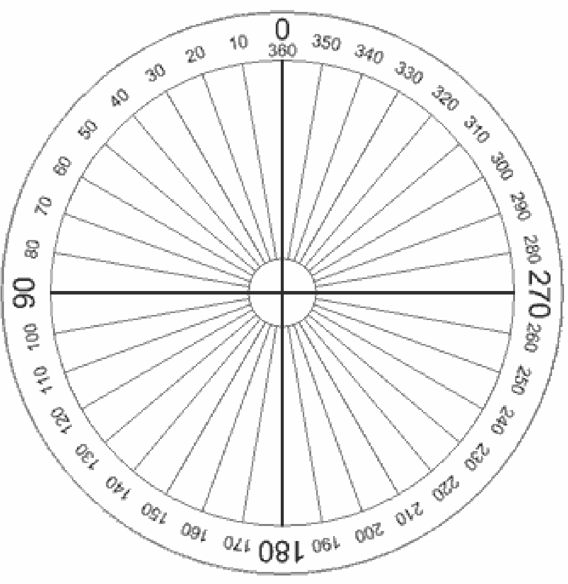
Math-1060
Lesson 1 (Section 4-1)

Measuring Angles

- Degrees and Radians
- Co-terminal Angles
- Arc Length
- Sector Area
- Linear and Angular Speeds

Why does the angular measure of a circle equal 360^{o}?



The idea of dividing a circle into 360 equal pieces dates back to the <u>sexagesimal (60-based</u>) counting system of the ancient Sumarians. Early astronomical calculations linked the sexagesimal system to circles.

<u>Degrees</u>: The measure of an angle as a portion of 360° (the angle measure of a circle).

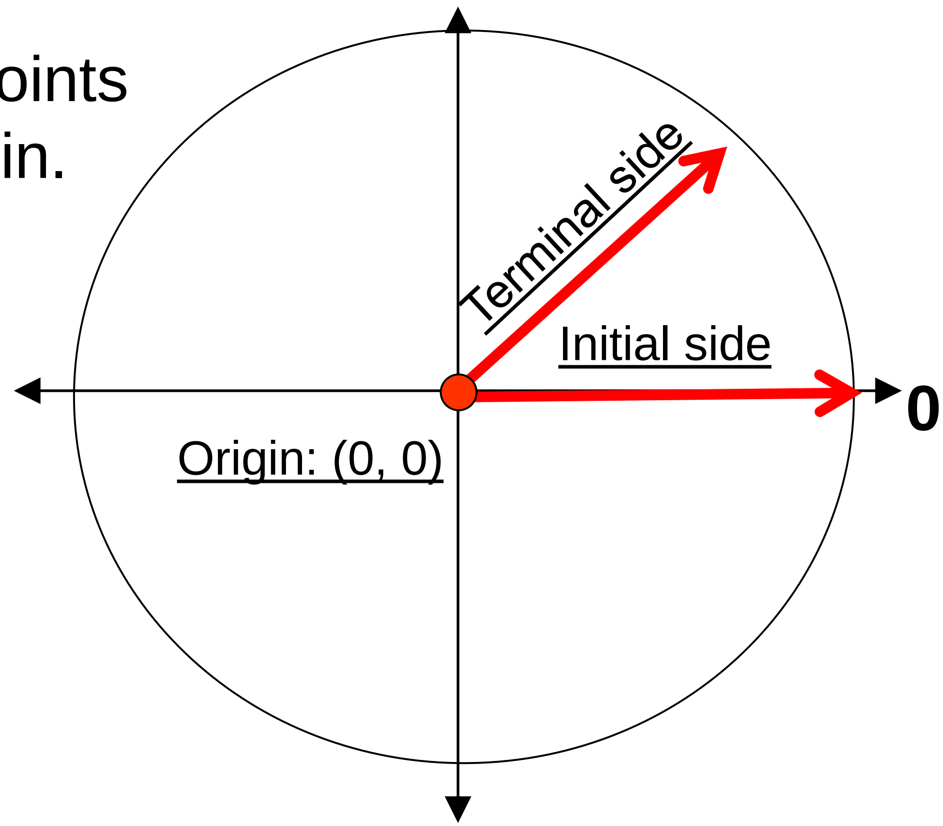
$$90^{\circ} = \frac{1}{4} * 360^{\circ}$$

Angle: two rays with a common end point

Initial side of the angle: Angles graphed on the x-y plane, have their vertex at the origin. The initial side always points in the positive ‘x’ direction.

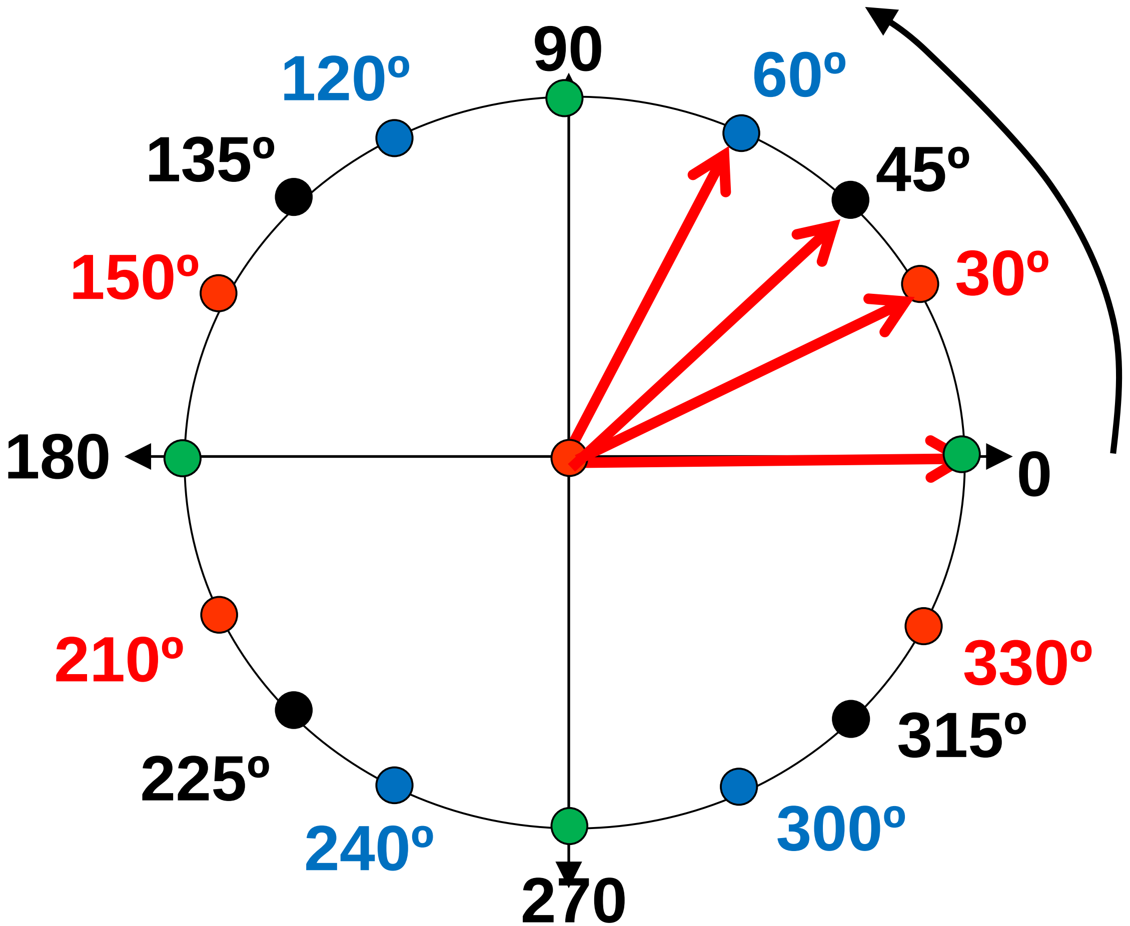
Terminal side of the angle: points in any direction from the origin.

