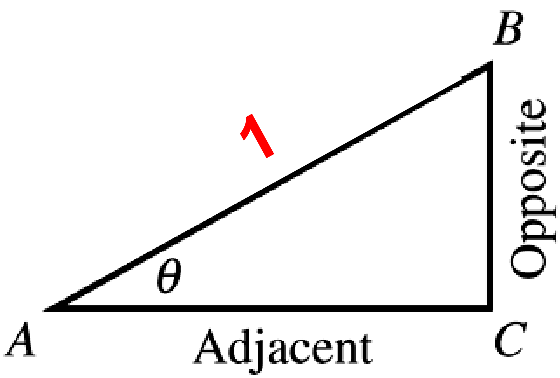
Math-1060
Lesson 4-3

Exact Trigonometric Ratios

hypotenuse = 1

Why is it “nice” to have a hypotenuse whose length is ‘1’?



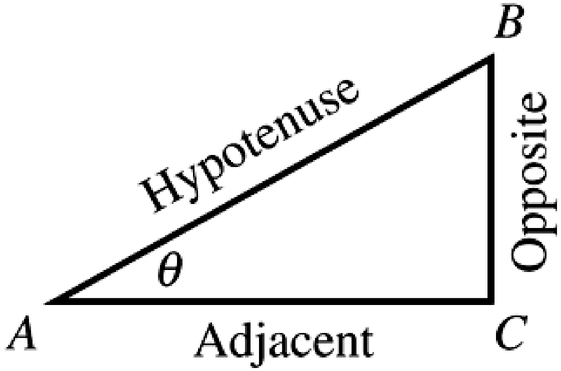
Sin Ɵ= opposite side

Cos Ɵ= adjacent side

Tan Ɵ= opp/adj

The length of the hypotenuse is no longer in the ratio!

Trigonometric Functions



Shot your cow: "Sha – Cho – Cao"

$$\sec A = \frac{h}{a} \qquad \csc A = \frac{h}{o} \qquad \cot A = \frac{a}{o}$$

$$\sin \rightarrow \sec$$

$$\cos \rightarrow \csc$$

The Sine, Cosine, and Tangent ratios are defined based upon ratios of side of a right triangle.

What happens if the angle is greater than 90 (triangles don't have angles this big)?

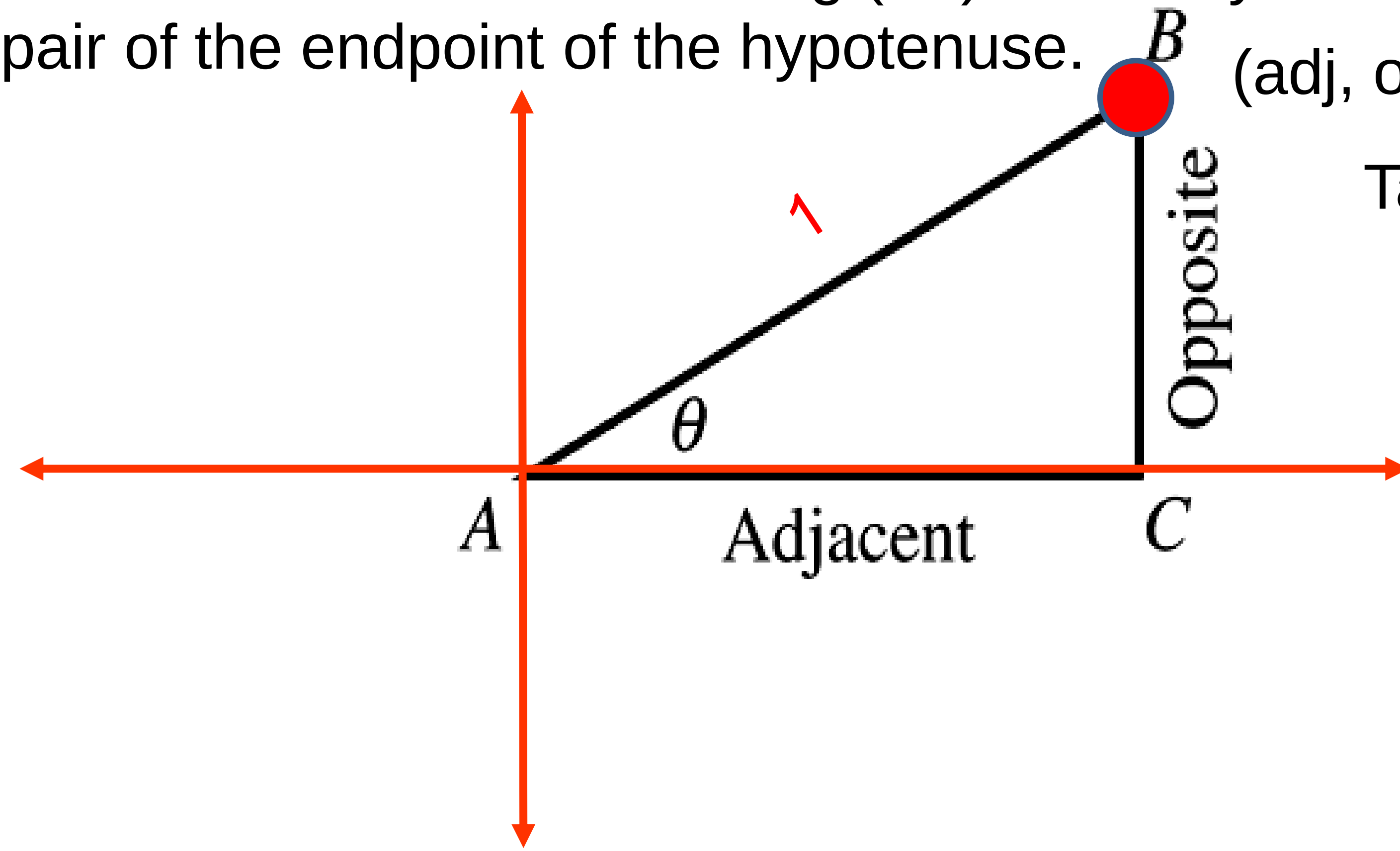
We still use a right triangle, but the trig ratios must now account for the sing (+/-) of the x-y pair of the endpoint of the hypotenuse.

Cos ϴ = x

Sin ϴ = y

(adj, opp) = (x, y)

Tan ϴ = y/x



Reference angle: The acute angle between the terminal side of the angle and the x-axis.

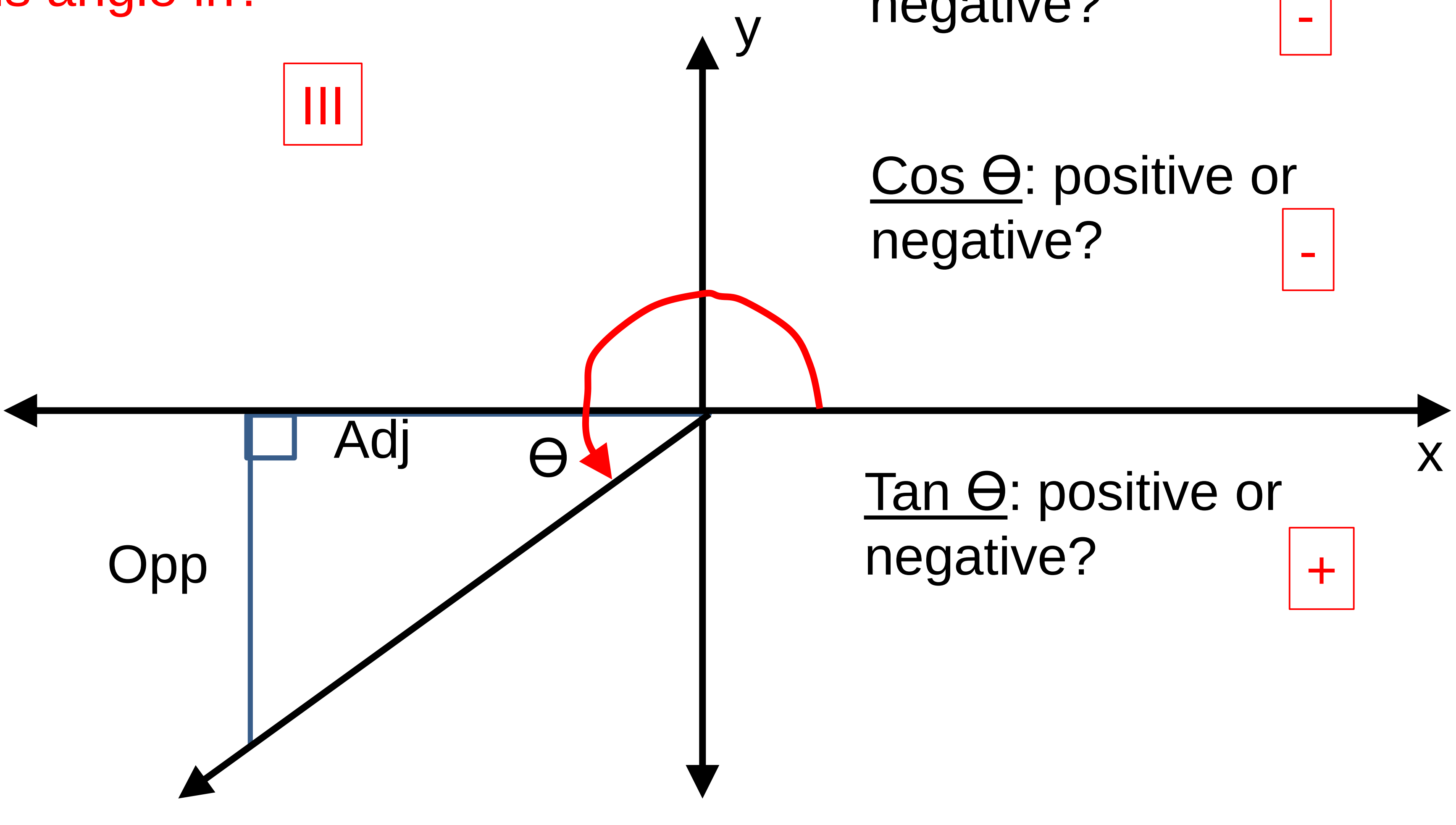
What quadrant of the x-y plane is this angle in?

