# PreCalculus Algebra

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## Chapter 4.2 Right Triangle Trigonometry

# TRIGONOMETRIC FUNCTIONS OF ANGLES

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- Solving Right Triangles

## 4.3 Trigonometric Functions of Angles

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# Chapter 4 Section 2
# Right Angle Trigonometry

**DEFINITION** **Trigonometric Functions (Alternate Form)**

For an acute angle $\theta$ in a right triangle:

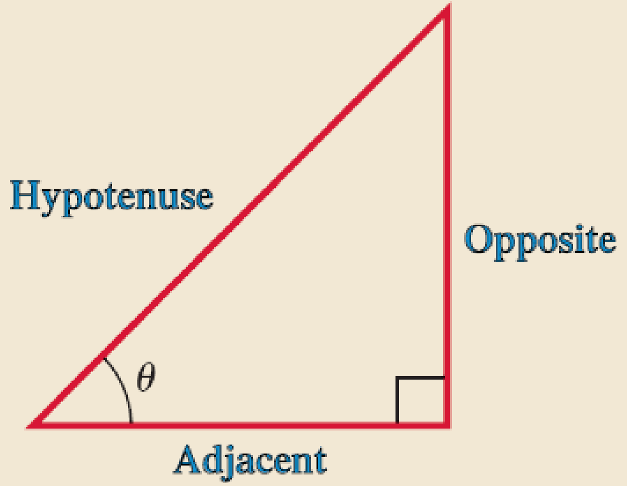
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \quad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \quad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

and their reciprocals:

$$\csc\theta = \frac{1}{\sin\theta} = \frac{\text{hypotenuse}}{\text{opposite}}$$

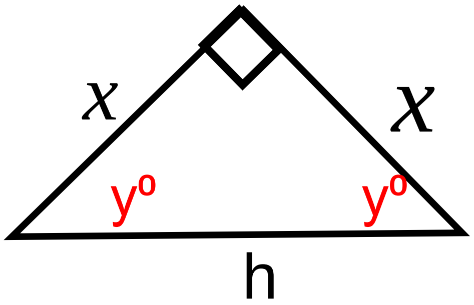
$$\sec\theta = \frac{1}{\cos\theta} = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{\text{adjacent}}{\text{opposite}}$$



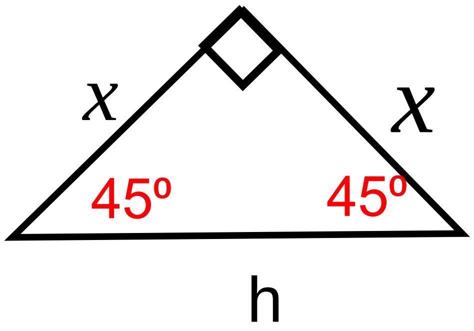
Isosceles Right Triangle: a right triangle with two sides that are congruent.

1) Find the measures of the base angles.



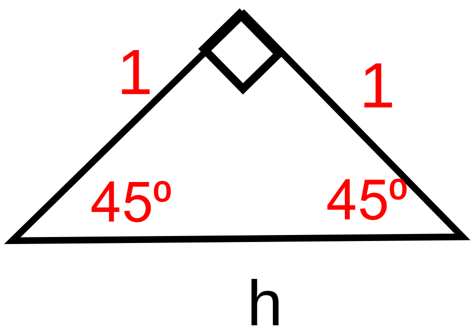

$$y^\circ + y^\circ + 90 = 180$$

$$2y^\circ = 90 \quad y = 45^\circ$$




2) "X" can be any number. To make it ***really*** easy, lets just make x = 1.