We think of these angles as “opening” as the terminal side moves around the circle.



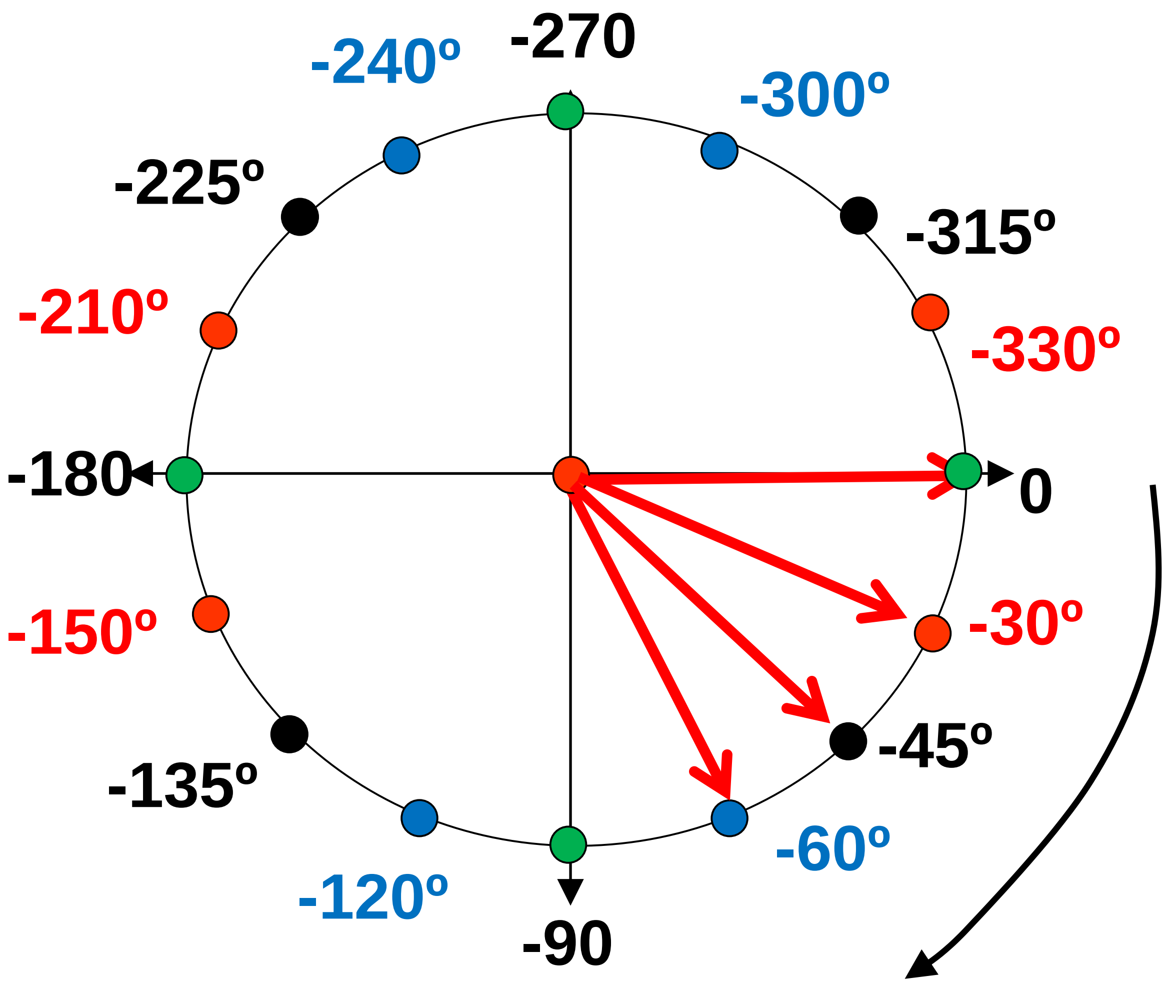
Positive Angles: The terminal side opened in the counter-clockwise direction from the initial side of the angle.

Angle measure: is put at the end of the terminal side rather than inside the angle.



Negative Angles: The terminal side opened in the clockwise direction from the initial side of the angle.

Large Angles: The terminal side may make more than one revolution. An angle whose terminal side makes one complete revolution *and then* an additional 30 degree would have a measure of 390 degrees.



Angles are classified based upon their degree measure.

Acute angle: $0° < m\measuredangle < 90°$

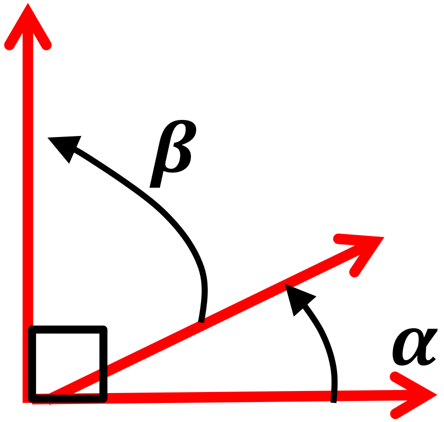
Right angle: $m\measuredangle = 90°$

Obtuse angle: $90° < m\measuredangle < 180°$

Straight angle: $m\measuredangle = 180°$

Adjacent angles share a common endpoint and one side.

Complimentary angles: two angles whose sum is 90 degrees. They need not be adjacent.



