

Math-1060

Lesson 6-3

“Double and Half-Angle Identities”

The Sine Sum Identity

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

Let B = A

$$\sin(A + A) = \sin A \cos(A) + \cos A \sin(A)$$

Sine Double Angle Identity

$$\sin(2A) = 2 \sin A \cos A$$

The Cosine Sum Identity

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

Let B = A

$$\cos(A + A) = \cos A \cos A - \sin A \sin A$$

Cosine Double Angle Identity

$$\cos(2A) = \cos^2 A - \sin^2 A$$

Cosine Double Angle Identity

$$\cos(2A) = \cos^2 A - \sin^2 A$$

$$\cos^2 A = 1 - \sin^2 A$$

Cosine Double Angle Identity

$$\cos(2A) = 1 - 2 \sin^2 A$$

$$\sin^2 A = 1 - \cos^2 A$$

$$\cos(2A) = \cos^2 A - (1 - \cos^2 A)$$

$$\cos(2A) = 2 \cos^2 A - 1$$

Looks somewhat like the Pythagorean Identity

$$\cos^2 A + \sin^2 A = 1$$

Rearrange the Pythagorean Identity

Use Substitution

Rearrange the Pythagorean Identity

Use Substitution

Cosine Double Angle Identity

Find $\sin(2x)$ given that $\cos(x) = \frac{1}{4}$ and $\sin(x) < 0$

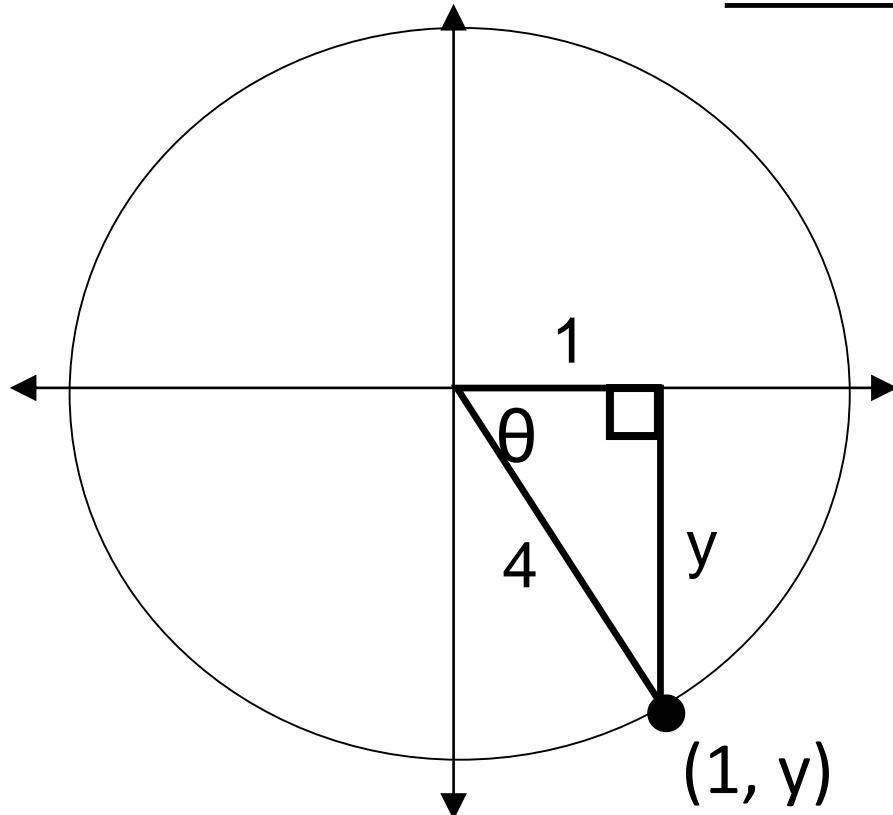
Only Quadrant IV has a positive cosine ratio and a negative sine ratio.

1. Find 'y' $y^2 + (1)^2 = 16$ $y = -\sqrt{15}$

2. Find: $\sin(2x)$

Sine Double Angle Identity

$$\sin(2x) = 2 \sin x \cos x$$



$$\cos(x) = \frac{1}{4} \quad \sin(x) = -\frac{\sqrt{15}}{4}$$

$$\sin(2x) = 2 * \frac{-\sqrt{15}}{4} * \frac{1}{4} = \frac{-\sqrt{15}}{8}$$

Find $\sin(2x)$, $\cos(2x)$, and $\tan(2x)$ given that $\sin(x) = \frac{-2}{3}$ and $\cos(x) < 0$

