rule



$\vec{RS}$ Initial point: (-4, 2)

terminal point: (-2, 6)

Represents the Vector:

$$v = \langle x_2 - x_1, y_2 - y_1 \rangle$$

$$v = \langle (-2-(-4)), (6-2) \rangle = \langle 2,4 \rangle$$

Are they the same vector?

T = (-4, 7), V = (-1, 5), K = (0, 0) Q = (3, -2)

Is <u>Vector TV</u> equivalent to <u>Vector KQ</u>?

Zero Vector: $v = \langle 0,0 \rangle$

(starts and ends where it began (at the origin))

Magnitude: (length of the vector) $|v|$

From (x_1, y_1) to (x_2, y_2) $|v| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(distance formula)

If the vector is: $v = \langle a, b \rangle$ (tail end at the origin)

then: $|v| = \sqrt{a^2 + b^2}$

Find the *magnitude* of the vector that begins at (-3, 4) and ends at (-5, 2).

(what is the distance between these two points)

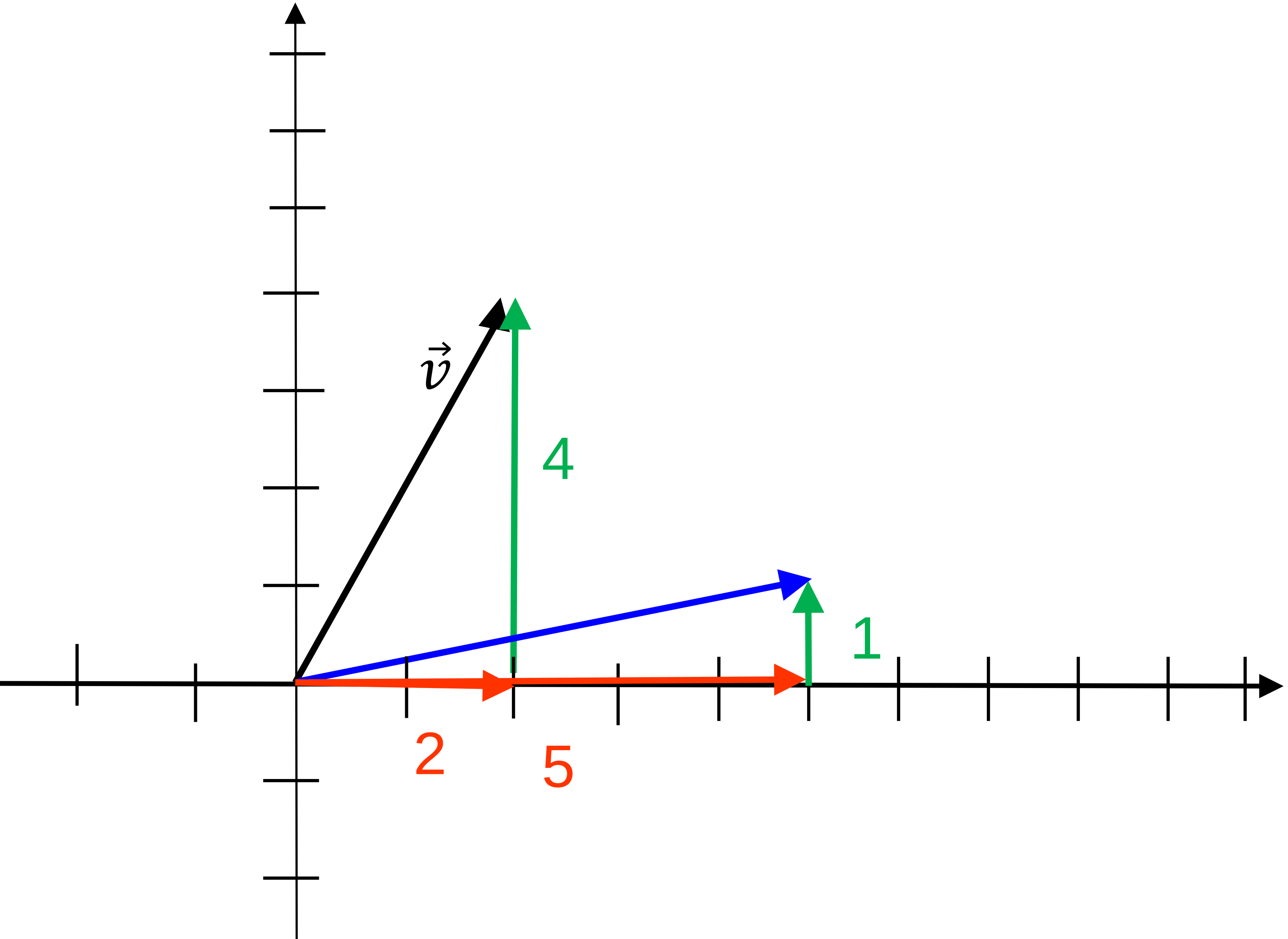
Find the *magnitude* of: $v = \langle 3,4 \rangle$

(begins at the origin and ends at (3, 4)

Find a vector that begins at the origin that is equivalent to the vector from (-3, 4) to (-5, 2)