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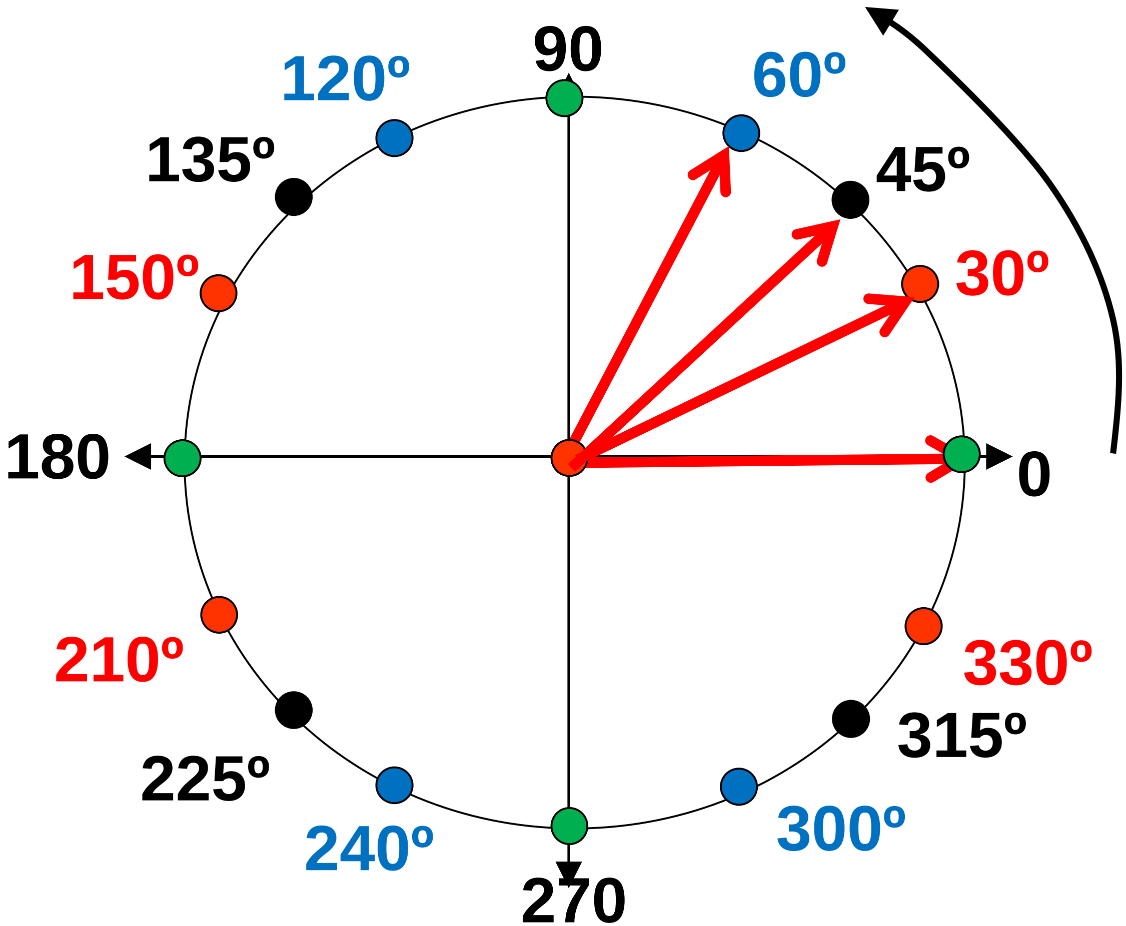
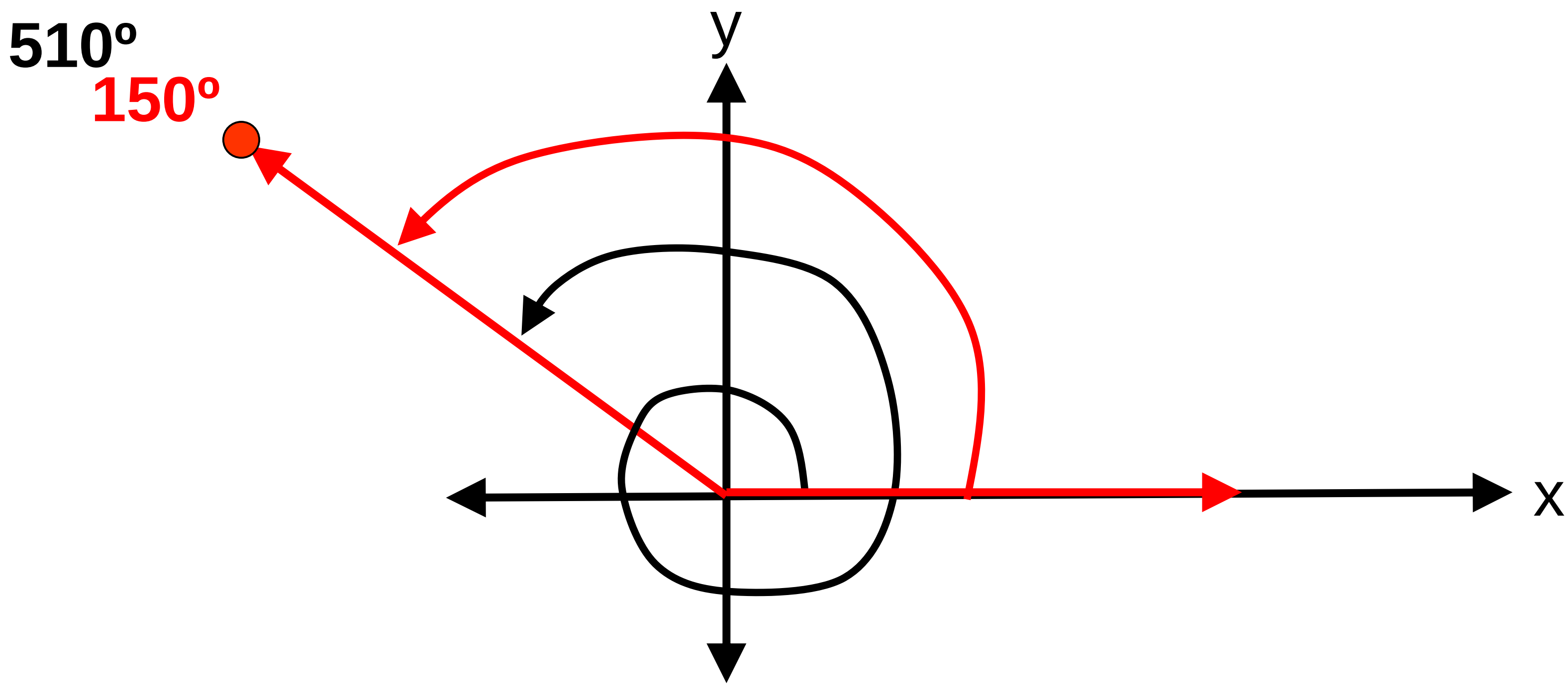


Co-terminal Angles: may have different measures but they share both the initial and terminal sides.



In general: co-terminal angles have measures defined by:

$$m\angle\theta = m\angle\beta \pm 360n$$

where ‘n’ is the number of revolutions beyond the 1st one.

Determining of angles are co-terminal:. $m\angle\theta = m\angle\beta \pm 360n$

$$m\angle\theta = 1550$$

$$m\angle\beta = 1550 - 360 = 1190$$

$$m\angle\beta = 1550 - 2(360) = 830$$

$$m\angle\beta = 1550 - 3(360) = 470$$

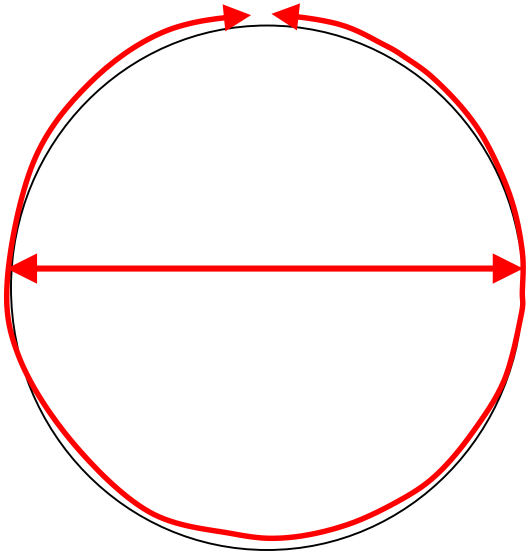
$$m\angle\beta = 1550 - 4(360) = 100$$

$$m\angle\beta = 1550 - 5(360) = -250$$

A typical co-terminal angle problem: will give the measure of an angle then ask for a different positive and negative co-terminal angle.