III

Sin Θ: positive or negative? -

Cos Θ: positive or negative? -

Tan Θ: positive or negative? +



Reference angle: The acute angle between the terminal side of the angle and the x-axis.

What quadrant of the x-y plane is this angle in?

IV

Sin Θ: positive or negative?

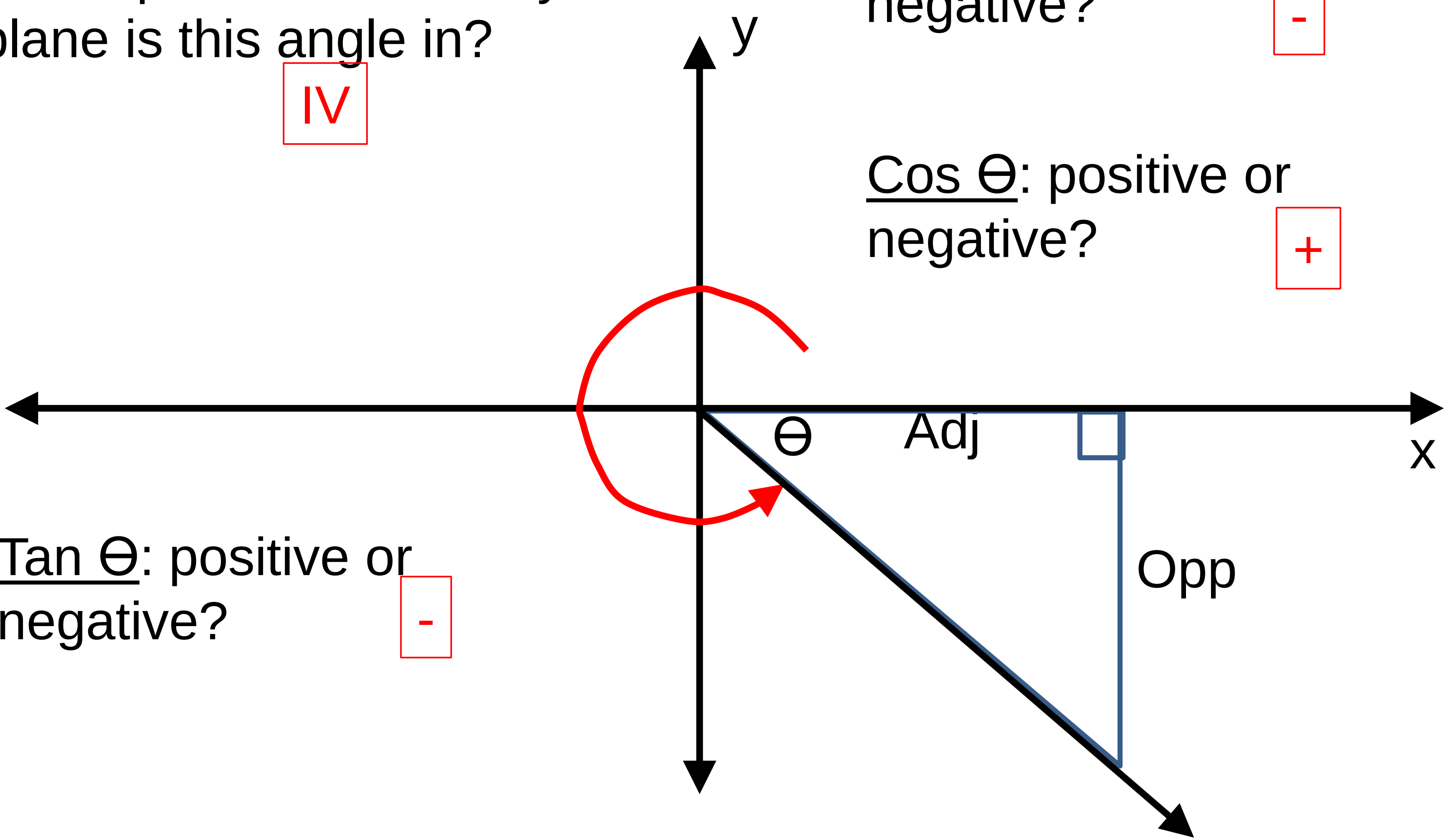
-

Cos Θ: positive or negative?

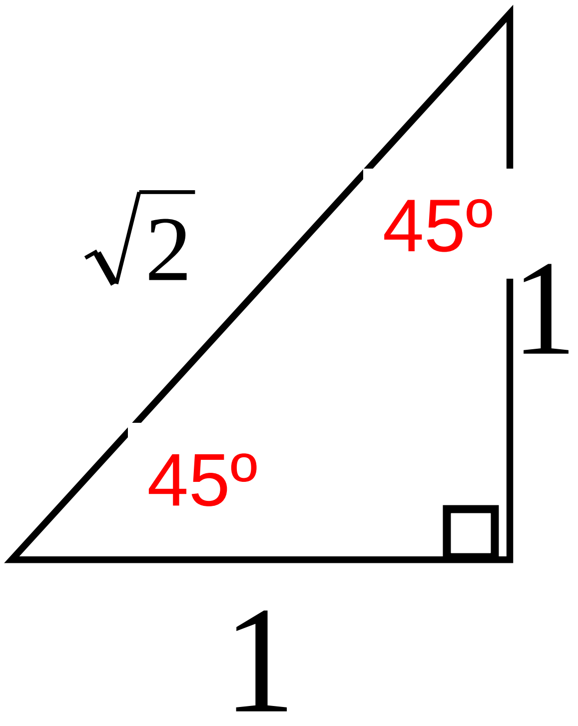
+

Tan Θ: positive or negative?

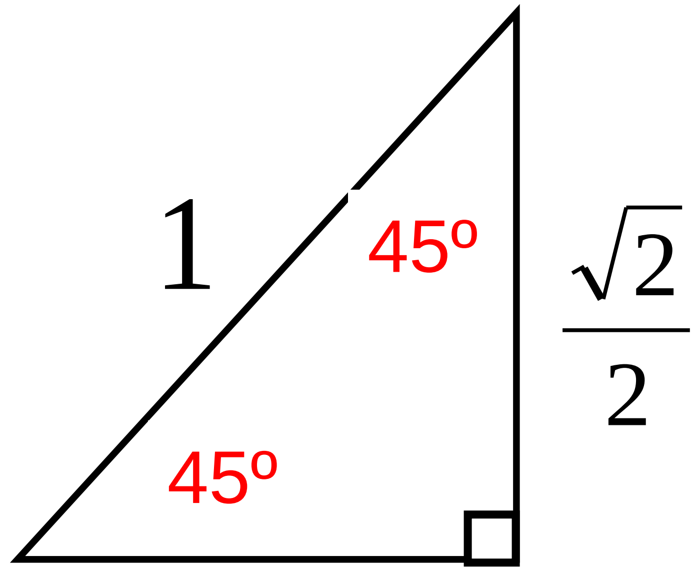
-



Do you remember the side lengths for a 45-45-90 triangle?