Vector Addition

$v = \langle 2,4 \rangle \qquad u = \langle 5,1 \rangle$

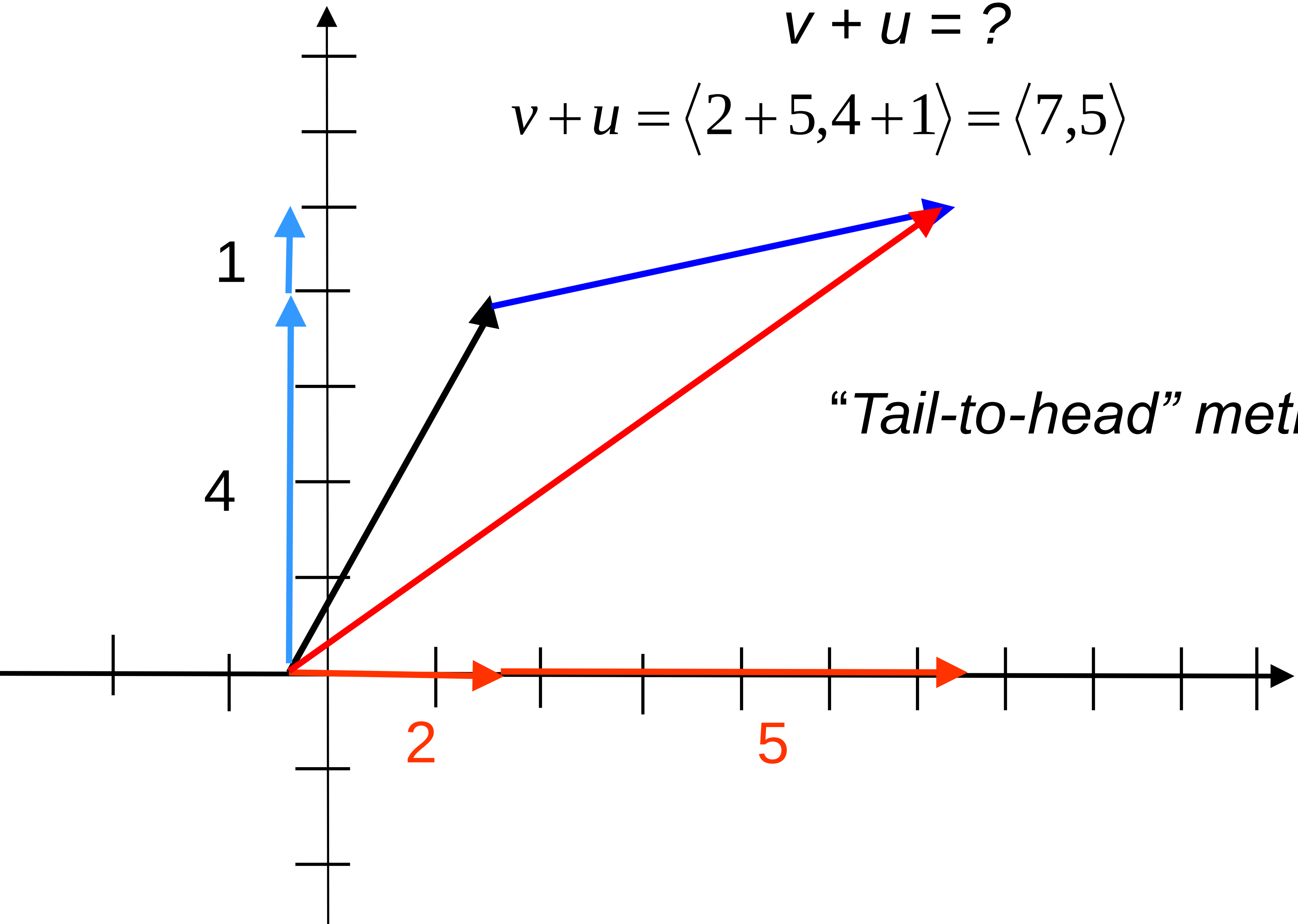


Vector Addition

$$v = \langle 2,4 \rangle \qquad u = \langle 5,1 \rangle$$

$$v + u = ?$$

$$v + u = \langle 2+5, 4+1 \rangle = \langle 7,5 \rangle$$



"Tail-to-head" method

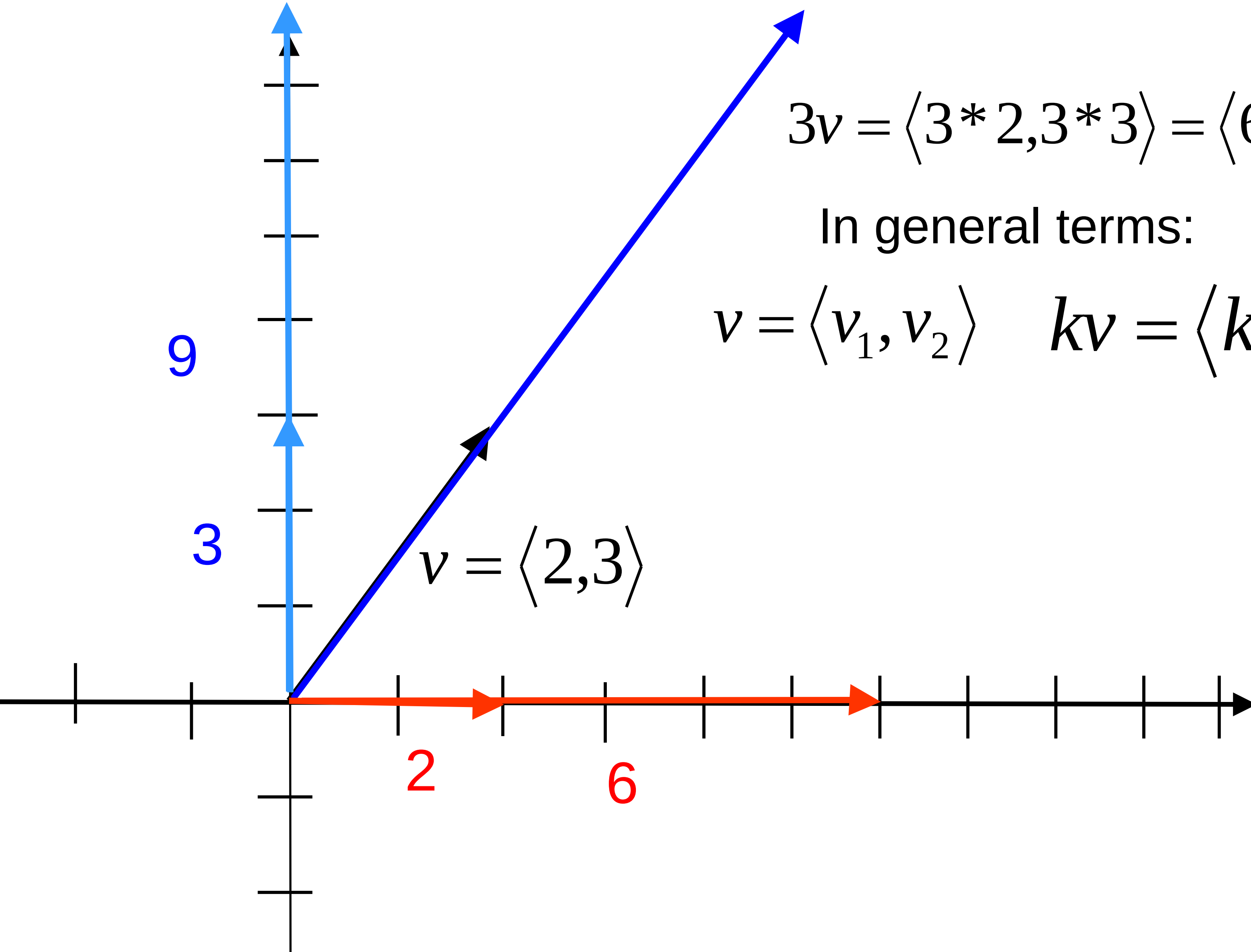
$$w = \langle -3, 7 \rangle \qquad v = \langle 4, -2 \rangle$$

For: w + v

Draw the two vectors (tail-to-head)

What is the resultant of the vectors “w” and “v”

Scalar Multiplication



$$v = \langle 2,3 \rangle$$

$$3v = \langle 3*2, 3*3 \rangle = \langle 6,9 \rangle$$

In general terms:

$$v = \langle v_1, v_2 \rangle \qquad kv = \langle kv_1, kv_2 \rangle$$

$v = \langle 3, -4 \rangle$ $m = \langle 2, 5 \rangle$ $n = \langle -1, 7 \rangle$

$$4v = ?$$

$$m + n = ?$$

$$2m - v = ?$$

<u>Unit Vector</u>: a vector with a magnitude of ‘1’.

$$|u| = 1$$

u is the “unit vector.”

(this “absolute value” symbol means magnitude)

A unit vector has a *<u>magnitude</u>* of ‘1’ but it <u>also has a direction.</u>