Pi: an irrational number that is the ratio of the distance around the circle to the distance across the circle.

$$\pi = \frac{C}{D}$$



$$\pi = \frac{C}{2r} \qquad C = 2\pi r$$

$$\boxed{\text{radian measure} = \frac{\textit{arc}\ \text{length}}{\text{radius}}}$$

$$\text{radian measure (of a circle)} = \frac{\textit{circumference}}{\text{r}}$$

$$\text{radian measure (of a circle)} = \frac{2\pi\cancel{r}}{\cancel{\text{r}}} = 2\pi$$

Units of radians = inches/inches

Radian measure has no units! (nice)

What is the radian measure of an angle that is ½ of the circle?

$$whole\ \text{circle} = 2\pi\ \text{radians}$$

Radians vs. Degrees

$$360^\circ = 2\pi\ (\text{radians})$$

$$half\ \text{circle} = \pi\ \text{radians}$$

Radians vs. Degrees

$$180^\circ = \pi\ (\text{radians})$$

Convert between radians and degrees using a “proportion”.

$$\boxed{\frac{angle_{degrees}}{360} = \frac{angle_{radians}}{2\pi}}$$

$$\frac{7}{8}\pi$$

$$\frac{angle_{degrees}}{360} = \frac{{}^{7}/_{8}\cancel{\pi}}{2\cancel{\pi}}$$

$$360 * \frac{angle_{degrees}}{360} = 0.4375 \ * 360$$

$$angle_{degrees} = 157.5^{\circ}$$

$$\boxed{180^\circ = \pi \text{ radians}}$$

Divide both sides by 180

$$\frac{180^\circ}{180^\circ} = \frac{\pi}{180^\circ}$$

$$1 = \frac{\pi}{180^\circ}$$

$$\left(\frac{\pi}{180^o}\right)$$

Divide both sides by π

$$\frac{180^\circ}{\pi} = \frac{\pi}{\pi}$$

$$1 = \frac{180^\circ}{\pi}$$

$$\left(\frac{180^o}{\pi}\right)$$

These are "unit conversion factors"

Unit Conversion factor: a ratio of equal measurements in different units that allow conversion of a one type of unit to another (feet → inches, degrees → radians, radians → degrees etc.)

When you multiply a number by one of these factors, (you are multiplying by "1") but the units are converted.

Converting from Degrees to Radian Measure

$$\cancel{140^{o}}\left(\frac{\pi}{\cancel{180^{o}}}\right)=\frac{140}{180}\pi=\frac{14}{18}\pi=\frac{7}{9}\pi$$

Converting from Radian Measure to Degrees

$$\frac{\cancel{\pi}}{2}\left(\frac{180^{o}}{\cancel{\pi}}\right)=90^{\circ}$$

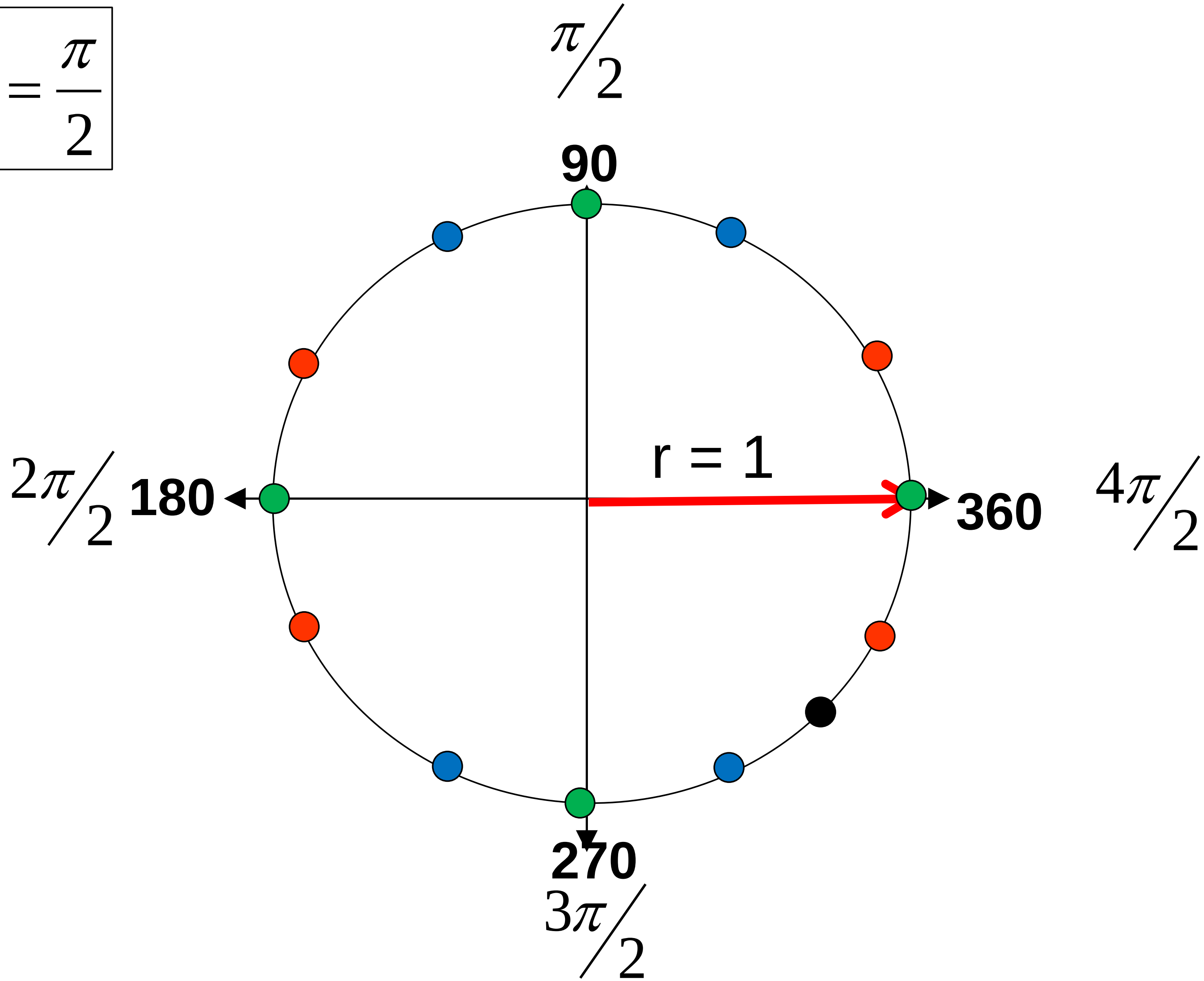
$$\frac{11}{3}\pi=?$$

$$270^{o}=?$$

$$90° \ast \left(\frac{\pi}{180^o}\right) = \left(\frac{90^\circ \pi}{180^o}\right)$$