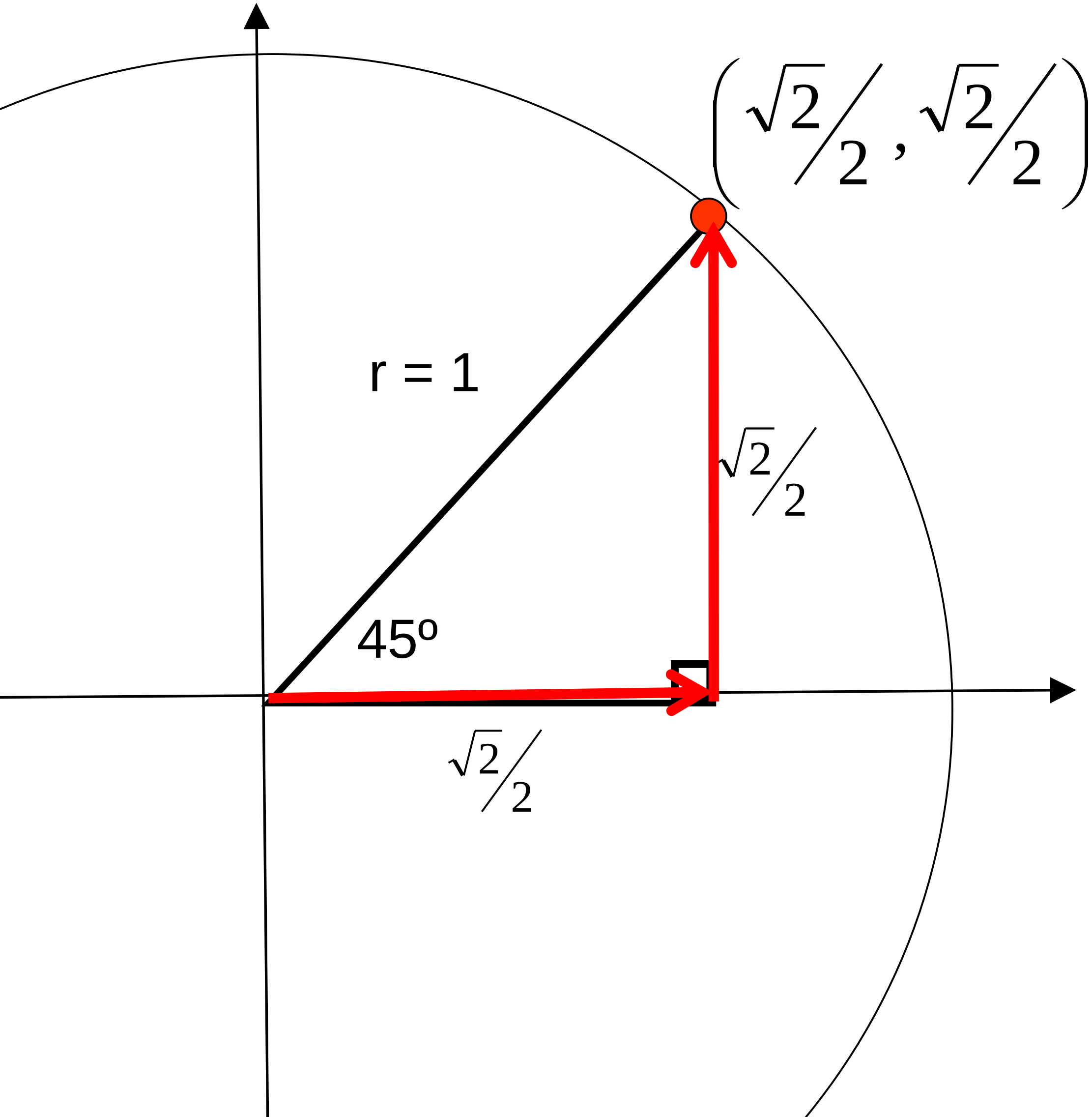


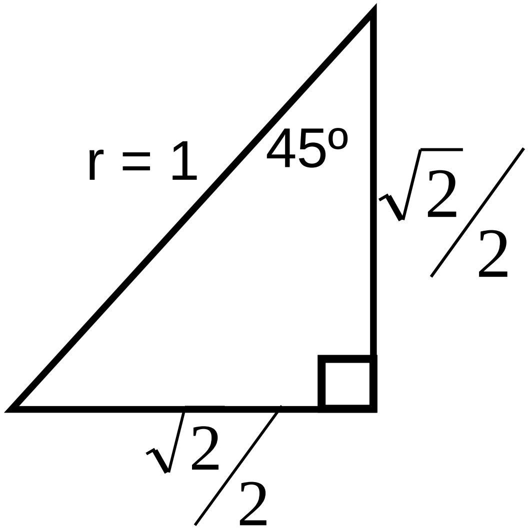
What are the leg lengths if the hypotenuse = 1?



$$\frac{1}{\sqrt{2}} * \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

Let’s put the triangle on top of a circle with radius = 1.