3) *Solve for 'h'.*

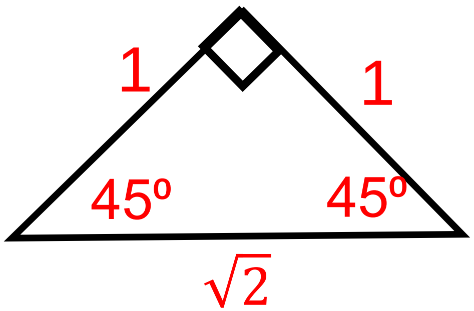



$$a^2 + b^2 = c^2$$

$$1^2 + 1^2 = c^2$$

$$2 = c^2$$

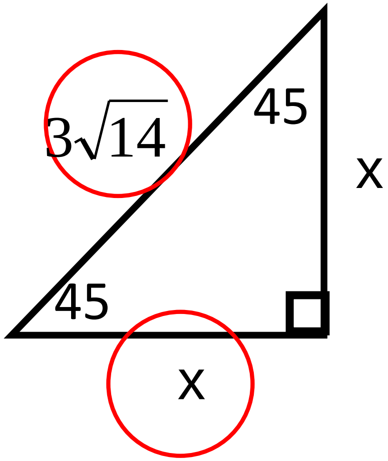
$$c = \sqrt{2}$$

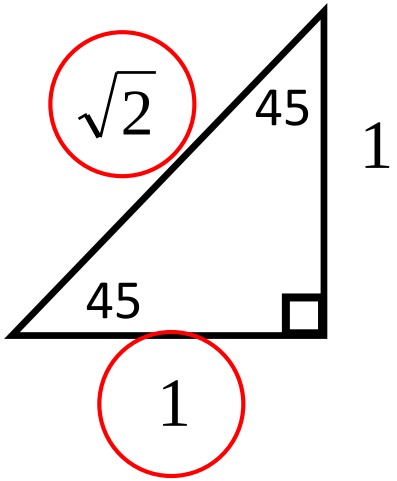



"One-One-Two-Root"

Use <u>scale factors</u> or proportions to solve for the lengths of sides of similar 45-45-90 right triangles.

Write a <u>proportion</u> (equation where a fraction equals a fraction)



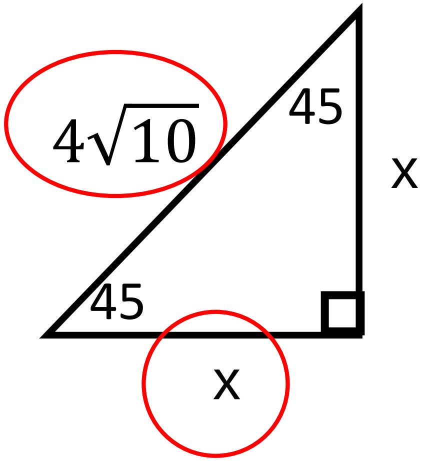

$$\frac{3\sqrt{14}}{\sqrt{2}} = \frac{x}{1}$$

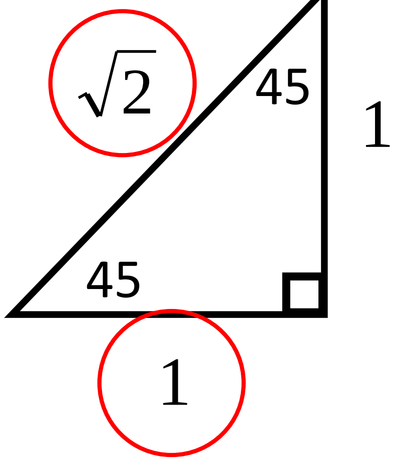
$$\frac{3\cancel{\sqrt{2}}\sqrt{7}}{\cancel{\sqrt{2}}} = x$$

$$x = 3\sqrt{7}$$

Use <u>scale factors</u> or proportions to solve for the lengths of sides of similar 45-45-90 right triangles.