If you have a vector whose magnitude is 3,
what happens if you multiply that vector by 1/3 ?

It’s magnitude becomes 1 → it becomes a unit vector.

$$u = \frac{m}{|m|}$$

Any vector divided by its magnitude becomes a unit vector.

Unit Vector

Find the unit vector for: $v = \langle 2,4 \rangle$

$$u = \frac{v}{|v|} \qquad |v| = \sqrt{2^2 + 4^2} \qquad |v| = \sqrt{20} = 2\sqrt{5}$$

$$u = \frac{1}{2\sqrt{5}}\langle 2,4 \rangle = \left\langle \frac{2}{2\sqrt{5}}, \frac{4}{2\sqrt{5}} \right\rangle = \left\langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle$$

This unit vector ‘u’ has the <u>same direction</u> as vector ‘v’ since it is a scalar multiple of vector ‘v’.

Does this “unit vector” have a magnitude of ‘1’?

So does this "unit vector" have a magnitude of '1'?

$$\left\langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle$$

$$|u| = \sqrt{\left(\frac{1}{\sqrt{5}}\right)^2 + \left(\frac{2}{\sqrt{5}}\right)^2}$$

$$|u| = \sqrt{\frac{1}{5} + \frac{4}{5}}$$

$$|u| = \sqrt{\frac{5}{5}} = 1$$

YES!!