"count around" the circle by adding 90 = π/2 to each angle

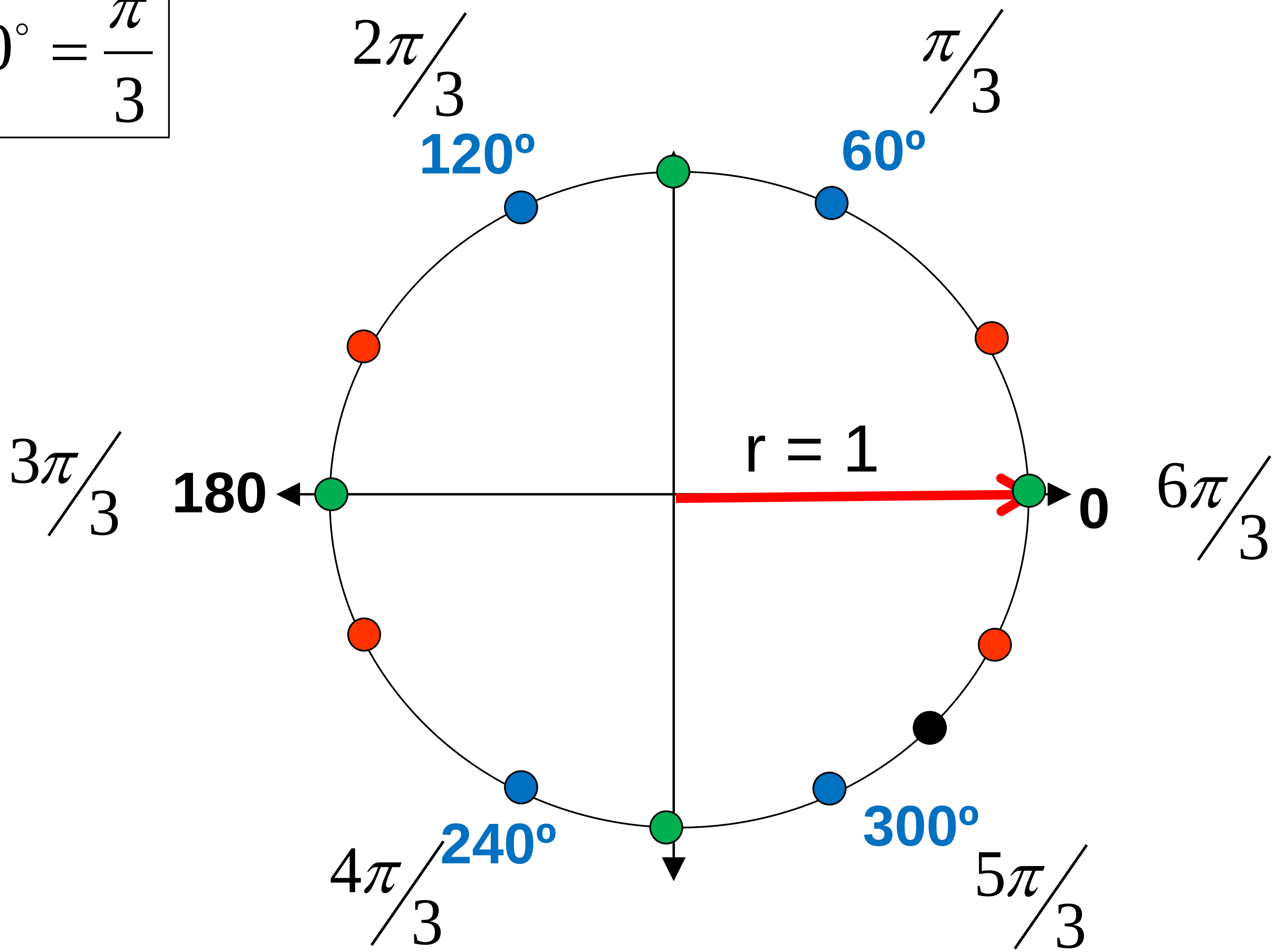
$$\boxed{90^\circ = \frac{\pi}{2}}$$



$$60° * \left(\frac{\pi}{180^o}\right) = \left(\frac{60^\circ \pi}{180^o}\right)$$

