Math-1060
Lesson 6-1

Verifying Trigonometric Identities

Domain of a single-variable equation. $$3x+2=6x+4$$

We identify the domain of the expression on the right side of the "=" sign separately from the domain of the left side.

The expression: $6x + 4$ is defined for all real numbers.

The expression: $3x + 2$ is defined for all real numbers.

Domain of Validity: the set of all values for 'x' that results in both sides of the equation being defined.

$$\frac{2x^2-4x}{x-2}=2x$$

Domain of validity: $x \neq 2$

Identity: an equation that is true for all values that are in the domain of both sides of the equation.

Identity: an equation that is true for all values that are in the domain of both sides of the equation.

Is it an Identity? If not, why not?

$$4x + 2 = 6x + 4$$

No. The domain of validity is "all real numbers" but the equation is true only when x = 1.

$$\frac{2x^2 - 4x}{x - 2} = 2x$$

When we factor the left side of the equation, we have $\frac{2x(x-2)}{x-2} = 2x$

Which simplifies to $2x = 2x$

Which is true for all real numbers.

AND it is true for all values in domain of validity (x ≠ 2). Therefore it is an identity.

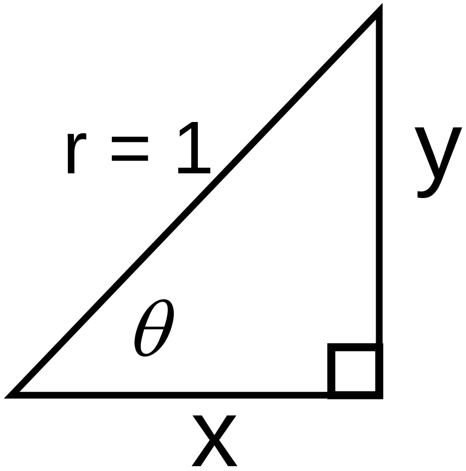
Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Quotient Identities

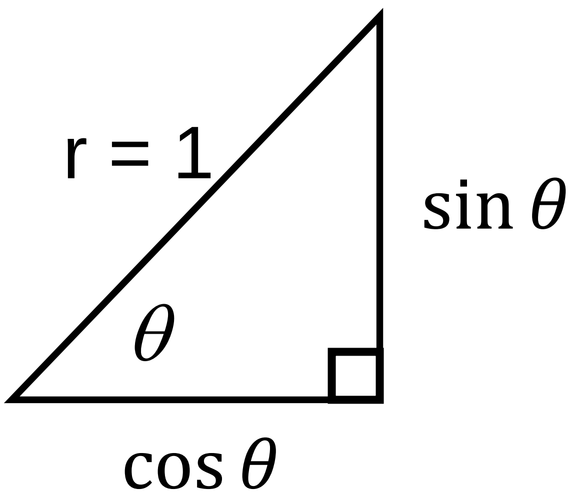
$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$



$$\sin\theta = \frac{opp}{hyp} = \frac{y}{1} = y$$

$$\cos\theta = \frac{adj}{hyp} = \frac{x}{1} = x$$

Using Substitution:



Using Pythagorean Theorem:

$$(\cos\theta)^2 + (\sin\theta)^2 = 1$$

The Pythagorean Identity

$$\boxed{\cos^2\theta + \sin^2\theta = 1}$$

Other Pythagorean Identities

$$\cos^2\theta + \sin^2\theta = 1$$

Divide both sides of the equation by $\cos^2\theta$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

This give us our 2nd "Pythagorean" identity.