Write a <u>proportion</u> (equation where a fraction equals a fraction)




$$\frac{4\sqrt{10}}{\sqrt{2}} = \frac{x}{1}$$

$$\frac{4\cancel{\sqrt{2}}\sqrt{5}}{\cancel{\sqrt{2}}} = x$$

$$x = 4\sqrt{5}$$

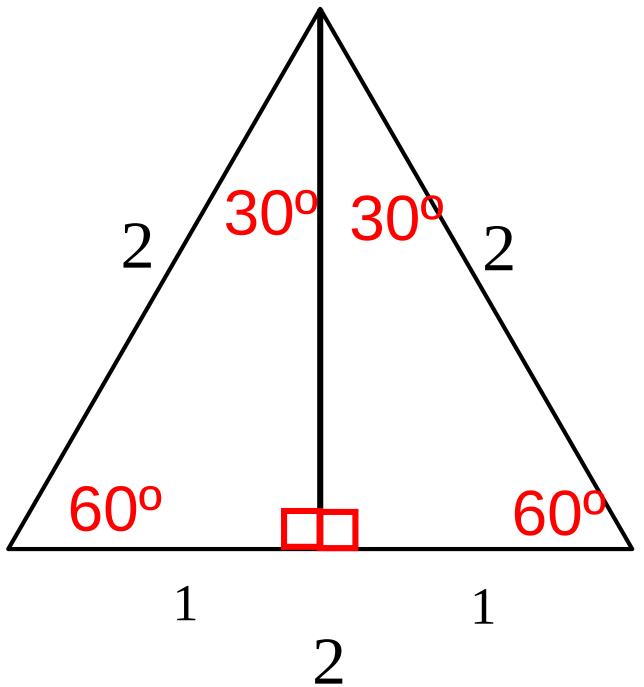
By constructing angle bisector of the top angle of a 60-60-60 equilateral triangle, two triangles are formed.

Are the two triangles congruent? Why? ASA

<u>Corresponding parts of congruent triangles are congruent (CPCTC)</u> (all remaining corresponding pairs of angles and sides are congruent).

Bottom legs (of the right triangles) are congruent so each is ½ the total of the original triangle's bottom length).

Length = 1 and length = 1



We now have a 30-60-90 triangle.

Solve for ‘x’.