“count around” the circle by adding 60 = π/3 to each angle

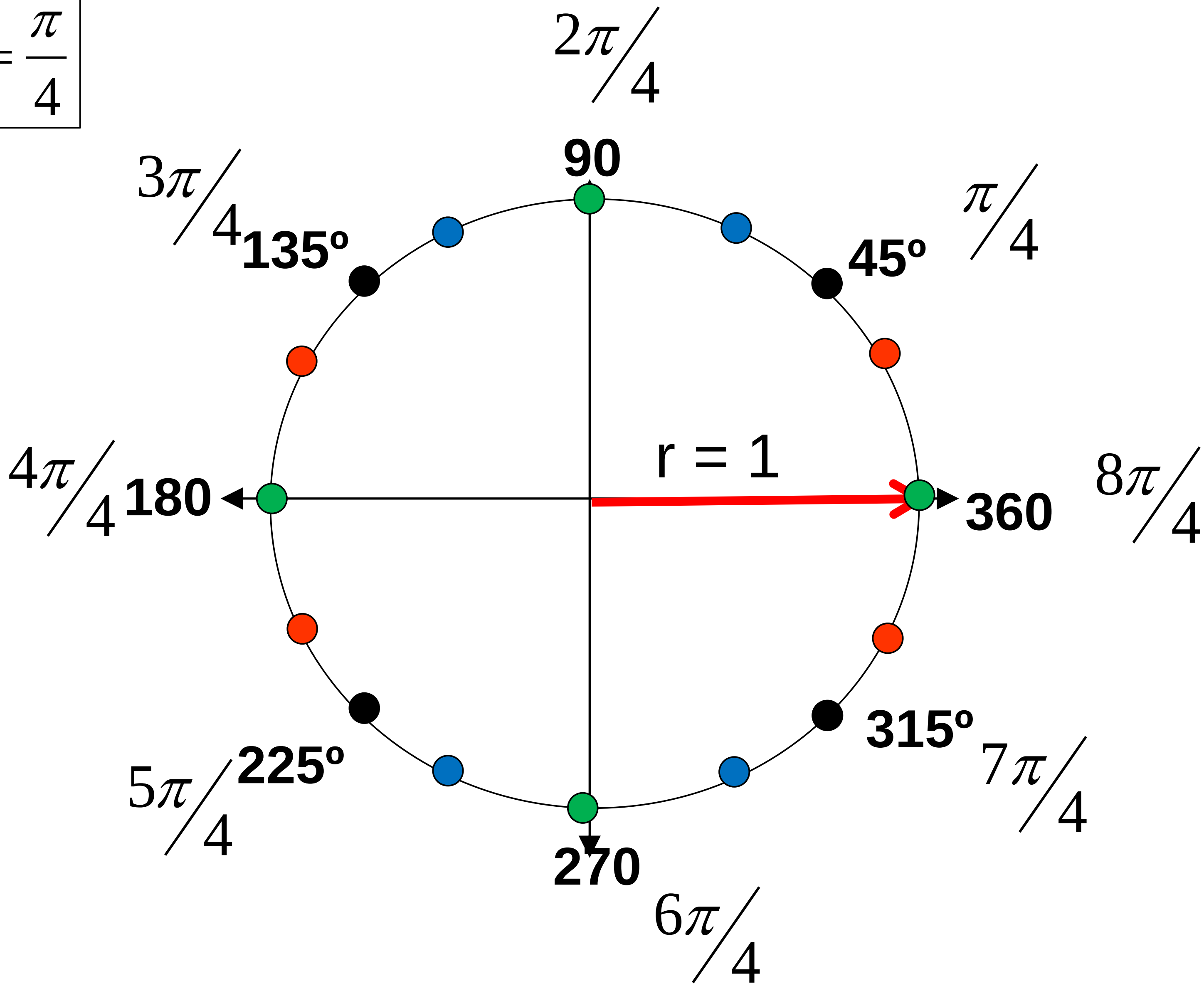
$$\boxed{60^\circ = \frac{\pi}{3}}$$



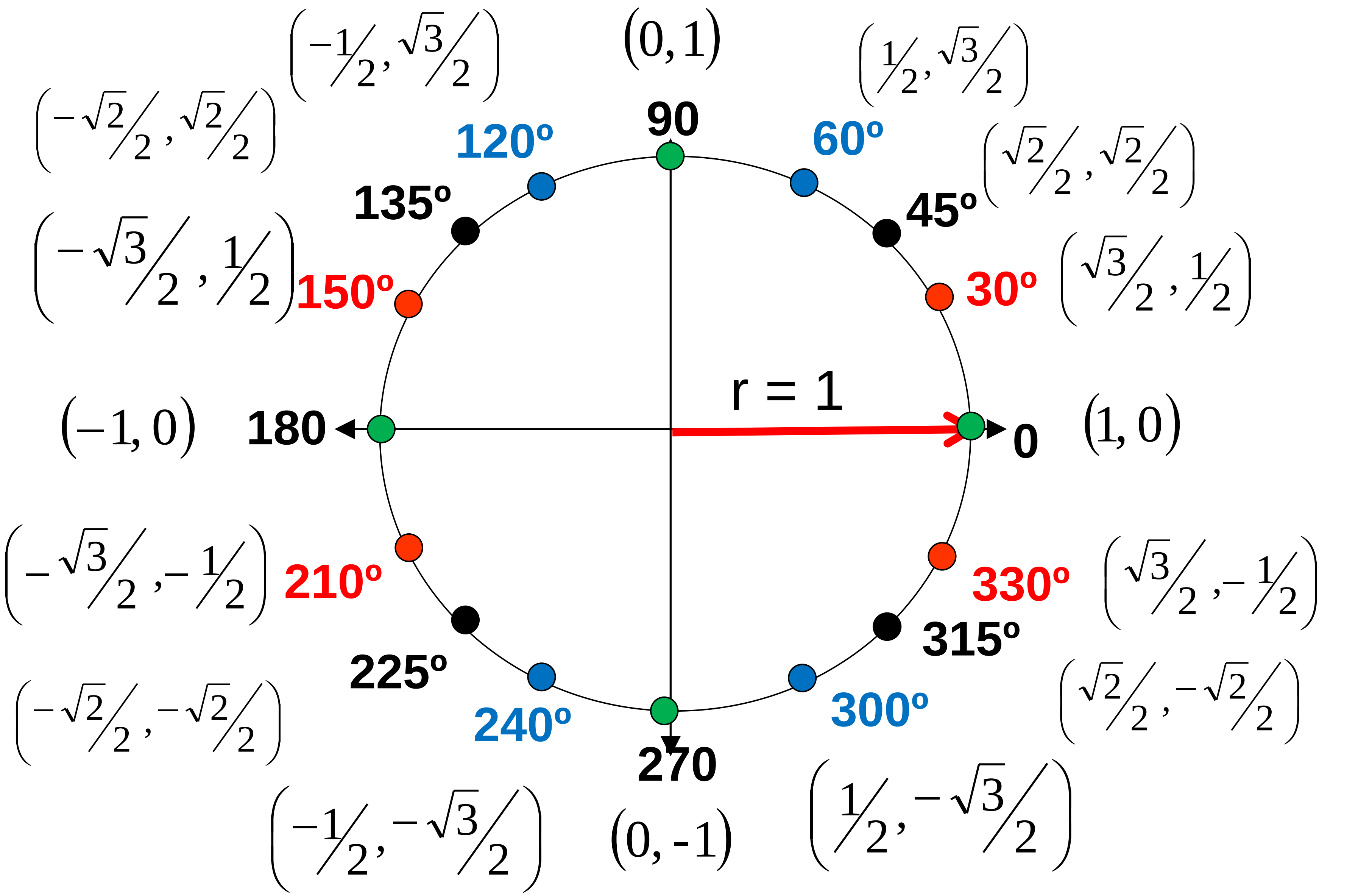
$$45° * \left(\frac{\pi}{180^o} \right) = \left(\frac{45^\circ \pi}{180^o} \right)$$

"count around" the circle by adding 45 = π/4 to each angle

$$\boxed{45^\circ = \frac{\pi}{4}}$$



The circle has a radius = 1. Fill in the measure of each angle (standard position angles). Fill in the x-y pairs each point.



Radian measure: the ratio of the arc length to the radius of the circle:

$$\boxed{\text{radian measure} = \frac{arc\ \text{length}}{\text{radius}}}$$

Symbolically, we write: $\theta = \dfrac{s}{\mathrm{r}}$

Using the property of equality, we can rewrite the formula as:

$$s = r\theta$$

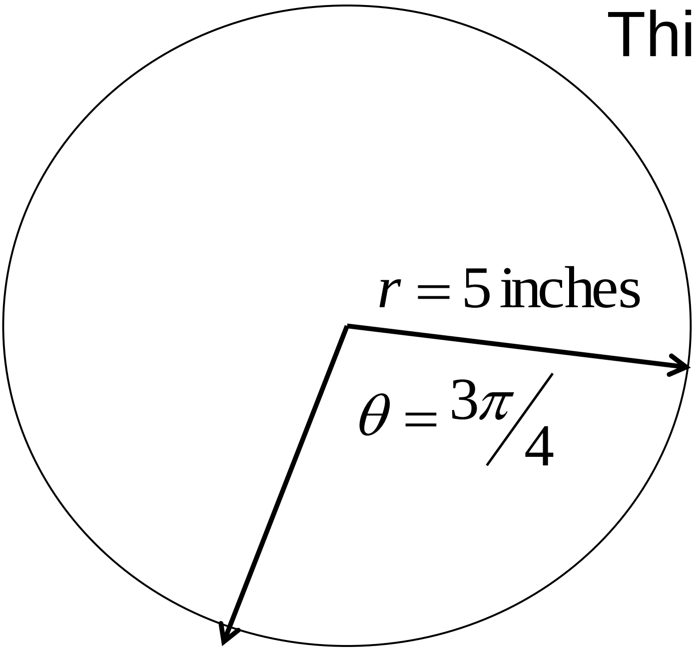
theta: a Greek letter. Traditionally, we use Greek letters as unknown values for the measure of an angle.

Problem types you'll see:

What is length of the subtended arc?

$$s = r\theta$$

This equation only works for radian measure!



$$s = 5 * 3\pi/4$$

$$s = \frac{15\pi}{4} \text{ inches}$$

$$s = r\theta$$ (This only works for radian measure!)