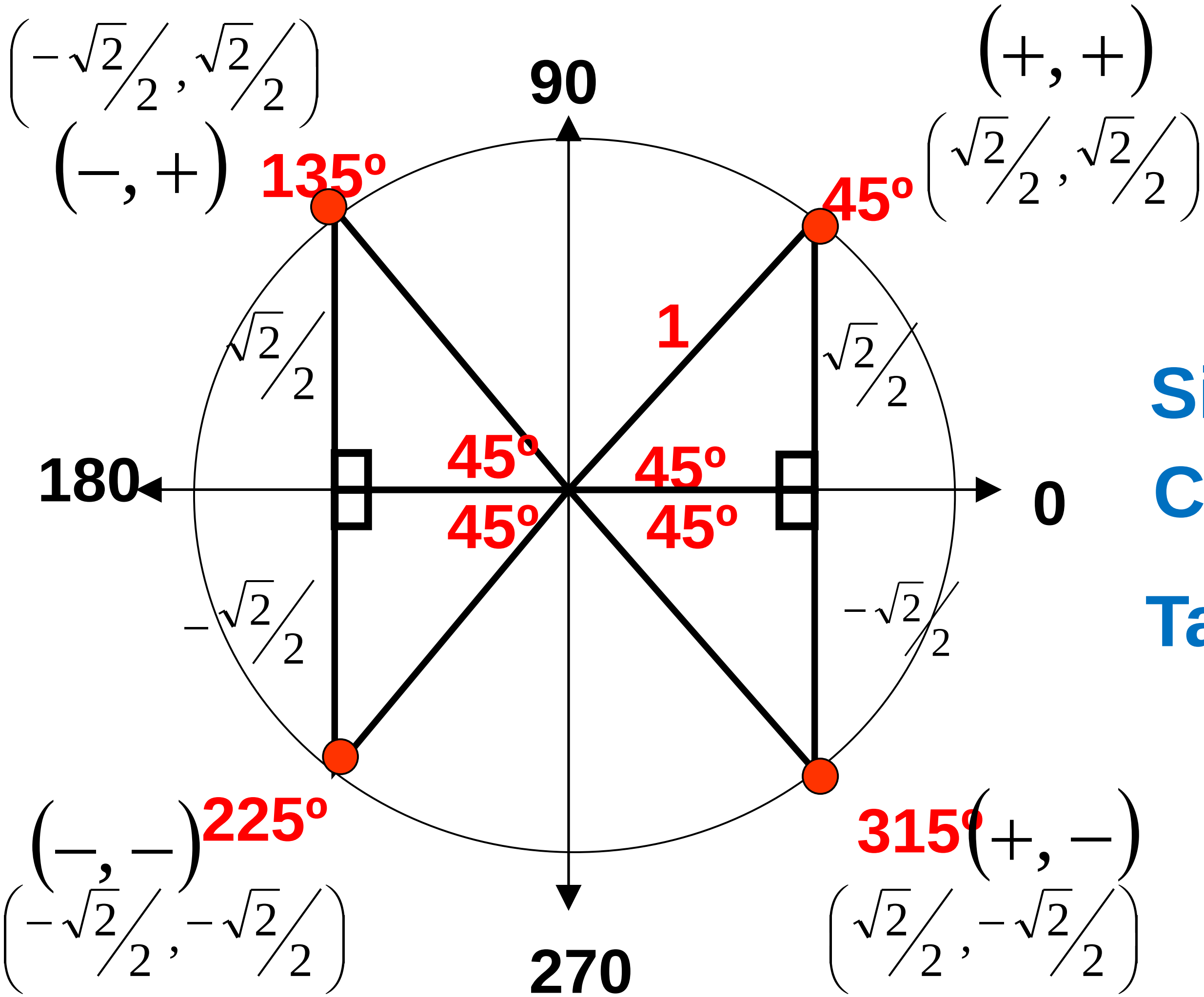


Sin ϴ = y

Cos ϴ = x

Tan ϴ = y/x

We can use a 45° reference angle 4 times

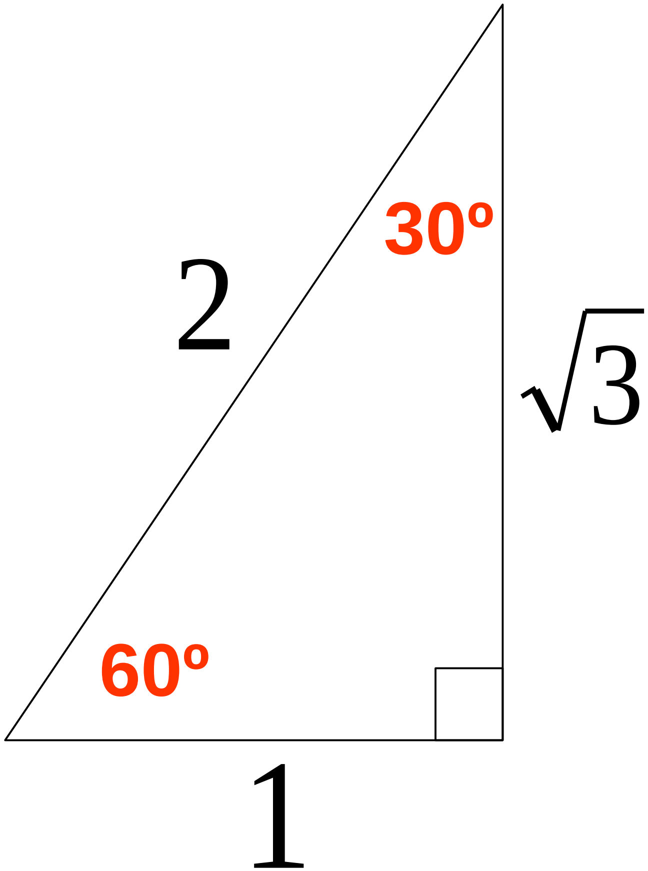


Sin ϴ = y

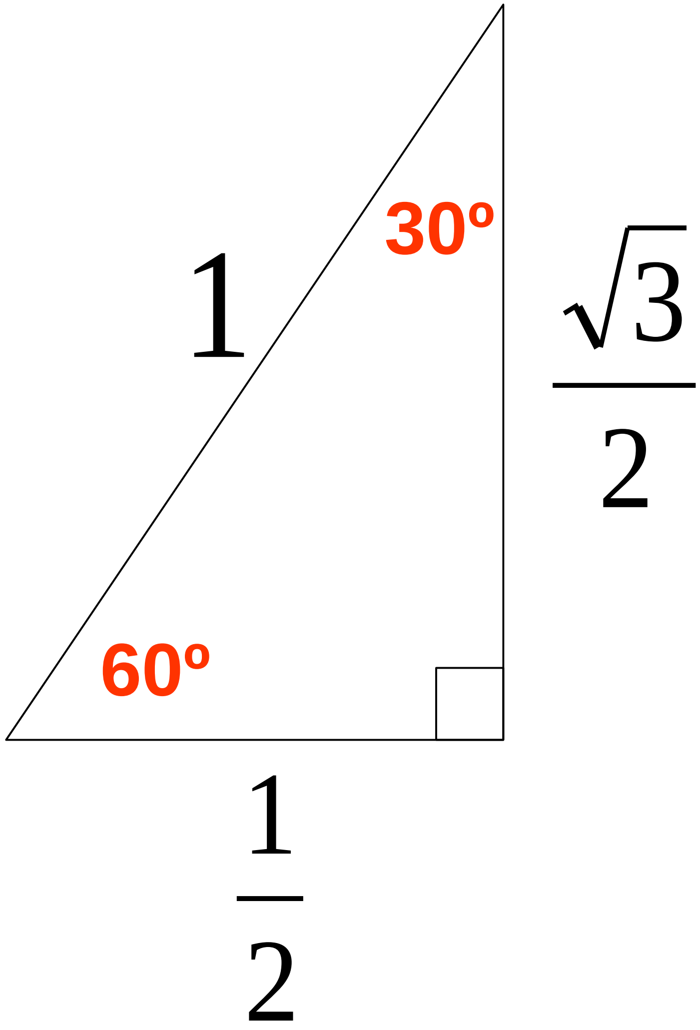
Cos ϴ = x

Tan ϴ = y/x

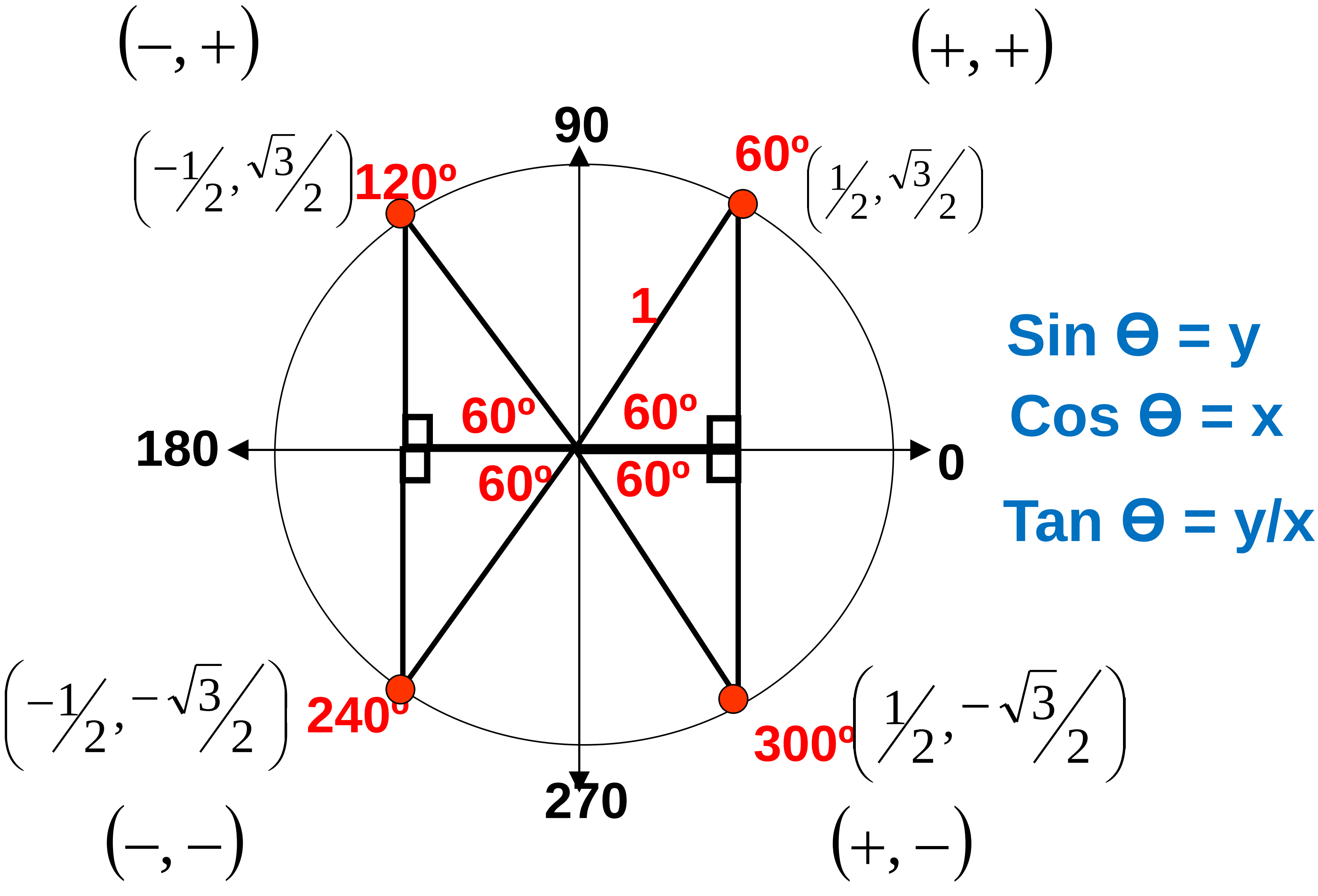
Do you remember the side lengths for the 30-60-90 triangle?