Find the unit vector for: $m = \langle 3,1 \rangle$

Standard unit vectors: unit vectors that have a magnitude of '1' and point to either:

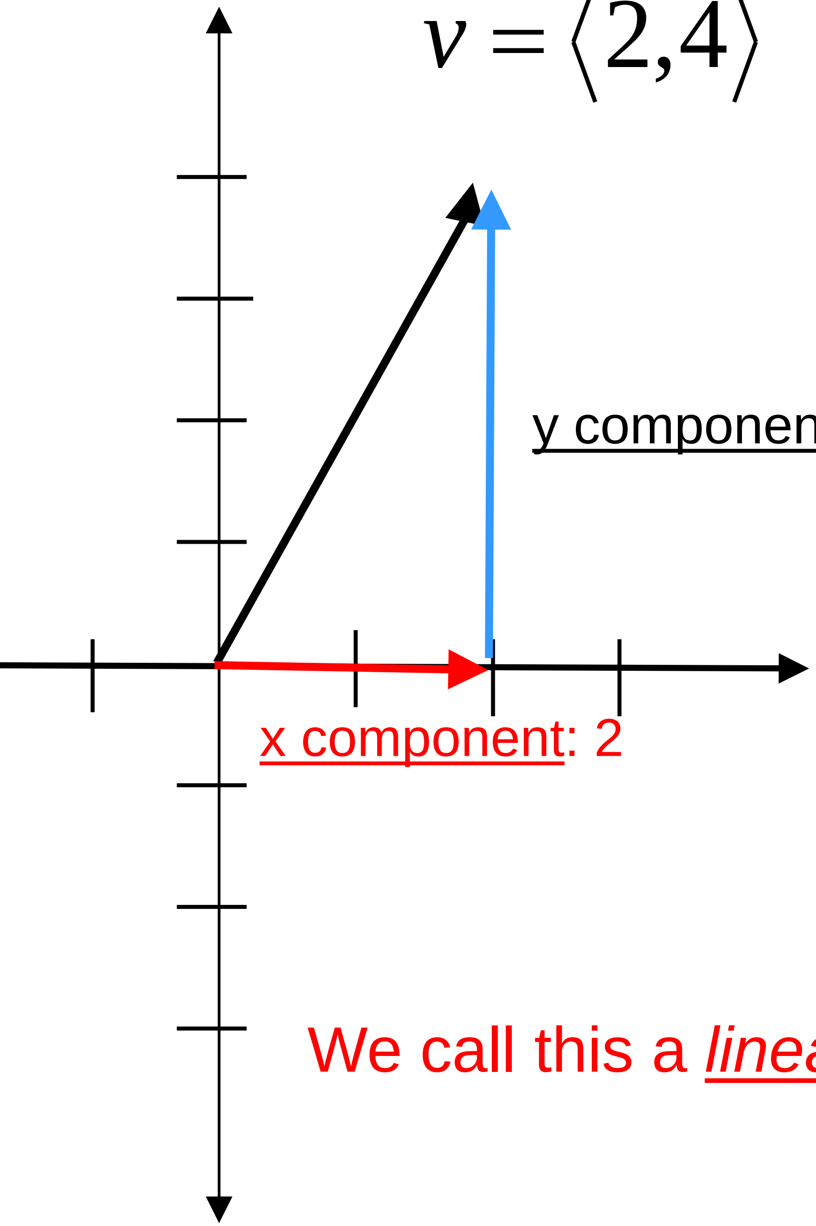
(1) + x direction $i = \langle 1,0 \rangle$

or

(2) + y direction. $j = \langle 0,1 \rangle$

Re-writing a vector as a scalar multiple of a standard vector.

$$v = \langle 2,4 \rangle$$



We could rewrite this as the addition of the x and y component vectors.

$$v = \langle 2,0 \rangle + \langle 0,4 \rangle$$

Each of these could be rewritten as a scalar times a standard unit vector.