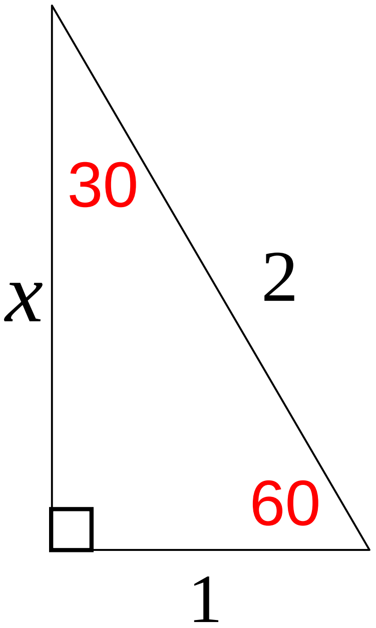
$$a^2 + b^2 = c^2$$

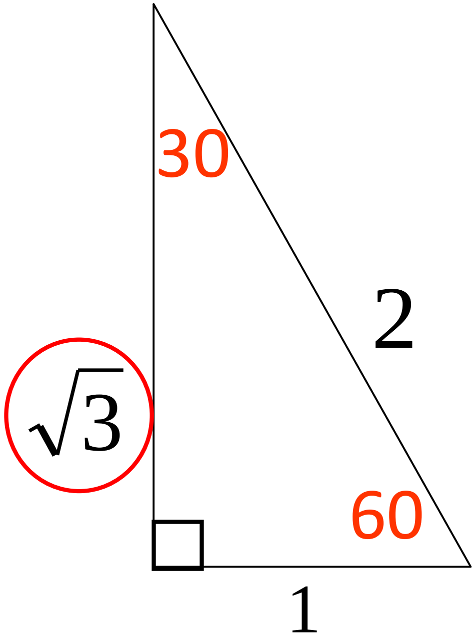
$$x^2 + 1^2 = 2^2$$

$$x^2 = 4 - 1$$

$$x^2 = 3$$

$$x = \sqrt{3}$$

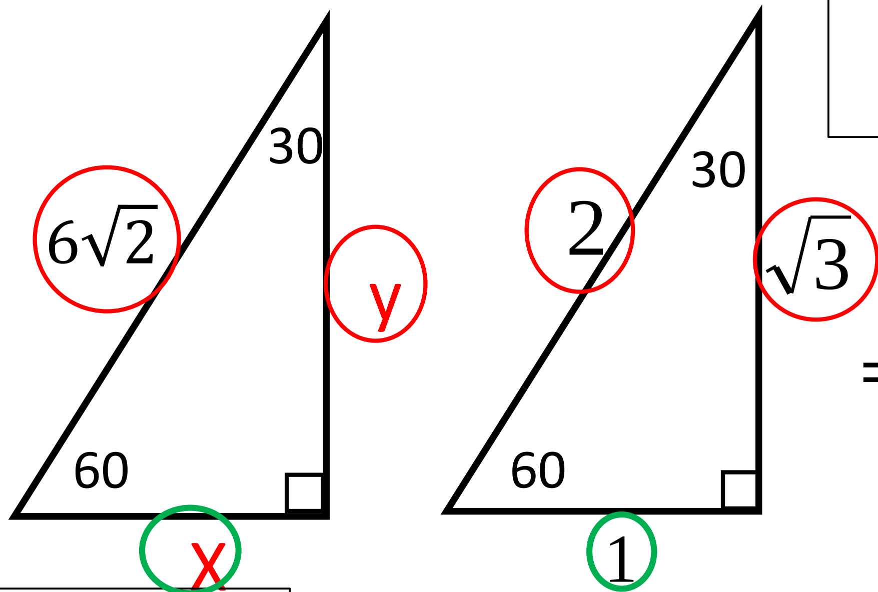



“one-two-three-root”

<u>30-60-90 Right Triangle</u>    Solve with a <u>proportion</u>

Write a <u>proportion</u> (equation where a fraction equals a fraction)




$$\boxed{\frac{6\sqrt{2}}{2} = \frac{y}{\sqrt{3}}} \Rightarrow \frac{\cancel{2} * 3 * \sqrt{2}}{\cancel{2}}$$

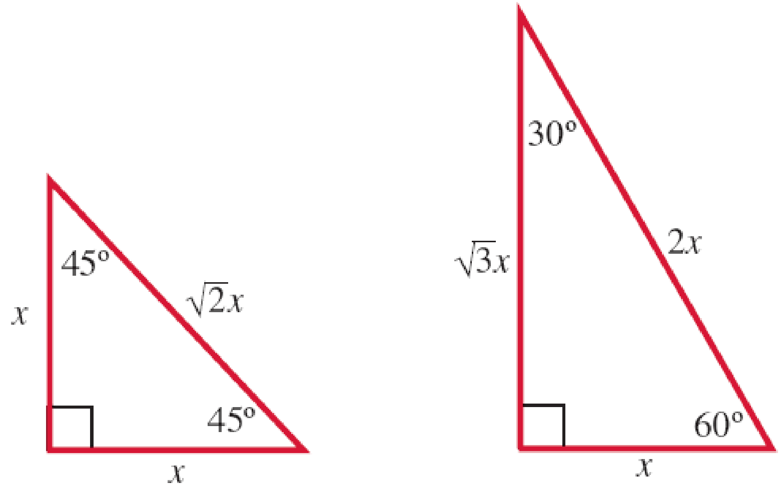
$$\Rightarrow 3\sqrt{2} = \frac{y}{\sqrt{3}}$$

$$\Rightarrow 3\sqrt{2} * \sqrt{3} = \frac{y}{\cancel{\sqrt{3}}} * \frac{\cancel{\sqrt{3}}}{1}$$

$$\Rightarrow \boxed{3\sqrt{6} = y}$$

$$\boxed{\frac{6\sqrt{2}}{2} = \frac{x}{1}} \Rightarrow \frac{\cancel{2} * 3 * \sqrt{2}}{\cancel{2}} = \frac{x}{1} \Rightarrow \boxed{3\sqrt{2} = x}$$