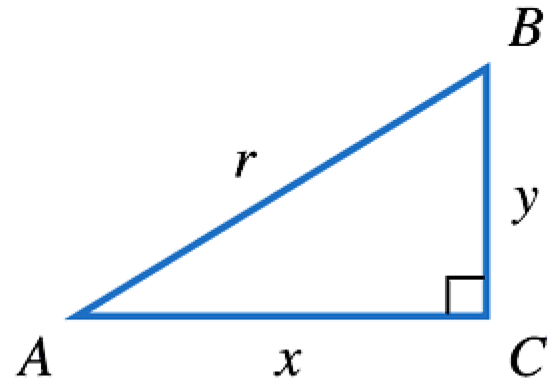
$$1 + \tan^2\theta = \sec^2\theta$$

Divide the first identity by $\sin^2\theta$ and simplify to find the 3rd Pythagorean Identity.

$$1 + \cot^2\theta = \csc^2\theta$$

Angle A: $\sin A = \dfrac{y}{r}$ $\tan A = \dfrac{y}{x}$ $\sec A = \dfrac{r}{x}$

$\cos A = \dfrac{x}{r}$ $\cot A = \dfrac{x}{y}$ $\csc A = \dfrac{r}{y}$



Angle B: $\sin B = \dfrac{x}{r}$ $\tan B = \dfrac{x}{y}$ $\sec B = \dfrac{r}{y}$

$\cos B = \dfrac{y}{r}$ $\cot B = \dfrac{y}{x}$ $\csc B = \dfrac{r}{x}$

$$\sin A = \cos B$$

$$\sin A = \cos(90 - A)$$

$$\boxed{\sin\theta = \cos(90 - \theta)}$$

function of angle A = "co-function" of angle B.

Co-function Identities

$\sin\theta = \cos(90 - \theta)$

$\cos\theta = \sin(90 - \theta)$

$\tan\theta = cot(90 - \theta)$

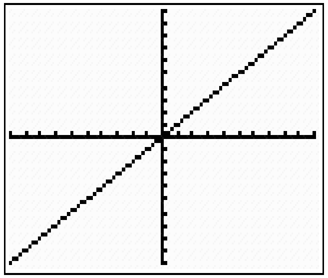
$\cot\theta = tan\,(90 - \theta)$

$sec\,\theta = \csc(90 - \theta)$

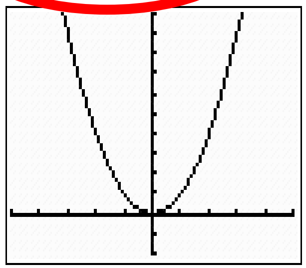
$\csc\theta = \sec(90 - \theta)$

Which functions are symmetric about the y-axis?