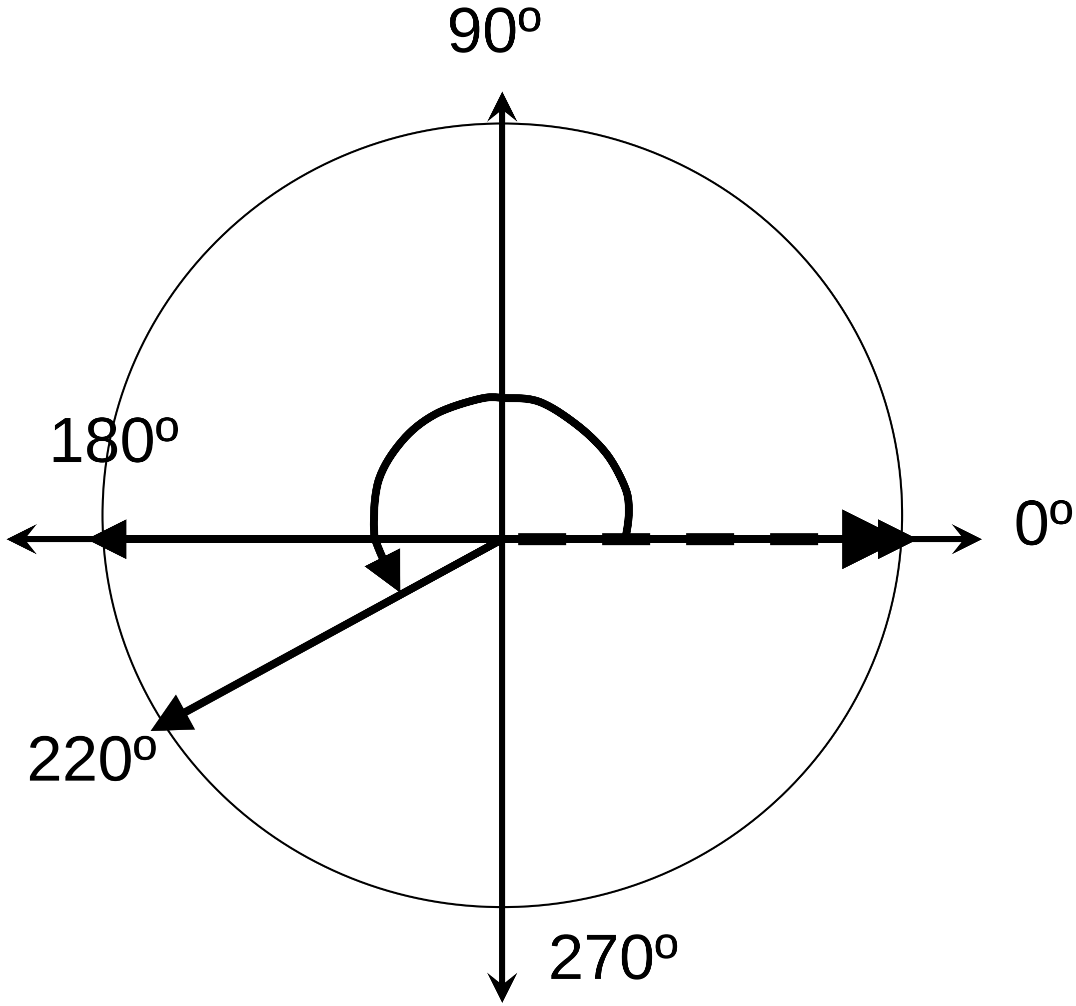
$$v = 2\langle 1,0 \rangle + 4\langle 0,1 \rangle$$

$$\boxed{v = 2i + 4j}$$

We call this a *linear combination* of the vectors "*i*" *and* "*j*"

Direction Angle: the *positive* angle between the x-axis and the position vector 'v' Think of this as an angle in Standard position.

Draw the standard position angle with the given measure. 220º



To find the direction angle of a vector:

$v = \langle -2, 4 \rangle$

1. The position vector is in Quadrant II.

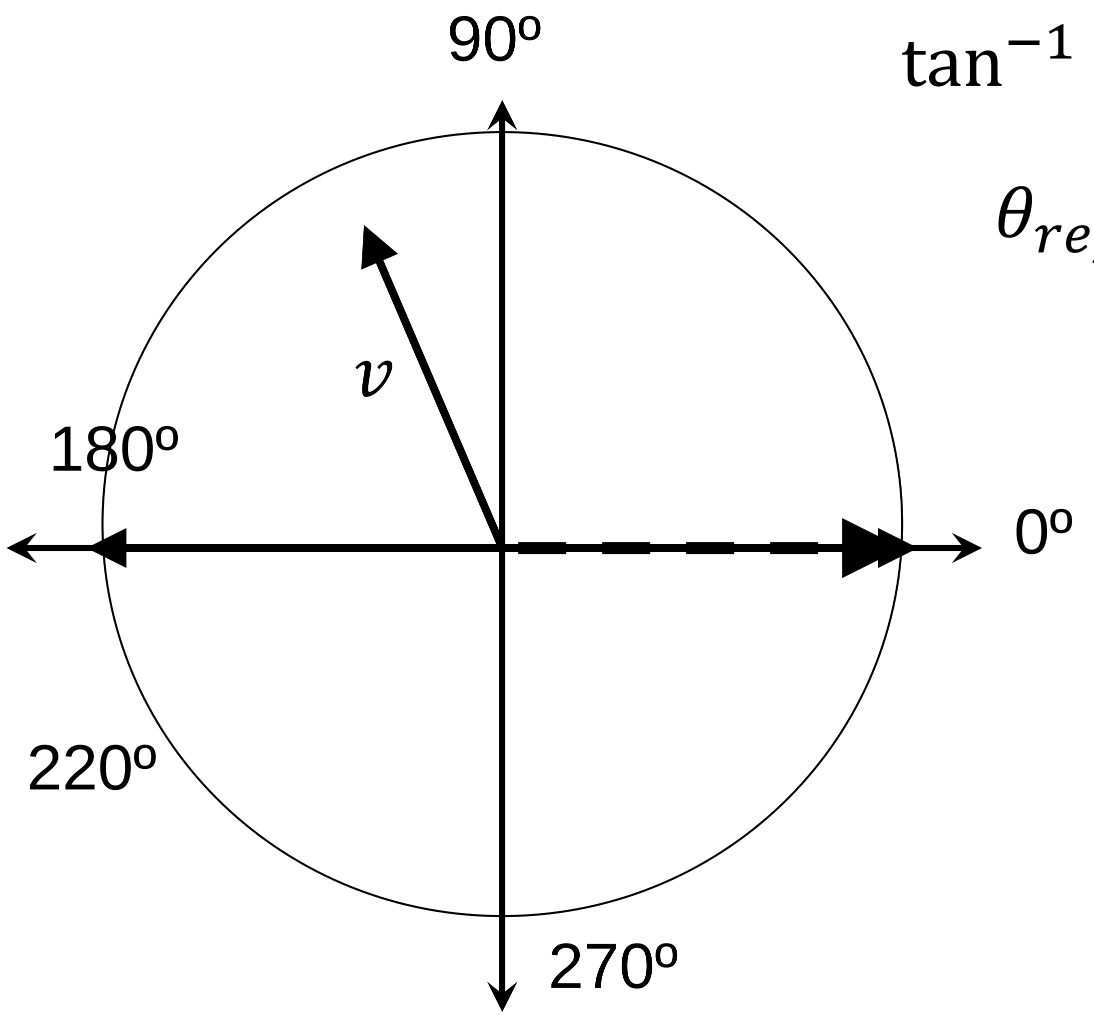
2. Find the reference angle. $\tan^{-1} \frac{y}{x} = \theta$