# Special Triangles

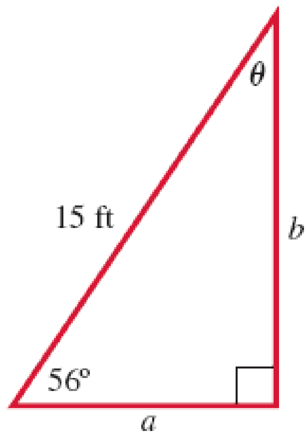



## Trigonometric Function Values for Special Angles

| θ Degrees | θ Radians | $\sin\theta$ | $\cos\theta$ | $\tan\theta$ | $\cot\theta$ | $\sec\theta$ | $\csc\theta$ |
|---|---|---|---|---|---|---|---|
| $30^\circ$ | $\frac{\pi}{6}$ | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{3}}{3}$ | $\sqrt{3}$ | $\frac{2\sqrt{3}}{3}$ | $2$ |
| $45^\circ$ | $\frac{\pi}{4}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{2}}{2}$ | $1$ | $1$ | $\sqrt{2}$ | $\sqrt{2}$ |
| $60^\circ$ | $\frac{\pi}{3}$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ | $\sqrt{3}$ | $\frac{\sqrt{3}}{3}$ | $2$ | $\frac{2\sqrt{3}}{3}$ |

# Solving Right Triangles

Solve the right triangle—find $a$, $b$, and $\theta$.




**Solution:**

**STEP 1** Solve for $\theta$.

$$\theta + 56^\circ = 90^\circ$$

$$\boxed{\theta = 34^\circ}$$

**STEP 2** Solve for $a$.

$$\cos 56^\circ = \frac{a}{15}$$

$$a = 15 \cos 56^\circ$$

$$a \approx 8.38789$$

$$\boxed{a \approx 8.4 \text{ ft}}$$

**STEP 3** Solve for $b$.

$$\sin 56^\circ = \frac{b}{15}$$

$$b = 15 \sin 56^\circ$$

$$b \approx 12.43556$$

$$\boxed{b \approx 12 \text{ ft}}$$

**STEP 4** Check.

$$\sin 34^\circ \stackrel{?}{=} \frac{8.4}{15}$$

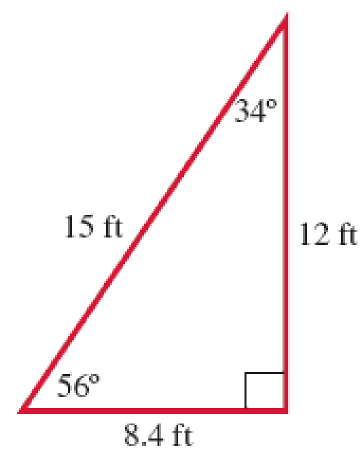
$$0.5592 \approx 0.56$$

$$\cos 34^\circ \stackrel{?}{=} \frac{12}{15}$$

$$0.8290 \approx 0.80$$

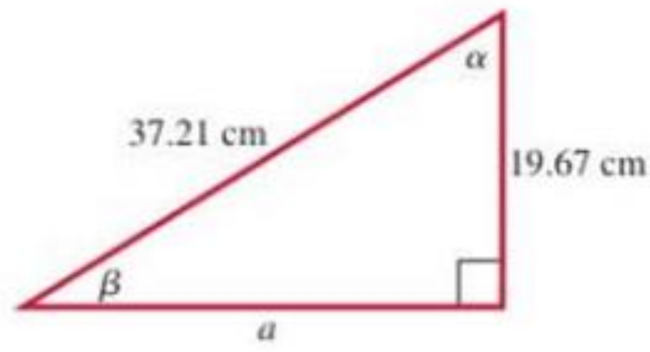
$$\tan 34^\circ \stackrel{?}{=} \frac{8.4}{12}$$

$$0.6745 \approx 0.70$$



# Solving Right Triangles given two sides

Solve the right triangle—find $a$, $\alpha$, and $\beta$.