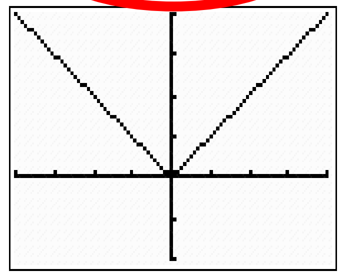
$f(x) = x$



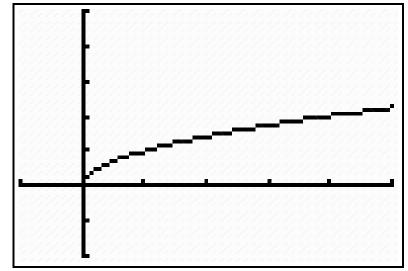
$f(x) = x^2$

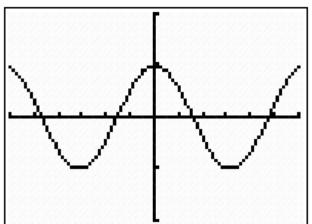


$f(x) = |x|$

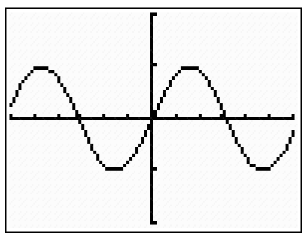


$f(x) = \sqrt{x}$

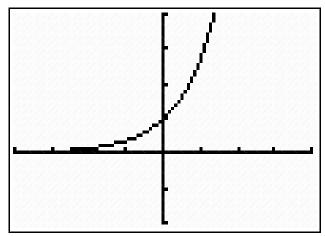




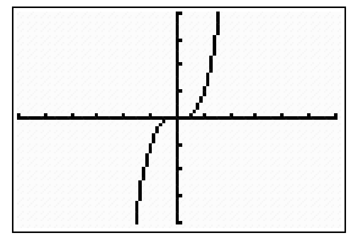
$f(x) = \cos x$



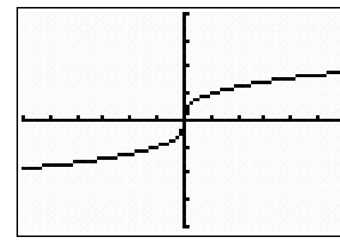
$f(x) = \sin x$



$f(x) = 2^x$



$f(x) = x^3$