Only Quadrant III has a positive cosine ratio and a negative sine ratio. This time we'll use the Pythagorean Identity

$$\cos^2 x + \sin^2 x = 1$$

1. Find $\cos(x)$ $\cos^2 x + \left(-\frac{2}{3}\right)^2 = 1$ $\cos^2 x = 1 - \frac{4}{9}$ $\cos^2 x = \frac{5}{9}$

$$\cos x = \frac{-\sqrt{5}}{3}$$

$$\sin(x) = \frac{-2}{3}$$

2. Find: $\sin(2x)$ Sine Double Angle Identity $\sin(2x) = 2 \sin x \cos x$

$$\sin(2x) = 2 * -\frac{2}{3} * \frac{-\sqrt{5}}{3}$$
$$= \frac{4\sqrt{5}}{9}$$

Find $\sin(2x)$, $\cos(2x)$, and $\tan(2x)$ given that $\sin(x) = \frac{-2}{3}$ and $\cos(x) < 0$

3. Find: $\cos(2x)$

$$\cos x = \frac{-\sqrt{5}}{3} \quad \sin(x) = \frac{-2}{3}$$

Cosine Double Angle Identity

$$\cos(2A) = \cos^2 A - \sin^2 A$$

$$\cos(2x) = \left(\frac{-\sqrt{5}}{3}\right)^2 - \left(\frac{-2}{3}\right)^2 = \frac{5}{9} - \frac{4}{9} = \frac{1}{9}$$

4. Find: $\tan(2x)$

$$\tan(2x) = \sin(2x)/\cos(2x) = \frac{\frac{-2}{3}}{\frac{-\sqrt{5}}{3}} = \frac{-2}{3} * \frac{3}{-\sqrt{5}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

Verifying Identities Using Double Angle Identities

$$(\cos x - \sin x)^2 =$$

$$\cos^2 x - 2 \cos x \sin x + \sin^2 x =$$

$$1 - 2 \cos x \sin x =$$

$$1 - \sin(2x) =$$

Choose a side → work on the left

Pythagorean Identity

$$\cos^2 x + \sin^2 x = 1$$

Sine Double Angle Identity

$$\sin(2x) = 2 \sin x \cos x$$

Verifying Multiple Angle Identities

Choose a side → work on the left

$$\cos(3x) = 1 - 4 \sin^2(x) \cos(x)$$

$$\cos(2x + x) =$$

$$\cos(2x) \cos(x) - \sin(2x) \sin(x) =$$

Cosine Sum Identity

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

Cosine Double Angle Identity

$$\cos(2A) = 1 - 2 \sin^2 A$$

$$(1 - 2 \sin^2(x)) \cos(x) - 2 \sin(x) \cos(x) * \sin(x) =$$

Sine Double Angle Identity

$$\cos(x) - 2 \sin^2(x) \cos(x) - 2 \sin^2(x) \cos(x) =$$

$$\sin(2x) = 2 \sin x \cos x$$

$$\cos(x) - 4 \sin^2(x) \cos(x) = 1 - 4 \sin^2(x) \cos(x)$$

Simplifying Expression Using Double Angle Identities

Graph: $y = \frac{\cot(x) - \tan(x)}{\cot(x) + \tan(x)}$ Convert to sines and cosines

$$y = \frac{\frac{\cos(x)}{\sin(x)} - \frac{\sin(x)}{\cos(x)}}{\frac{\cos(x)}{\sin(x)} + \frac{\sin(x)}{\cos(x)}} \quad \text{Obtain a Common denominator}$$

$$y = \frac{\frac{\cos^2(x) - \sin^2(x)}{\sin(x)\cos(x)}}{\frac{\cos^2(x) + \sin^2(x)}{\sin(x)\cos(x)}} \quad y = \frac{\cos^2(x) - \sin^2(x)}{\cos^2(x) + \sin^2(x)}$$