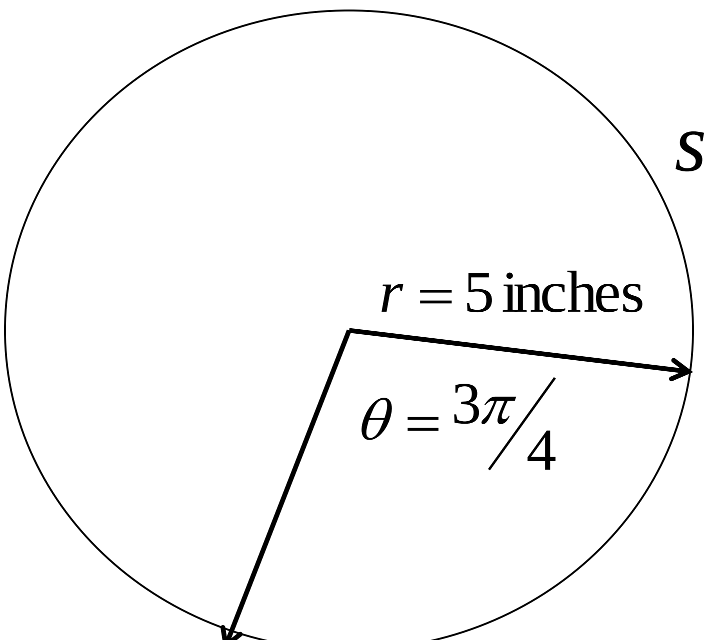
Solving “subtended arc” problems: (1) use a proportion OR (2) convert the angle measure to radians and use the formula.

$$\frac{\text{part}}{\text{whole}_{(\text{arc lengths})}} = \frac{\text{part}}{\text{whole}_{(\text{angles})}}$$

$$\frac{s}{2 * \pi * r} = \frac{\theta}{360 \text{ or } 2\pi}$$

$$\frac{\text{arc of the sector}}{\text{total arc of the circle}} = \frac{\text{angle of the sector}}{\text{total angle of the circle}}$$

$$\frac{s}{2 * \pi * 5} = \frac{3\pi/4}{2\pi}$$



$$s = \cancel{2\pi} * 5 * \frac{3\pi/4}{\cancel{2\pi}}$$

$$s = \frac{5 * 3}{4}\pi$$

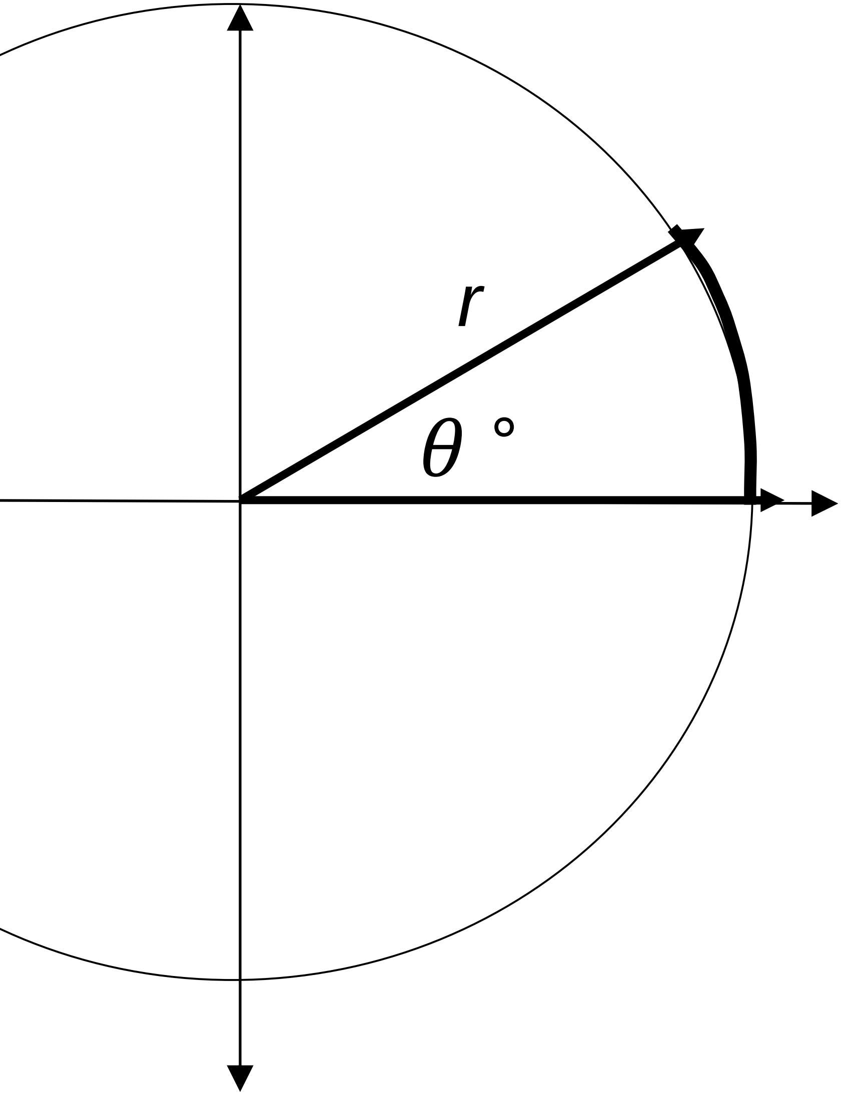
We want our answers in reduced fraction form, with (Pi) π in the answer.

$$s = \frac{15\pi}{4} \text{ in}$$

The area of a Sector, is a fraction of the area of a circle.

Write a proportion.

$$\frac{\text{part}}{\text{whole}_{(\text{areas})}} = \frac{\text{part}}{\text{whole}_{(\text{angles})}}$$



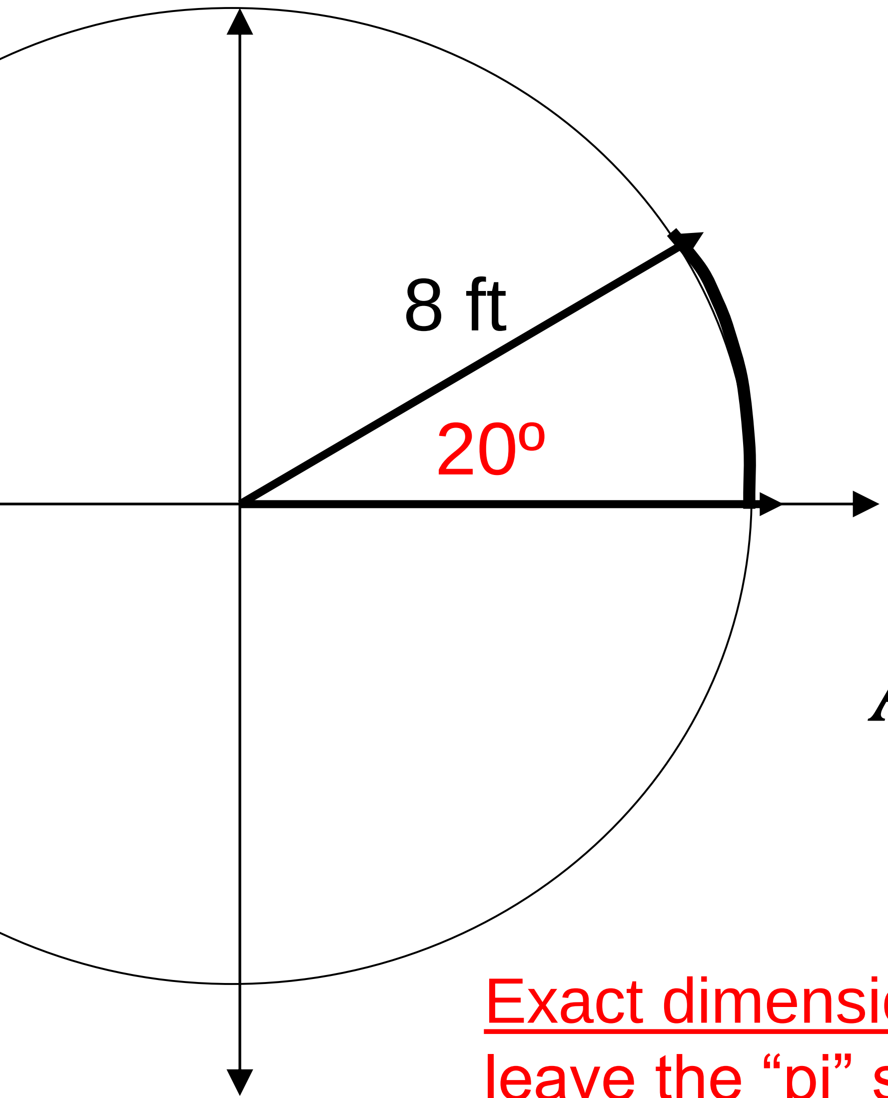
$$\frac{A_{\text{sector}}}{\pi * r^2} = \frac{\theta}{360 \text{ or } 2\pi}$$