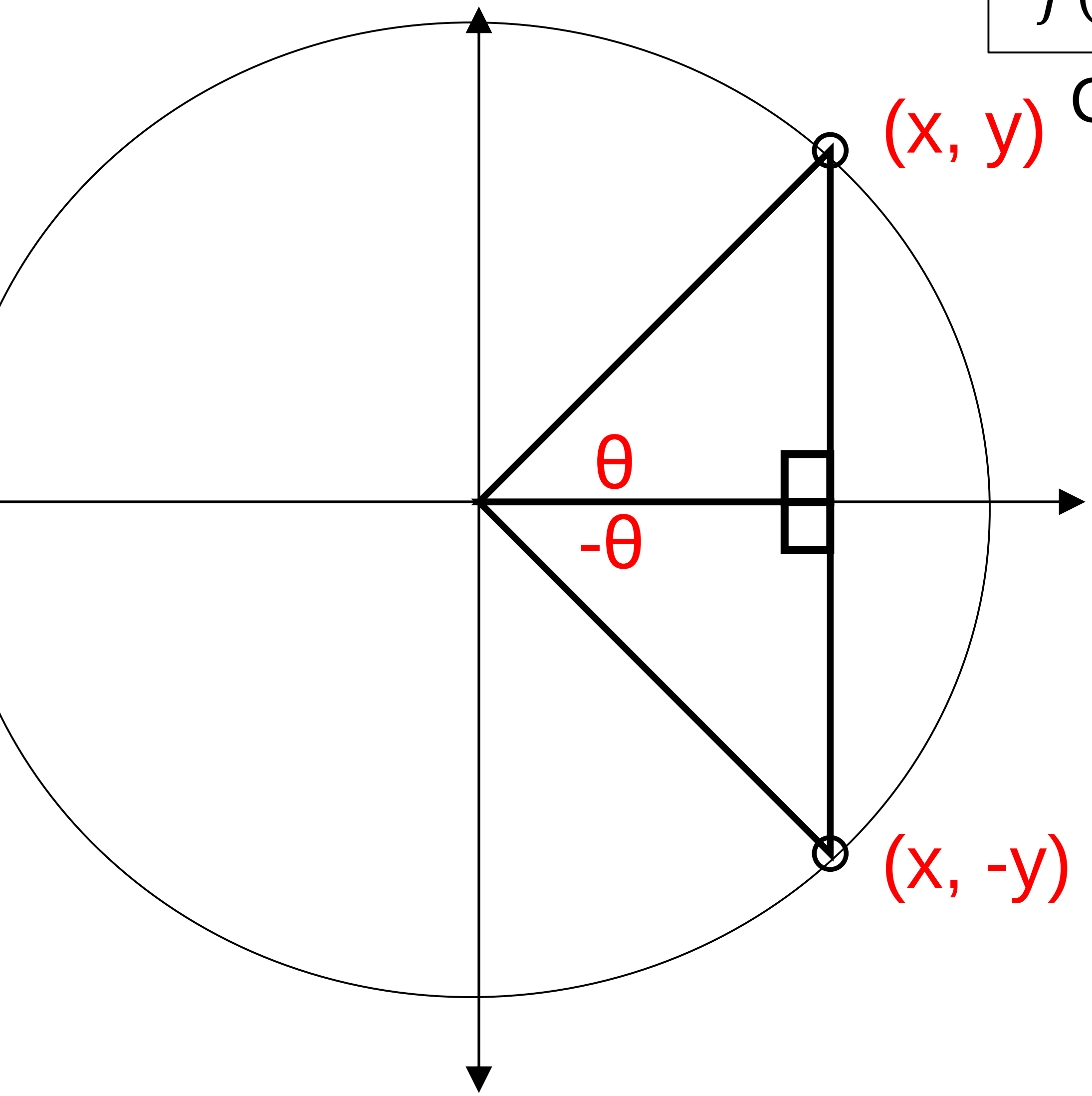


$f(x) = \sqrt[3]{x}$

We call functions that are
symmetric about the 'y'-axis,
even functions.

The mathematical definition of an *even function*.

$$\boxed{f(x) = f(-x)}$$

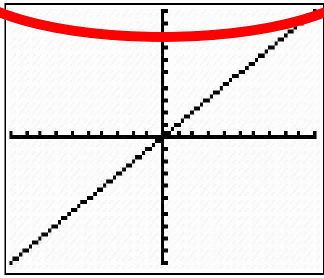


Cos θ = x coordinate of
the point on the circle.

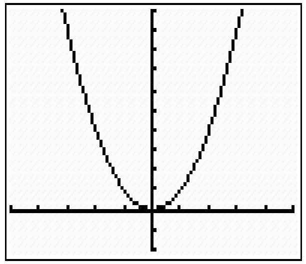
cos(θ) = cos(-θ)

Which functions are symmetrical across the origin?