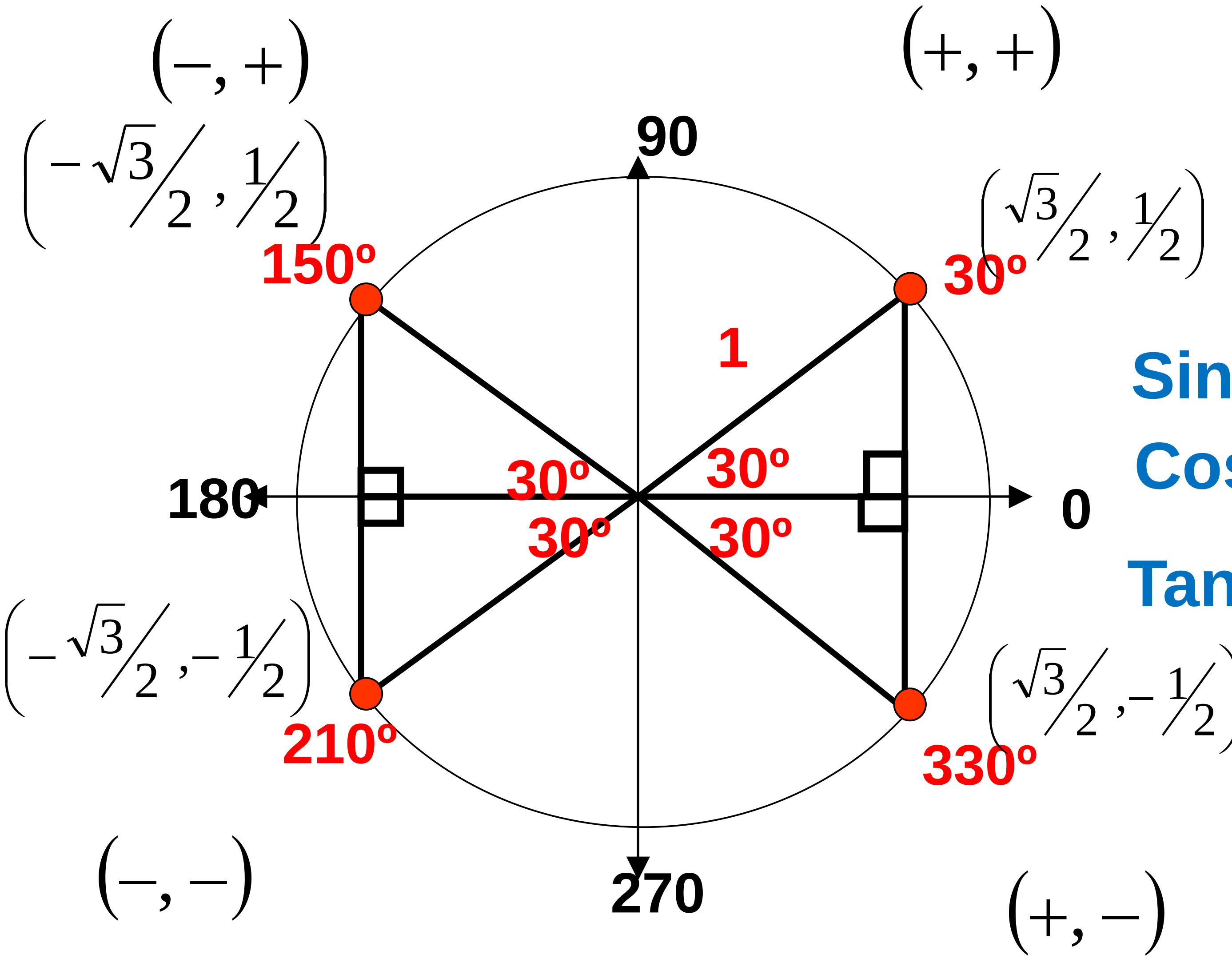
What are the leg lengths if the hypotenuse = 1?



We can use a 60° reference angle 4 times



We can use a 30° reference angle 4 times



Sin ϴ = y

Cos ϴ = x

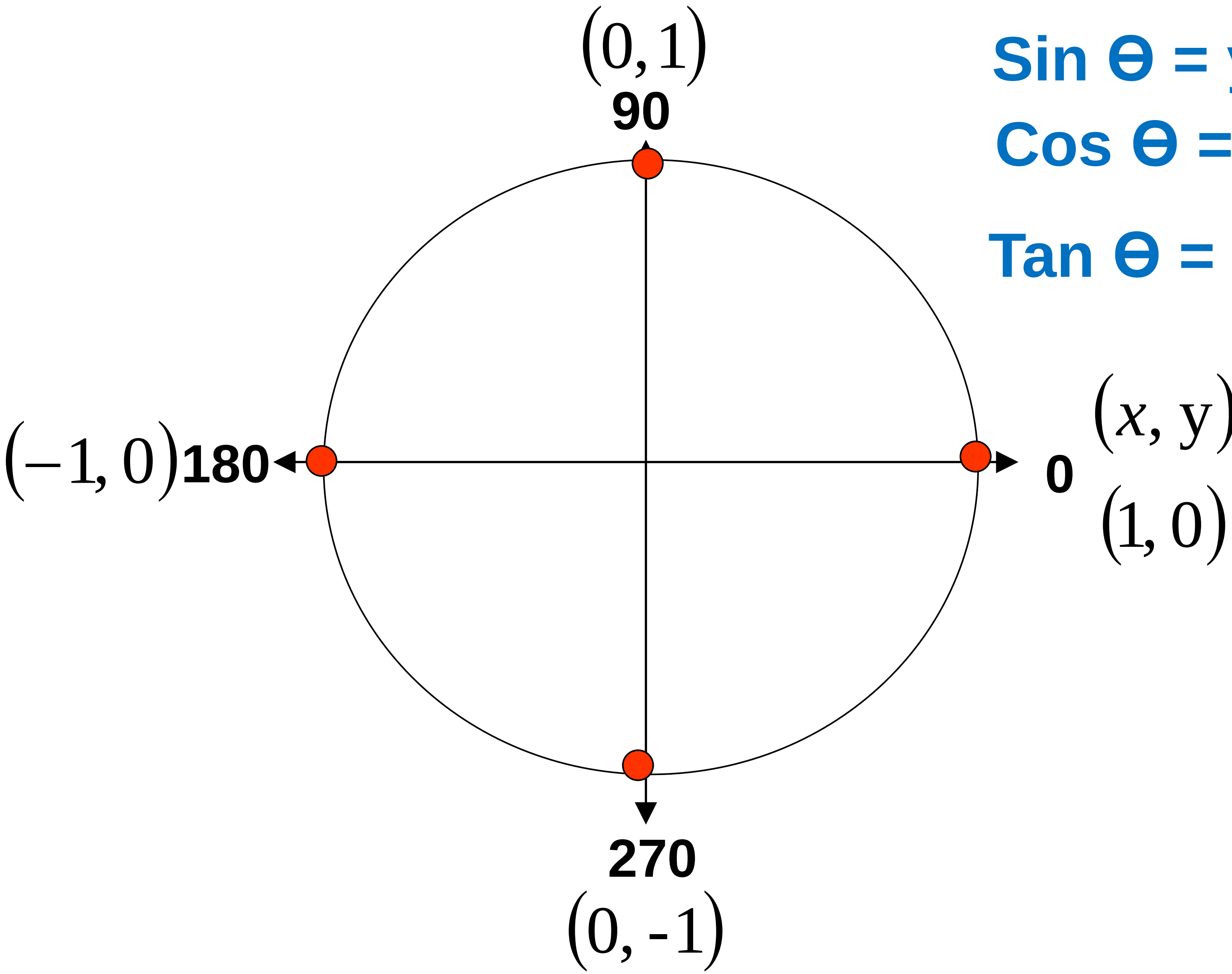
Tan ϴ = y/x

What about the "cardinal angles"?

Sin ϴ = y

Cos ϴ = x

Tan ϴ = y/x



We know the exact ratios for the following angles.