$$\tan^{-1}\left(\frac{4}{2}\right) = \theta$$

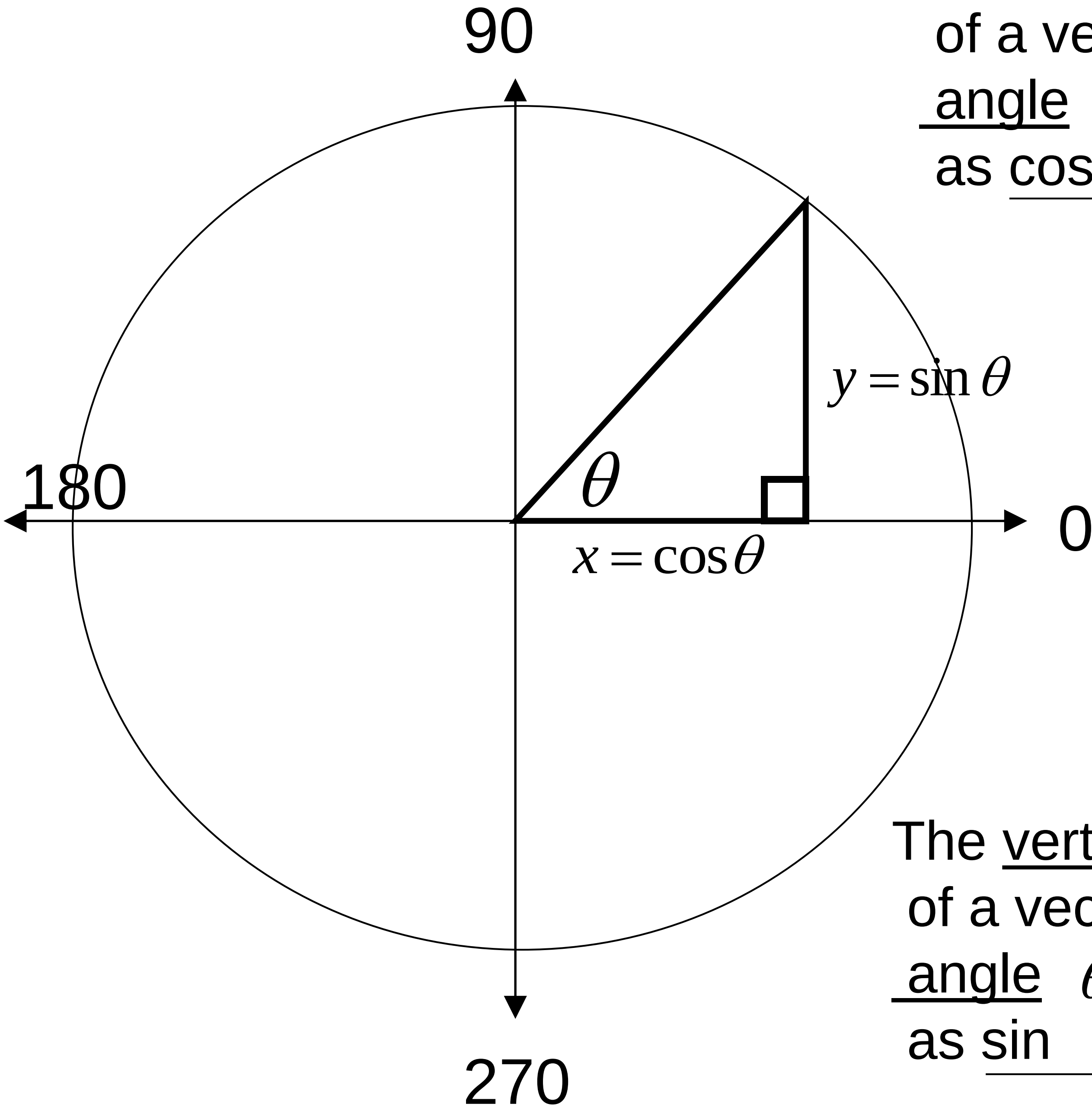
$$\theta_{ref} = 63.4$$

$$\theta_{position} = 180 - 63.4$$

$$\boxed{\theta_{position} = 116.6}$$



The horizontal component of a vector with a direction angle θ can be written as cos θ