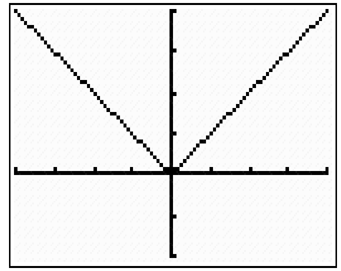
$f(x) = x$



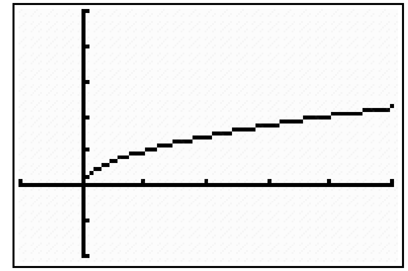
$f(x) = x^2$

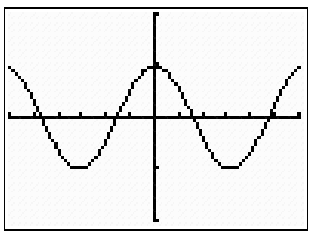


$f(x) = |x|$

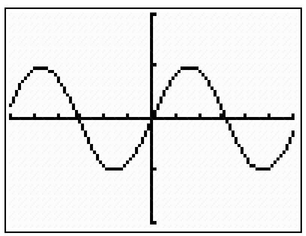


$f(x) = \sqrt{x}$

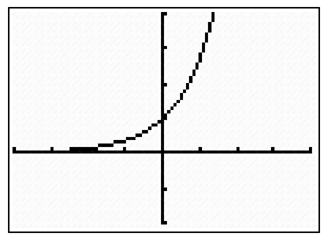




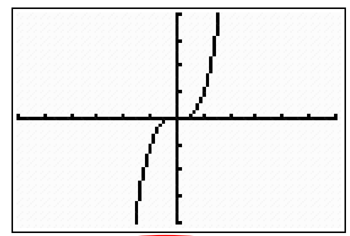
$f(x) = \cos x$



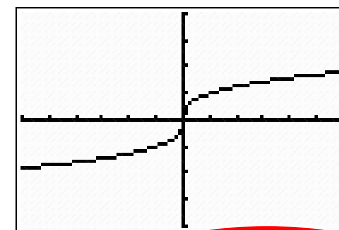
$f(x) = \sin x$



$f(x) = 2^x$



$f(x) = x^3$

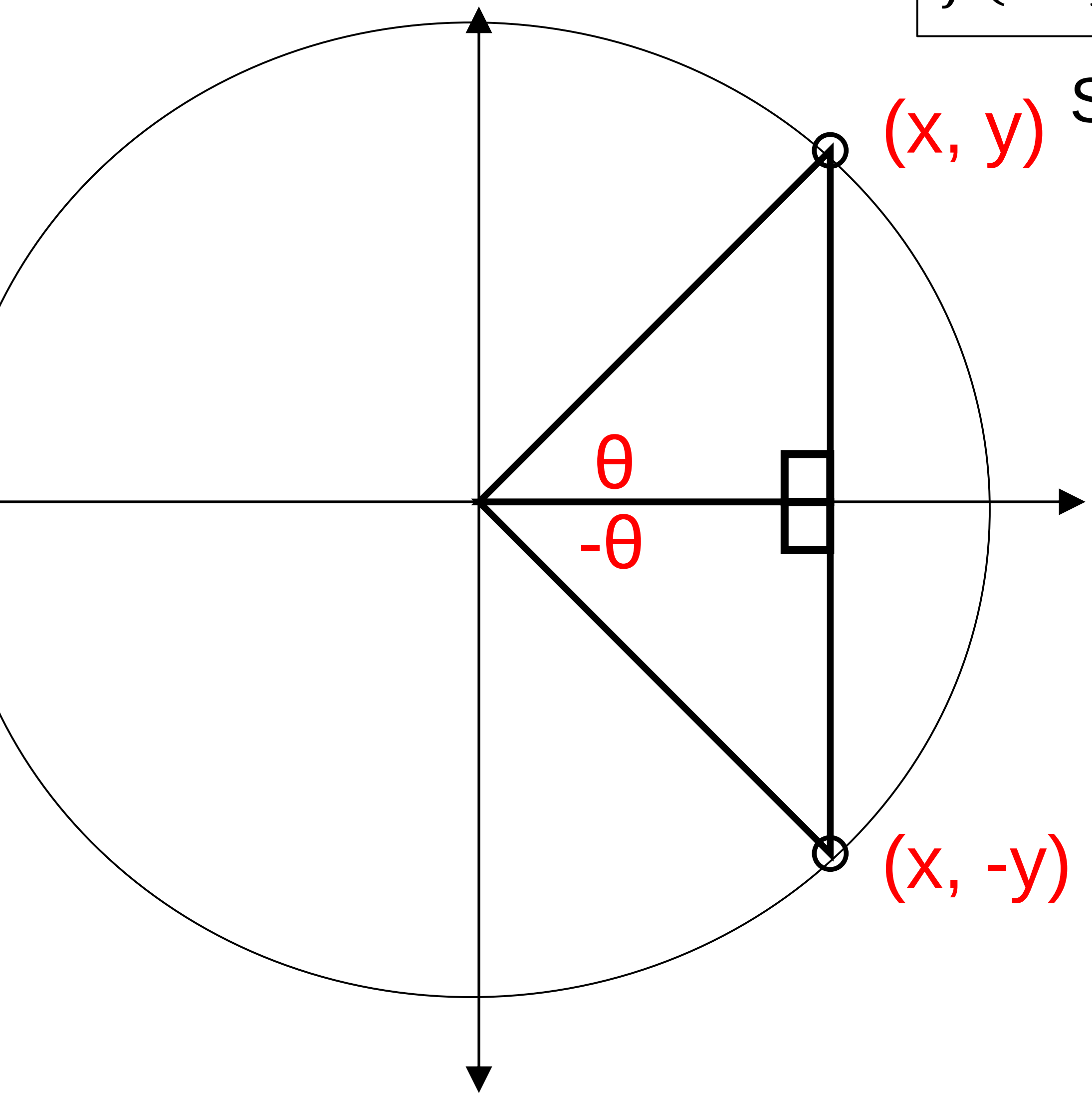


$f(x) = \sqrt[3]{x}$

We call these functions “odd” functions.