Angle	Sine	Cosine	Tangent
0	0	1	0
90	1	0	$undef$
180	0	-1	0
270	-1	0	$undef$

Tan 0° $= {}^{y}/_{x} \quad = \dfrac{0}{1} \quad = 0$

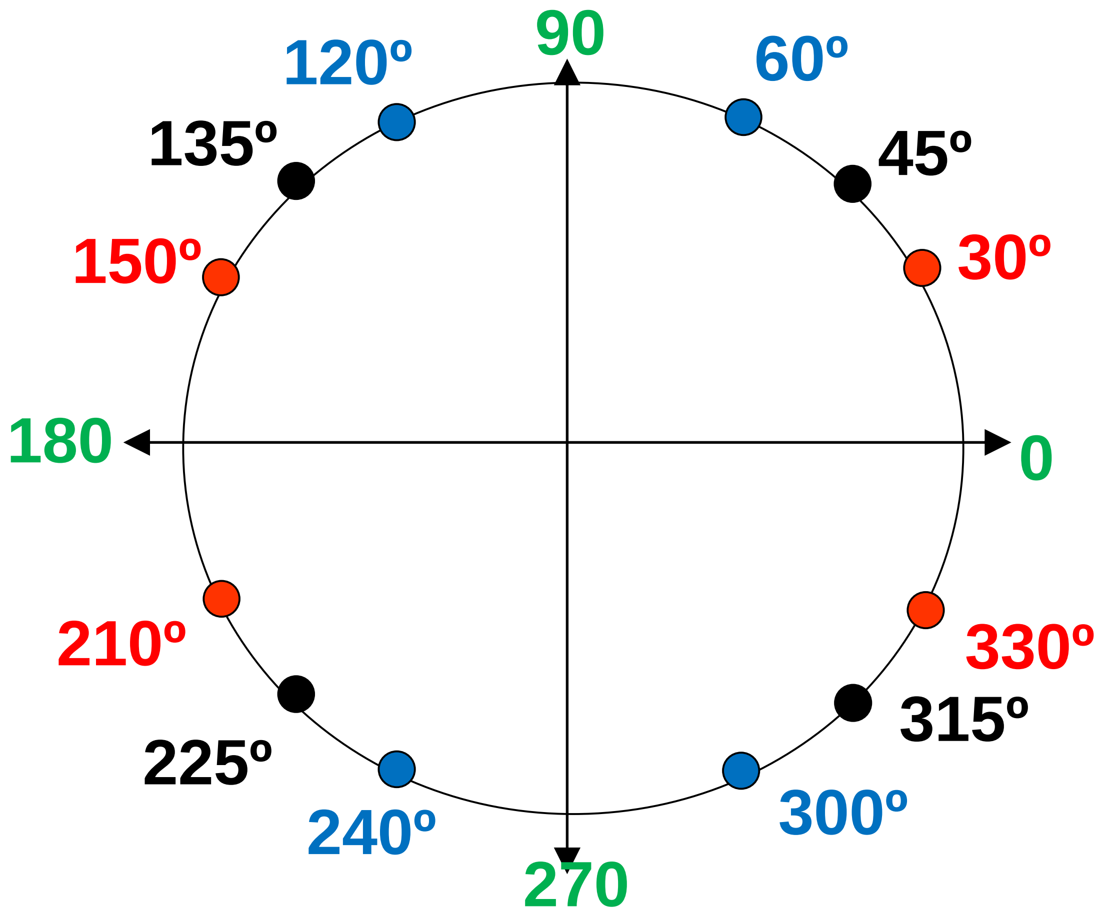
Tan 90° $= {}^{y}/_{x} \quad = \dfrac{1}{0} \quad = undefined$

The tangent function does **NOT** have a domain of "all real numbers".

Can you quickly come up with the exact ratio?

Green colored angles use a reference angle of 90.

Red colored angles use a reference angle of 30.



Sin ϴ = y

Cos ϴ = x

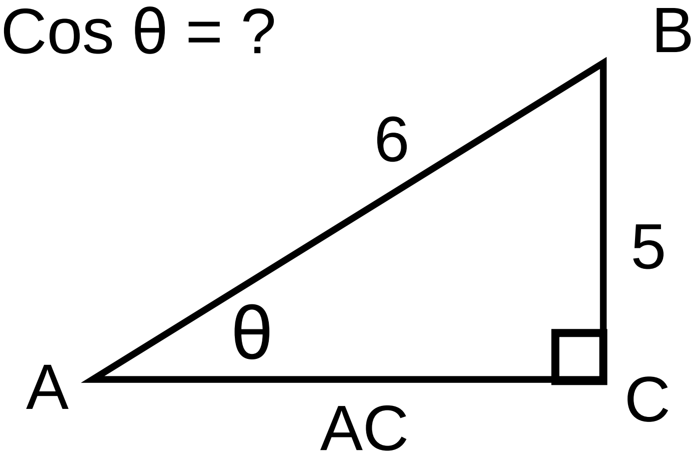
Tan ϴ = y/x

Black colored angles use a reference angle of 45.

Blue colored angles use a reference angle of 60.

What happens if the hypotenuse does not equal 1?

Cos θ = ?



$$cos\theta = \frac{AC}{6}$$

AC = ?

$$AC^2 + 5^2 = 6^2$$

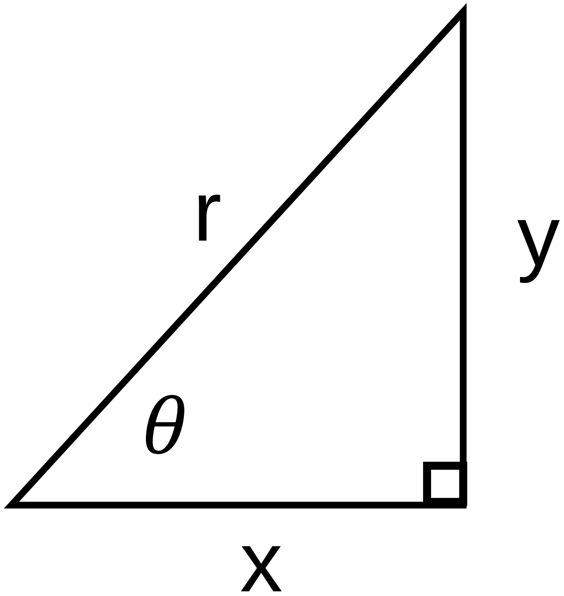
$$AC^2 = 36 - 25$$

$$AC = \sqrt{11}$$

$$\cos\theta = \sqrt{11}/6$$

For trig problems on the x-y plane:

We must use the complete ratio (opp/hyp, adj/hyp)