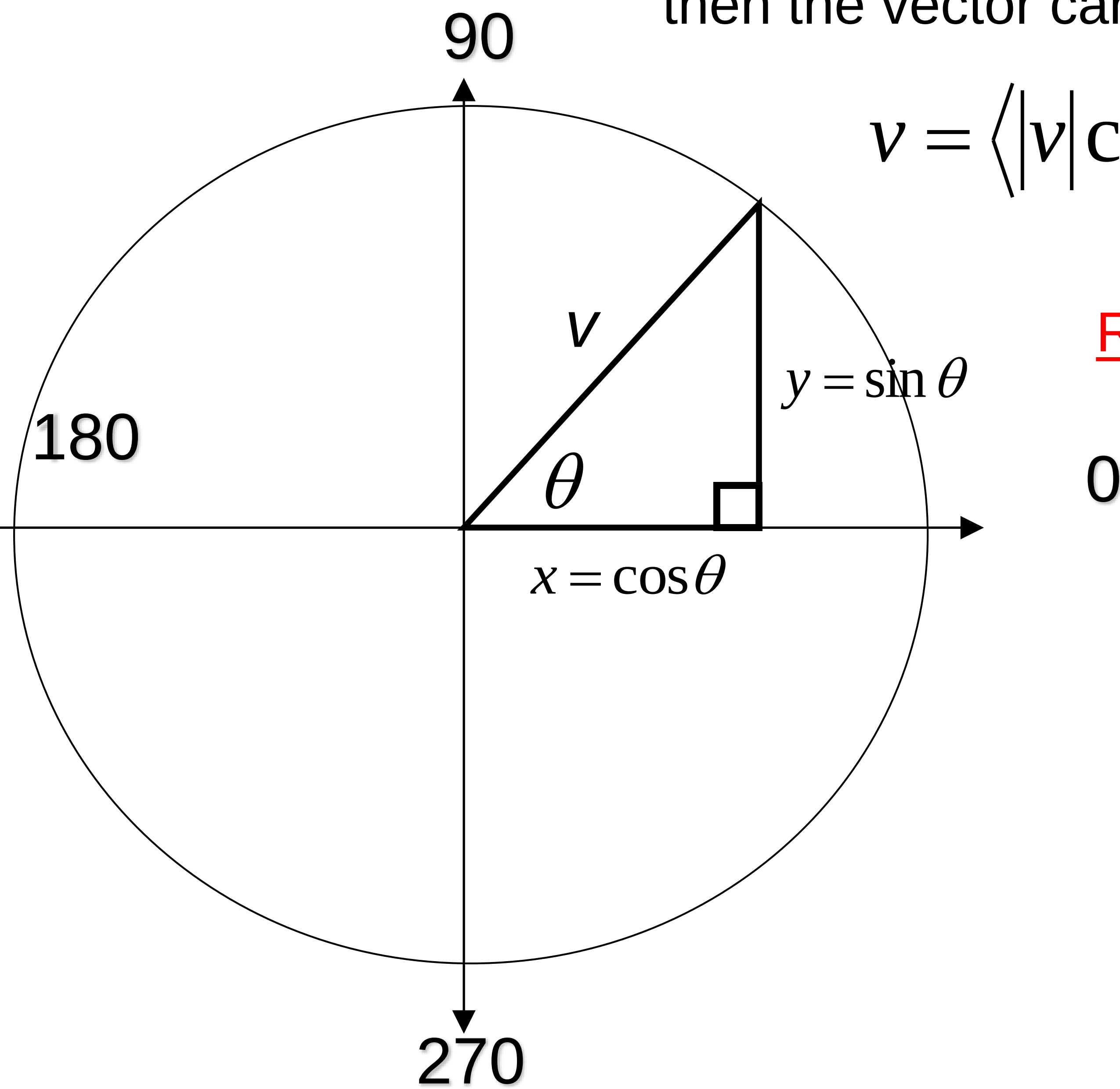


The vertical component of a vector with a direction angle θ can be written as sin θ

If a vector has a direction angle θ
then the vector can be rewritten:

$$v = \langle |v|\cos\theta, |v|\sin\theta \rangle$$

We call this
Resolving the Vector.



Resolving Vectors

Find the "*i*" and "*j*" components of a vector (resolve the vector) with a direction angle of 120º, and a magnitude of 6.

$$v = \langle |v|\cos\theta, |v|\sin\theta \rangle \qquad v = \langle 6\cos 120^\circ, 6\sin 120^\circ \rangle$$

We convert standard position angles that are *not in quadrant I*, into acute reference angles and we account for +/-.

$$v = \langle 6(-\cos 60^\circ), 6\sin 60^\circ \rangle$$

(BUT a decent calculator will take care of the +/-)