Numerator multiplied by the reciprocal of the denominator

$$y = \frac{\cos^2(x) - \sin^2(x)}{\cos^2(x) + \sin^2(x)}$$

Pythagorean Identity

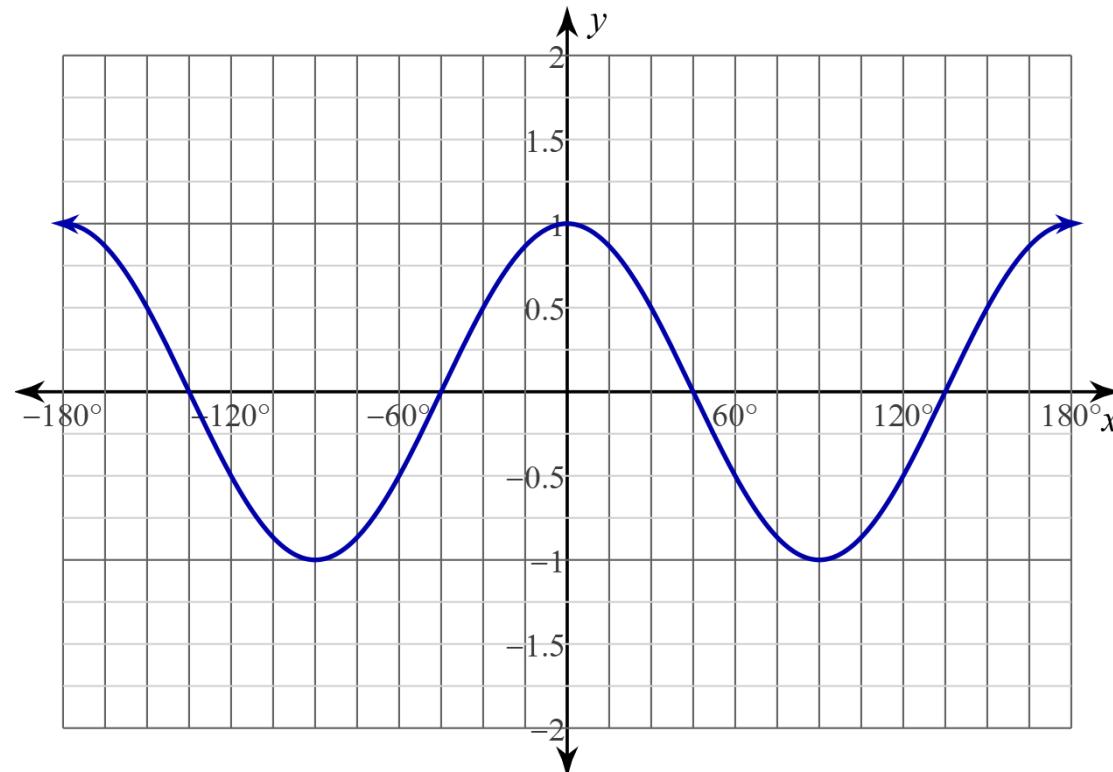
$$\cos^2 x + \sin^2 x = 1$$

$$y = \cos^2(x) - \sin^2(x)$$

Cosine Double Angle Identity

$$\cos(2A) = \cos^2 A - \sin^2 A$$

$$y = \cos(2x)$$



Half-Angle Identities

Cosine Double Angle Identity

$$\cos(2A) = 1 - 2 \sin^2 A$$

$$2\sin^2 A = 1 - \cos(2A)$$

$$\sin^2 A = \frac{1 - \cos(2A)}{2}$$

Almost there

Rearrange

Rearrange

Cosine Double Angle Identity

$$\cos(2A) = 2 \cos^2 A - 1$$

$$2\cos^2 A = 1 + \cos(2A)$$

$$\cos^2 A = \frac{1 + \cos(2A)}{2}$$

$$\tan^2 A = \frac{1 - \cos(2A)}{1 + \cos(2a)}$$

Half-Angle Identities

$$\sin^2 x = \frac{1 - \cos(2x)}{2}$$

Let: $x = \frac{A}{2}$

$$\cos^2 A = \frac{1 + \cos(2A)}{2}$$

$$\sin^2\left(\frac{A}{2}\right) = \frac{1 - \cos\left(2\left(\frac{A}{2}\right)\right)}{2}$$

Sine Half-Angle Identity

$$\sin\left(\frac{A}{2}\right) = \pm \sqrt{\frac{1 - \cos(A)}{2}}$$

Cosine Half-Angle Identity

$$\cos\left(\frac{A}{2}\right) = \pm \sqrt{\frac{1 + \cos(A)}{2}}$$

Tangent Half-Angle Identity

$$\tan\left(\frac{A}{2}\right) = \pm \sqrt{\frac{1 - \cos(A)}{1 + \cos(A)}}$$

(Lesson 6-2) Find the exact trig ratios for the angle. $\sin(15)$

$$\rightarrow \sin(60 - 45) \rightarrow \sin(60)\cos(45) - \cos(60)\sin(45)$$

$$\rightarrow \frac{\sqrt{3}}{2} * \frac{\sqrt{2}}{2} - \frac{1}{2} * \frac{\sqrt{2}}{2} * \rightarrow \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \rightarrow \frac{\sqrt{6} - \sqrt{2}}{4}$$

≈ 0.2588190451

Finding Exact Values of Angles Using Half-Angle Identities

$$\sin\left(\frac{A}{2}\right) = \pm \sqrt{\frac{1 - \cos(A)}{2}}$$

$$\sin(15) = \sin\left(\frac{30}{2}\right) = \sqrt{\frac{1 - \cos(30)}{2}}$$

Are they equivalent?

$$= \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} - \sqrt{\frac{\frac{2}{2} - \frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2 - \sqrt{3}}{4}} \rightarrow \frac{\sqrt{2 - \sqrt{3}}}{2}$$

≈ 0.2588190451