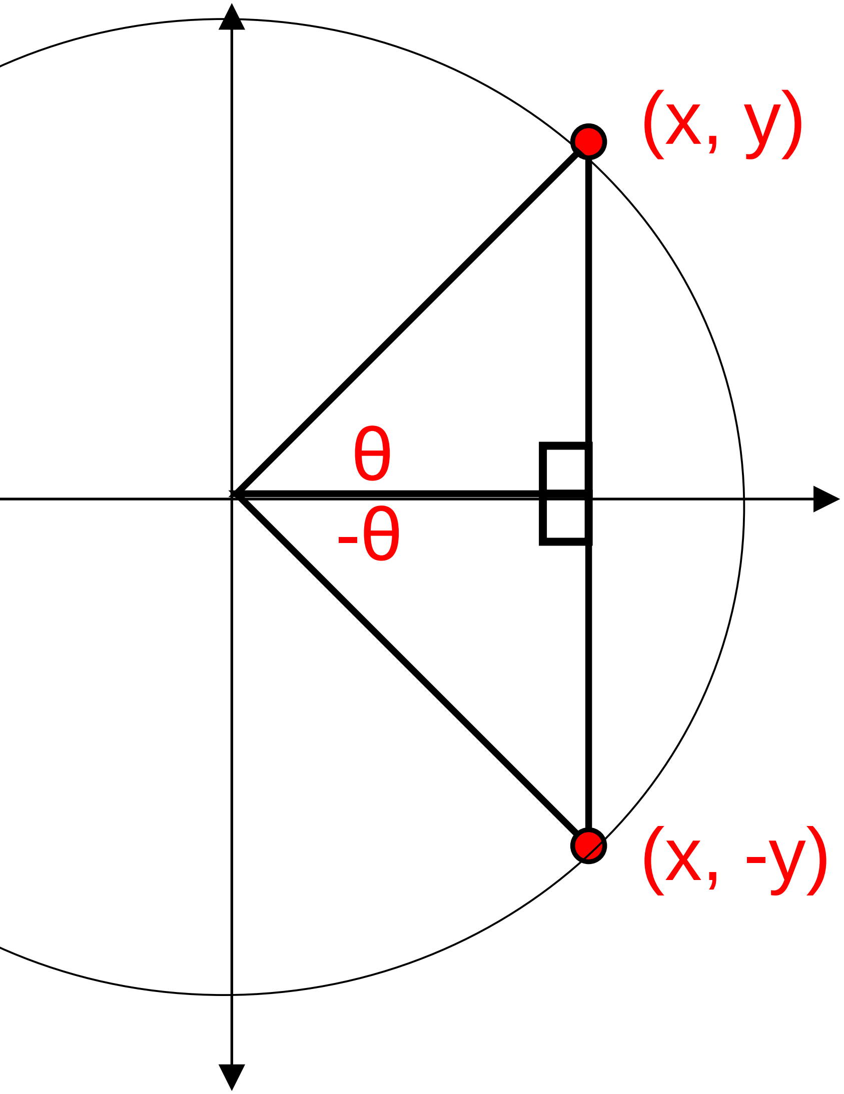
The mathematical definition of an *odd function*.

$$\boxed{f(-x) = -f(x)}$$



Sin θ = y coordinate of the point on the circle.

sin(-θ) = -sin(θ)



sin (-θ) = -sin (θ)

$$\csc\theta = \frac{1}{\sin\theta}$$

csc (-θ) = - csc (θ)