$$sin\beta = \frac{opp}{hyp} = \frac{19.67\ \cancel{cm}}{37.21\ \cancel{cm}}$$

$$sin\beta = 0.5286$$

$$\sin^{-1}(sin\beta) = \sin^{-1}(0.5286)$$

$$\boxed{\beta = 37.9}$$

$$\alpha = 180 - \beta$$

$$\boxed{\alpha = 148.1}$$

$$cos\beta = \frac{adj}{hyp}$$

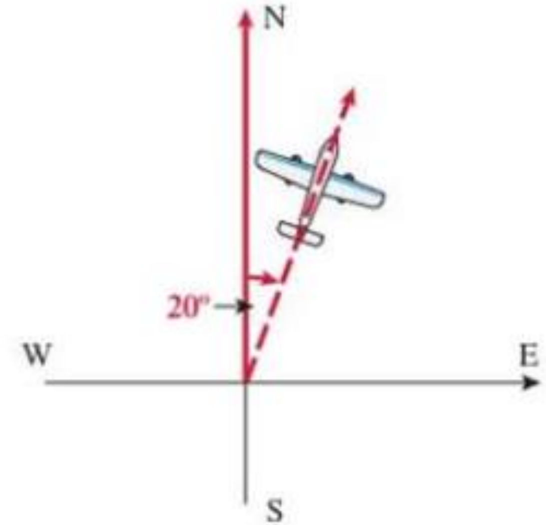
$$cos37.9 = \frac{a}{37.21\ cm}$$

$$0.7891 = \frac{a}{37.21\ cm}$$

$$\boxed{a = 29.4\ cm}$$

# Using Bearing

In navigation, the word **bearing** means the direction in which a vessel is pointed. **Heading** is the direction in which the vessel is actually traveling. Heading and bearing are only synonyms when there is no wind. Direction is often given as a bearing, which is the measure of an acute angle with respect to the north–south vertical line. "The plane has a bearing of N 20° E" means that the plane is pointed 20° to the east of due north.



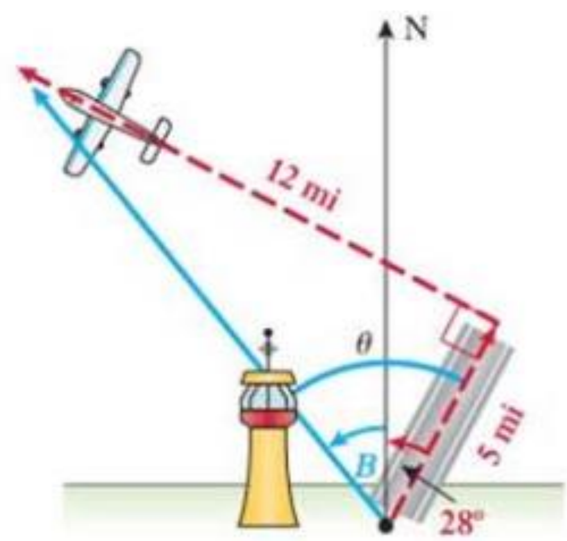
A jet takes off bearing N 28° E and flies 5 miles, and then makes a left (90°) turn and flies 12 miles further. If the control tower operator wants to locate the plane, what bearing should she use?

-Notice that the bearing (direction the plane is pointed) is relative to north. We can assume that the position of the control tower is at the location where the plane took off.

-We can draw a right triangle with leg lengths 5 and 12 miles since the plane turned 90 degrees to the left between the initial bearing of 28 N and the next bearing.

-We can calculate the bottom angle of the right triangle using the tangent ratio.

$$\theta = \tan^{-1}\left(\frac{12}{5}\right) = 67.4$$




To find the plane's bearing from the control tower, we must subtract the plane's initial bearing from theta. $\beta = 67.4 - 28$ $\beta = 39.4$

We can describe the bearing as 39.4 degrees to the west of north: (N 39.4 W) or using the true direction it is 39.4 degrees to the left of north (39.4 less than 360) or 320.6 "true."