$$v = \langle -0.5, 0.86 \rangle$$

$$v = -0.5i + 0.866j$$

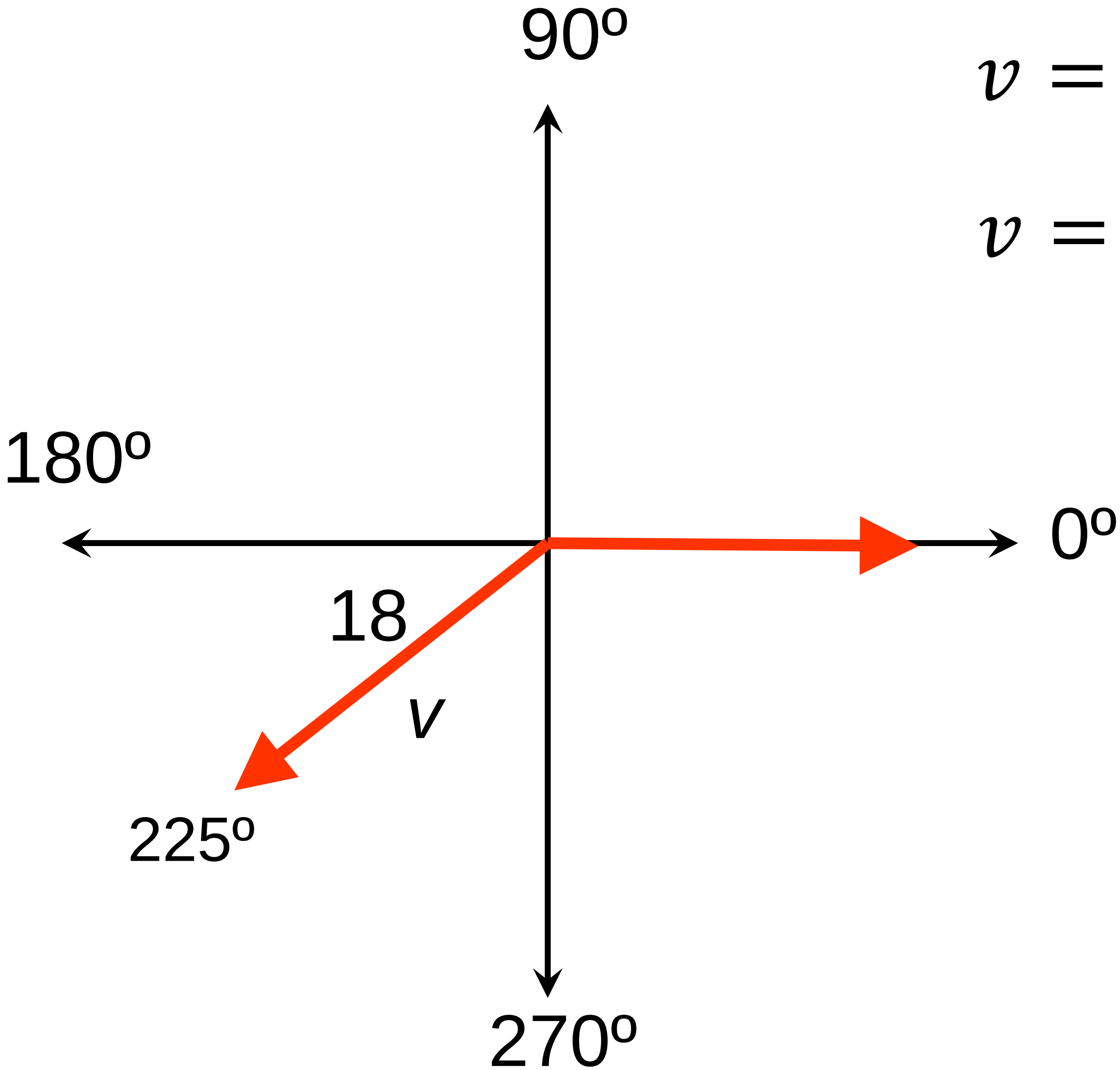
Find the component form of the vector 'v' (v = ai + bj)

$$v = \langle |v| \cos\theta, |v| \sin\theta \rangle$$

$$v = \langle 18 \cos 225 , 18 \sin 225 \rangle$$

$$v = \langle -12.73, -12.73 \rangle$$

$$v = -12.73i - 12.73j$$



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Resolving a Vector

Find: Magnitude and the Direction Angle of a Vector

$$v = \langle 5,7 \rangle$$ (which quadrant?)

$$|v| = \sqrt{5^2 + 7^2} = \sqrt{74}$$ (Magnitude)

$$v = \langle |v|\cos\theta, |v|\sin\theta \rangle$$

$$5 = \sqrt{74}\cos\theta$$

$$\theta = \cos^{-1}\frac{5}{\sqrt{74}} = 54.5^\circ$$ (Direction angle)

Find: Magnitude and the Direction Angle of a Vector

$$v = \langle -1, -3 \rangle$$ (quadrant III)

$$|v| = \sqrt{(-1)^2 + (-3)^2} \quad = \sqrt{10}$$ (Magnitude)

$$v = \langle |v| \cos\theta, |v| \sin\theta \rangle$$

$$1 = \sqrt{10} \cos\theta$$ (use + side lengths)

$$\theta_{ref} = \cos^{-1} \frac{1}{\sqrt{10}} = 71.6^\circ$$ (reference angle)

$$\theta_{position} = 180 + 71.6$$

$$\boxed{\theta_{position} = 251.6}$$

Properties of Vectors

$v + u = u + v$ Vector Addition is “commutative”

$(v + u) + w = u + (v + w)$ Vector Addition is “associative”

$(k_1 k_2)u = k_1(k_2 u)$ Scalar multiplication is “associative”

$k_1(u + v) = k_1 u + k_1 v)$

Scalar multiplication is “distributive”

$(k_1 + k_2)v = k_1 v + k_2 v)$

$0u = 0$ Zero Product Property

$1u = u$ Identity Property of Multiplication

$u + (-u) = 0$ Inverse Property of Addition