Exact dimension → Exact answer

leave the "pi" symbol in your answer

A circle has an 8 foot radius.
What is the area of a 20° sector?

Write a proportion.



$$\frac{\text{part}}{\text{whole}_{(\text{areas})}} = \frac{\text{part}}{\text{whole}_{(\text{angles})}}$$

$$\boxed{\frac{A_{\text{sector}}}{\pi * r^2} = \frac{\theta}{360 \text{ or } 2\pi}}$$

$$\frac{A_{sector}}{\pi * 8^2} = \frac{20}{360}$$

$$\frac{A_{sector}}{64\pi} = 0.5555\bar{5}$$

$$A_{sector} = 64 * 0.5555\bar{5} * \pi$$

$$A_{\text{sector}} = \frac{32}{9}\pi \text{ ft}^2$$

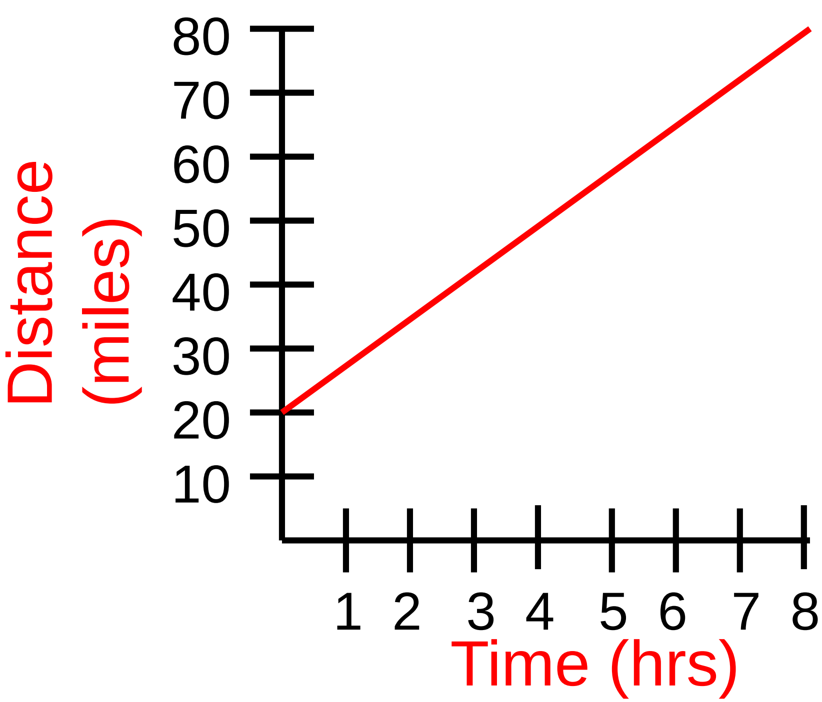
Exact dimension → Exact answer
leave the “pi” symbol in your answer

Linear Speed: the ratio of change in distance to the time required to travel that distance.

$$\boxed{speed = \frac{\Delta\, distance}{\Delta\, time}}$$

Rate: a ratio of two real world quantities → speed is a rate.

Sometimes the formula is written as: $\boxed{D = r * t}$



Speed: slope of the distance as a function of time graph.

What are the “units” of slope for this graph?

Now think of a point traveling around a circle.

Linear Speed: the ratio of arc-length traveled to the time required to travel that arc length. $\boxed{v = \frac{s}{t}}$

A car travels around a circular track with a circumference of 2 miles. If the car records a time of 15 minutes for 9 laps, what is the linear speed of the car in miles per hour?

$$\boxed{v = \frac{s}{t}} \qquad v = \frac{9\ \cancel{laps} * \frac{2\ miles}{\cancel{lap}}}{15\ min}$$

$$v = \frac{18\ miles}{15\ \cancel{min}} * \frac{60\ \cancel{min}}{1\ hour} \qquad v = \frac{1080\ miles}{15\ hour} \boxed{= \frac{72\ miles}{1\ hour}}$$

Angular Speed: As a point moves around a circle at a constant linear speed, the angular speed will also be constant. The angular speed is the ratio of change in the angle (in radian measure) to the change in time .

$$angular\ speed = \frac{change\ in\ the\ angle}{change\ in\ time}$$

$$\boxed{w = \frac{\theta}{t}}$$

$$\boxed{\text{"w"}\ is\ the\ Greek\ letter\ Omega}$$

A light house has a rotating light. The light rotates one complete revolution every 10 seconds. What is the angular speed of rotation?

$$\boxed{w = \frac{\theta}{t}}$$

$$\theta = 2\pi$$

$$w = \frac{2\pi}{10\ \cancel{sec}} * \frac{60\ \cancel{sec}}{1\ min}$$

$$\boxed{w = \frac{12\pi\ radians}{sec}}$$

Relating Linear Speed to Angular Speed: As a point moves around a circle at a constant linear speed, the angular speed will also be constant.

Similar to the arc-length formula, $s = r\theta$ The two are related as follows:

The arc-length distance (S) has been replaced with the time rate of chance of distance (velocity) (v) and the angle (theta) has been replace with the time rate of change of the angle (omega).

$$\boxed{v = r\omega}$$

Solving for omega, we have:

$$\boxed{w = \frac{v}{r}}$$

A standard truck wheel (metal rim) has a diameter of 17 inches. With the tire installed on the rim a truck tire has a diameter of 25.7 inches.

The owner decides to upgrade to 19 inch rims. He buys new tires. After installation, the tire diameter has increase to 28. 2 inches.

Without having the onboard computer updated, how fast will the speedometer read if he is actually traveling at 75 mph?

1. Calculate the angular speed for 75 miles per hour on the old tires.

$$\boxed{w = \frac{v}{r}} \qquad w = \frac{75\ miles/hr}{25.7\ in/2} = \frac{75\ miles}{12.85\ in*hr} * \frac{63360\ in}{1\ mile}$$

$$w \approx 369{,}805\,\frac{radians}{hr} \qquad \boxed{v = r\omega} \qquad v = \frac{28.2\ in}{2}\omega$$

$$v = \frac{28.2\ in}{2} * 369{,}805\,\frac{radians}{hr} = \frac{521425\ in}{hr} = \frac{82.3\ miles}{hr}$$