Cos (-θ) = cos (θ)

$$\sec\theta = \frac{1}{\cos\theta}$$

sec (-θ) = sec (θ)

Sin (-θ) = -sin (θ)

Cos (-θ) = cos (θ)

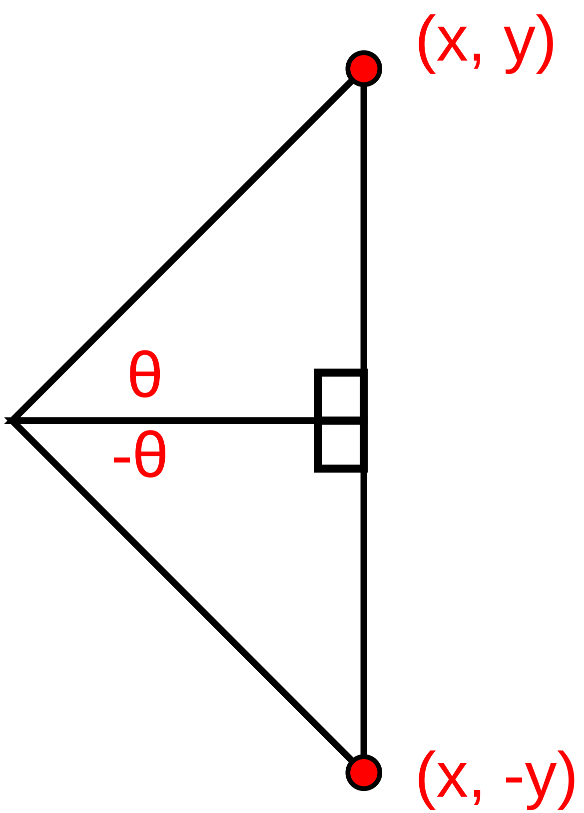
$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\tan(-\theta) = \frac{\sin(-\theta)}{\cos(-\theta)}$$

$$\tan(-\theta) = \frac{-\sin(\theta)}{\cos(\theta)}$$

tan (-θ) = - tan (θ)

cot (-θ) = - cot (θ)

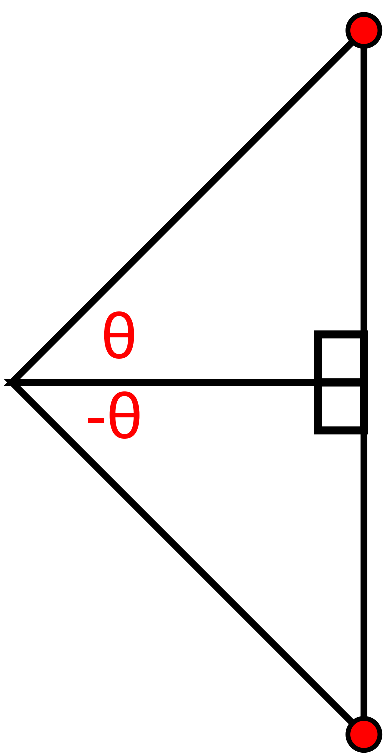


Even-Odd Identities

Even: Cos (-θ) = cos (θ)   sec (-θ) = sec (θ)

Odd: sin (-θ) = -sin (θ)   csc (-θ) = - csc (θ)

tan (-θ) = - tan (θ)   cot (-θ) = - cot (θ)



Find:

$$\sin(-\theta)\sec(-\theta) = -\sin(\theta)\sec(\theta)$$

$$= -\sin(\theta)\frac{1}{\cos(\theta)} = \frac{-\sin(\theta)}{\cos(\theta)}$$

$$= -\tan\theta$$

Simplifying by Using Properties and Trigonometric Identities

Simplify: $$\cot x \sin^3 x \sec x + \tan x \cos^3 x \csc x$$

Try converting tan, cot, sec, csc into functions of sin and cos

Use properties of exponents:

$$\frac{\cos}{\sin x}\sin^3 x \frac{1}{\cos x} + \frac{\sin x}{\cos x}\cos^3 x \frac{1}{\sin x}$$