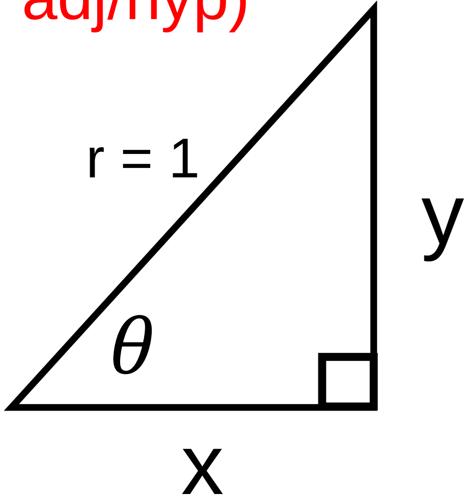
$$r^2 = x^2 + y^2$$

$$r = \sqrt{x^2 + y^2}$$

$$\boxed{sin\theta = \frac{y}{\sqrt{x^2 + y^2}}}$$

$$\boxed{cos\theta = \frac{x}{\sqrt{x^2 + y^2}}}$$

$$\boxed{tan\theta = \frac{y}{x}}$$



Sin Θ = y

Cos Θ = x

Tan Θ = y/x

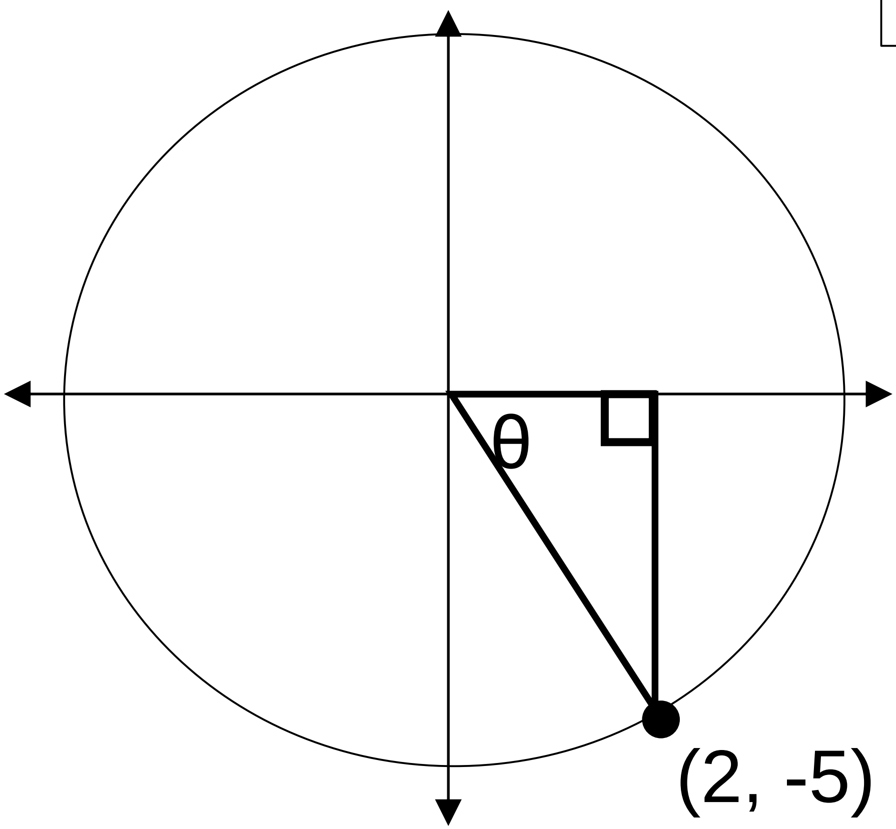
A typical x-y plane trig. problem:

What is the sine ratio of an angle whose terminal side passes through the point (2, -5) on the xy-plane?

1. Build a right triangle with the reference angle being “theta” (the angle whose terminal side passes through (2, -5)

2. Use the sine equation:

$$\boxed{sin\theta = \frac{y}{\sqrt{x^2 + y^2}}}$$



$$sin\theta = \frac{-5}{\sqrt{(2)^2 + (-5)^2}}$$

$$sin\theta = \frac{-5}{\sqrt{29}} * \frac{\sqrt{29}}{\sqrt{29}}$$

3. Rationalize the denominator.

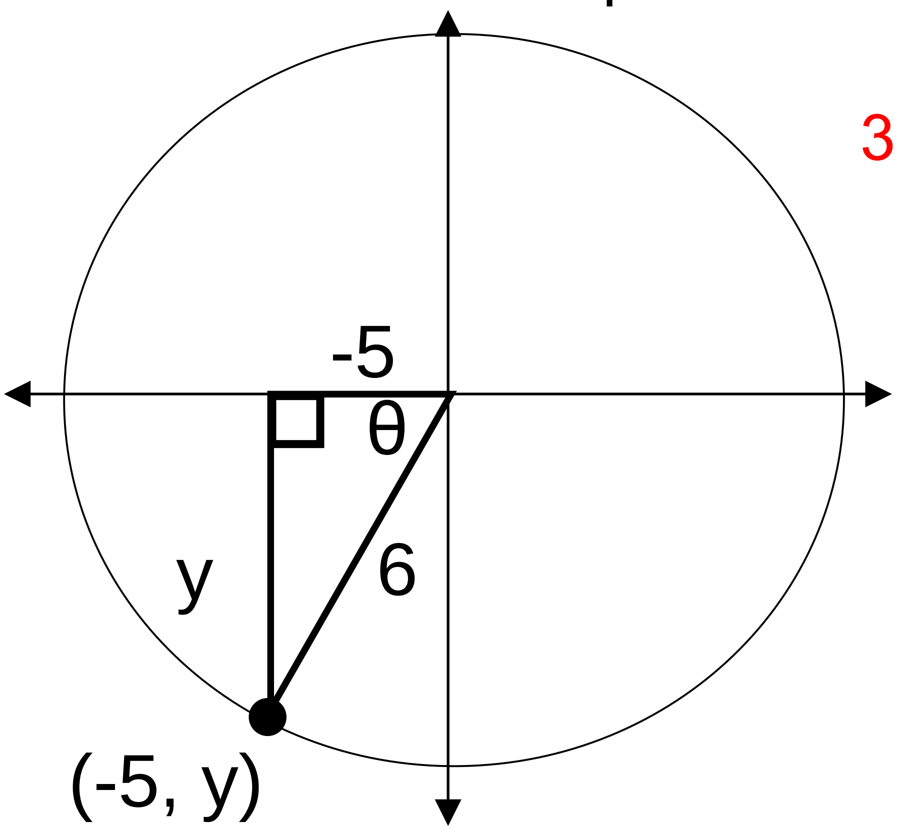
$$\boxed{sin\theta = \frac{-5\sqrt{29}}{29}}$$

Another typical x-y plane trig. problem:

If the cosine ratio is 5/6, and the terminal side of the angle is in quadrant III, what is the sine ratio of the angle?

1. Build a right triangle with the reference angle being “theta” (the angle whose terminal side passes through (-5, y) that has a hypotenuse of 6.

2. Use the sine equation: $\boxed{sin\theta = \frac{y}{r}}$ $\quad sin\theta = \frac{y}{6}$



3. Solve for ‘y’.

$$r^2 = x^2 + y^2$$

$$36 = 25 + y^2$$

$$y = \sqrt{11}$$

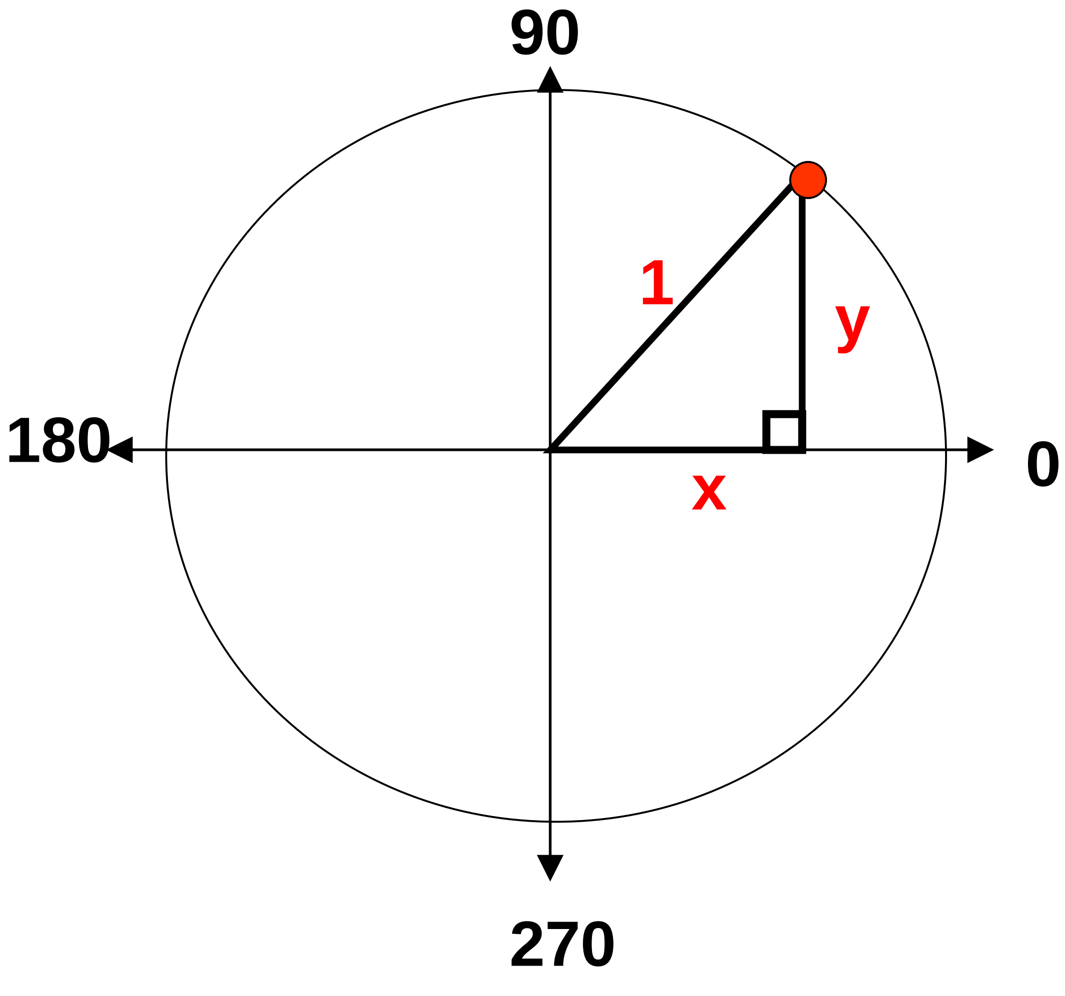
4. y-values are negative in Q-III

$$\boxed{sin\theta = \frac{-\sqrt{11}}{6}}$$

Ranges of the Trigonometric Functions

Sin ϴ = y

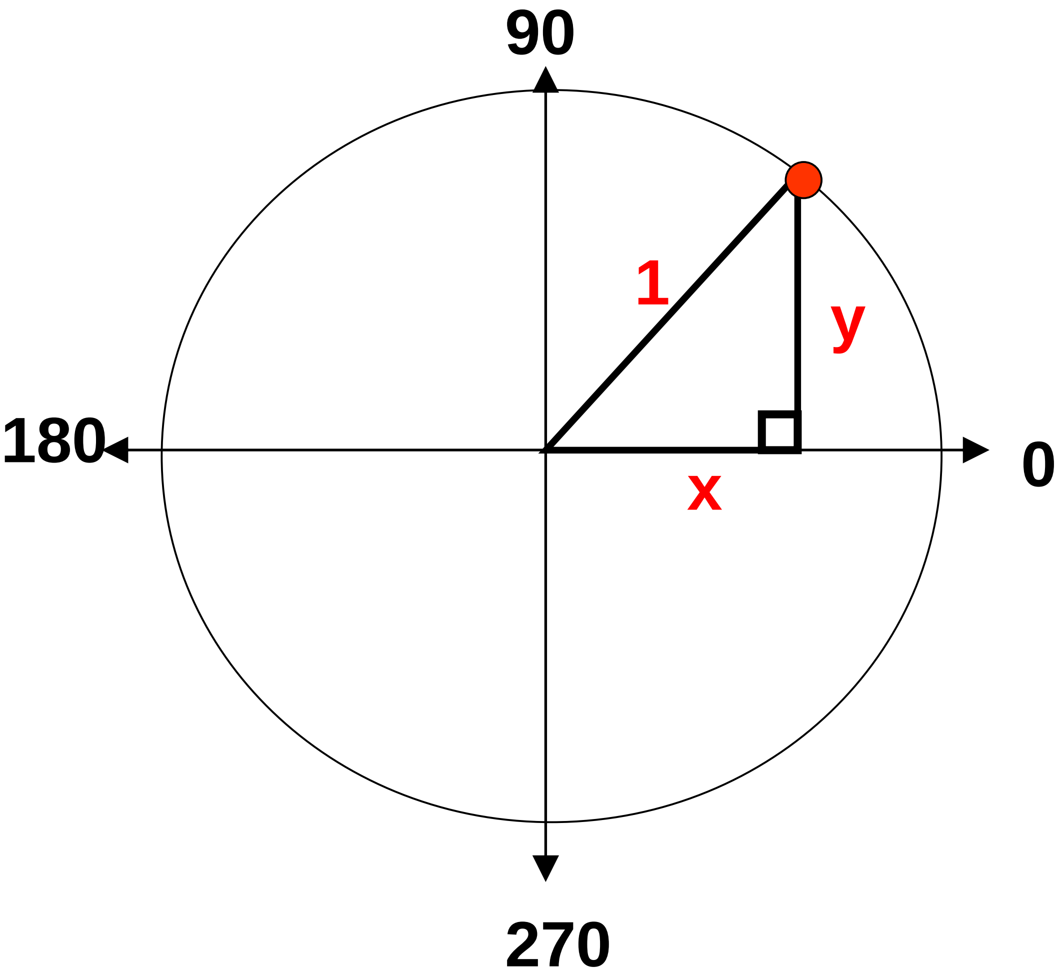
The sine of an angle is the y-value of the point as it rotates around the unit circle. **-1 < Sin ϴ < 1**



Ranges of the Trigonometric Functions

$$\cos \theta = x$$

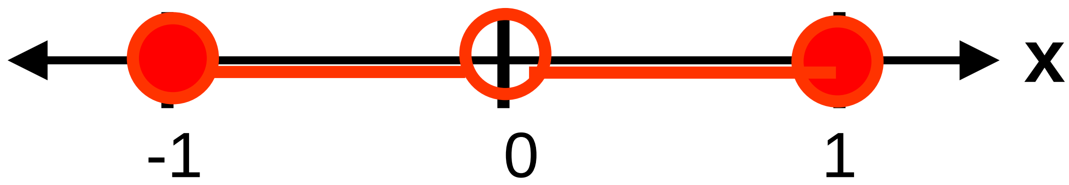
The cosine of an angle is the x-value of the point as it rotates around the unit circle. $-1 < \cos \theta < 1$



Ranges of the Trigonometric Functions

sec Ɵ = 1/x

The secant of an angle is the reciprocal of the cosine ratio. Therefore x cannot equal zero.

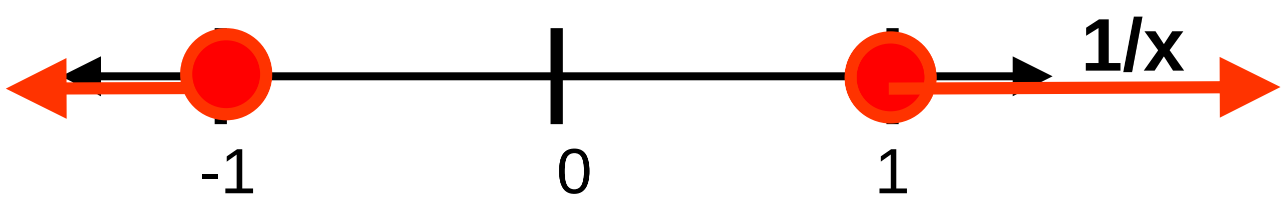


$$-1 \le x \le 1$$

And

$$x \neq 0$$

The reciprocal of a number between 0 and 1 is greater than 1.



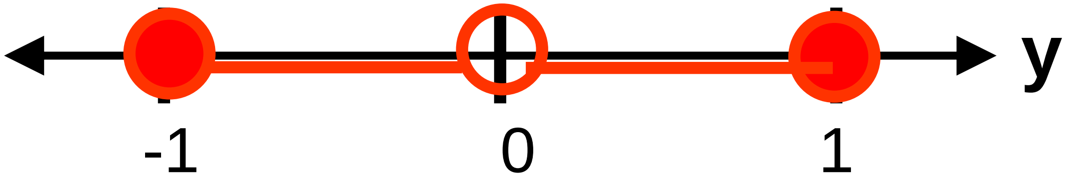
The reciprocal of a number between 0 and -1 is less than -1.

Interval notation equivalent: $x = (-\infty, -1] U [1, \infty)$

Ranges of the Trigonometric Functions

csc Ө = 1/y

The secant of an angle is the reciprocal of the cosine ratio. Therefore x cannot equal zero.



$-1 \leq y \leq 1$

And

y$\neq$ 0

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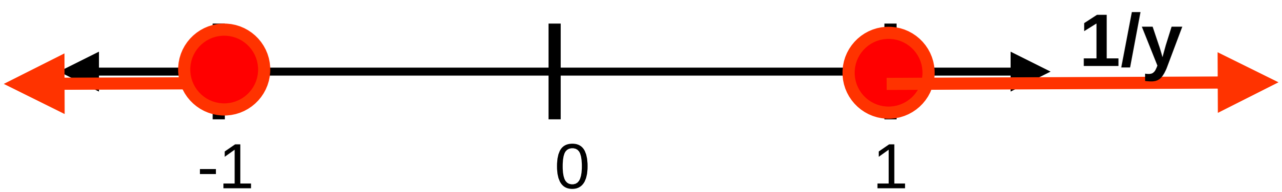

The reciprocal of a number between 0 and -1 is less than -1.

Interval notation equivalent: y$= (-\infty, -1] U [1, \infty)$

<u>Which is possible?</u>

1. $\sin \Theta = 0.568$ yes
2. $\sin \Theta = -2.1$ no
3. $\cos \Theta = 1.6$ no
4. $\sec \Theta = 0.5$ no
5. $\sec \Theta = -1.568$ yes
6. $\csc \Theta = -3.1$ yes
7. $\csc \Theta = -0.561$ no