Use the inverse Property of multiplication:

$$\cancel{\cos x}\sin^2 x \frac{1}{\cancel{\cos x}} + \cancel{\sin x}\cos^2 x \frac{1}{\cancel{\sin x}}$$

Use the Pythagorean Identity

$$\sin^2 x + \cos^2 x$$

$$\boxed{=1}$$

Simplifying by Expanding and Using Identities

Simplify: $\dfrac{(\csc x - 1)(\csc x + 1)}{\cos^2 x}$ These are conjugate pairs!

$$\frac{\csc^2 x - 1}{\cos^2 x}$$

It looks like the factorization of a "difference of 2 squares".

For example:

$$(x+2)(x-2) = x^2 - 4$$

Looks similar to a Pythagorean identity:

$$1 + \cot^2\theta = \csc^2\theta \quad \rightarrow \quad \cot^2\theta = \csc^2\theta - 1$$

Use Substitution: Convert to Sines/Cosines:

$$\frac{\cot^2 x}{\cos^2 x} = \frac{\cos^2 x}{\sin^2 x} * \frac{1}{\cos^2 x} = \frac{1}{\sin^2 x} \boxed{= \csc^2 x}$$

Simplifying by Factoring and Using Identities

Simplify: $$\sin^3 x + \sin x \cos^2 x$$

$$= \sin x(\sin^2 x + \cos^2 x)$$

Use Substitution:

Pythagorean Identity: $\sin^2 x + \cos^2 x = 1$ $\boxed{= \sin x}$

Using these guidelines may help to "simplify" simplification:

1. Try factoring
2. Look for things that look like: $1 + (???)^2 x$ or $(???)^2 x$
3. Convert tan, cot, sec, csc into functions of sin and cos
4. Combine fractions (rational expressions) if you can.

Simplify:

$$\tan x \cos x \cot x \csc x$$

$$\sin x(\tan x + \cot x)$$

Identity: an equation that is true for all values that are in the domain of both sides of the equation.

Is it an Identity? If not, why not?

$$4x + 2 = 6x + 4$$

No. The domain of validity is "all real numbers" but the equation is true only when x = 1.

$$\frac{2x^2 - 4x}{x - 2} = 2x$$

When we factor the left side of the equation, we have
$$\frac{2x(x-2)}{x-2} = 2x$$

Which simplifies to $2x = 2x$

Which is true for all real numbers.

AND it is true for all values in domain of validity (x ≠ 2). Therefore it is an identity.

Verifying Trigonometric Identities

We are *not* trying to solve the equation. We are trying to verify that the left side of the equation is equivalent to the right side.

$$\frac{2x^2 - 4x}{x - 2} = 2x$$

We work on one side of the equal sign to simplify the expression or to use substitution to try to make it equal to the other side.

$$\frac{2x^2 - 4x}{x - 2} = 2x$$

Usually, we pick the side to work on that is the most complicated expression.

$$\frac{2x\cancel{(x-2)}}{\cancel{x-2}} = 2x$$

$$2x = 2x$$

This is an identity

Verify the Identity

$$\frac{2-\cos^2 x}{\sin x} = \csc x + \sin x$$

$\cos^2 x$

Try Pythagorean

$$\cos^2\theta + \sin^2\theta = 1$$

$$\sin^2\theta = 1-\cos^2\theta$$

$$\frac{1+1-\cos^2 x}{\sin x} = \csc x + \sin x$$

$$\frac{1+\sin^2 x}{\sin x} = \csc x + \sin x$$

Substitution

$$\frac{1}{\sin x} + \frac{\sin^2 x}{\sin x} = \csc x + \sin x$$

Two terms on the right; split the left into two terms.

$$\csc x + \sin x = \csc x + \sin x$$

Simplify.

Yes: it's an identity