Math-1050
Session #8

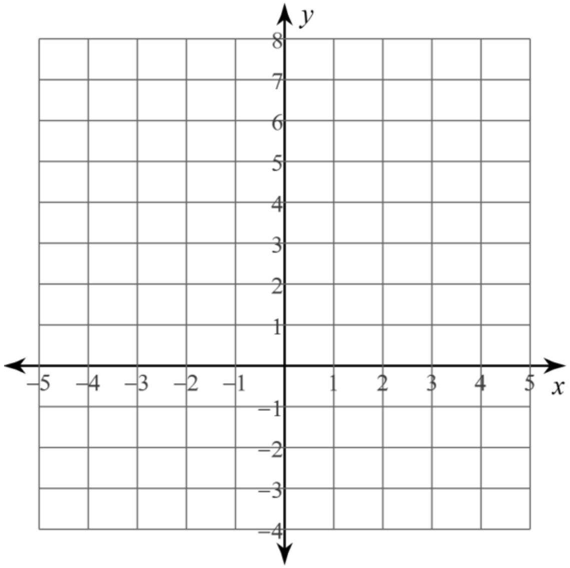
Textbook:

4.3: Quadratic Functions and Their Properties, Quadratic Models;

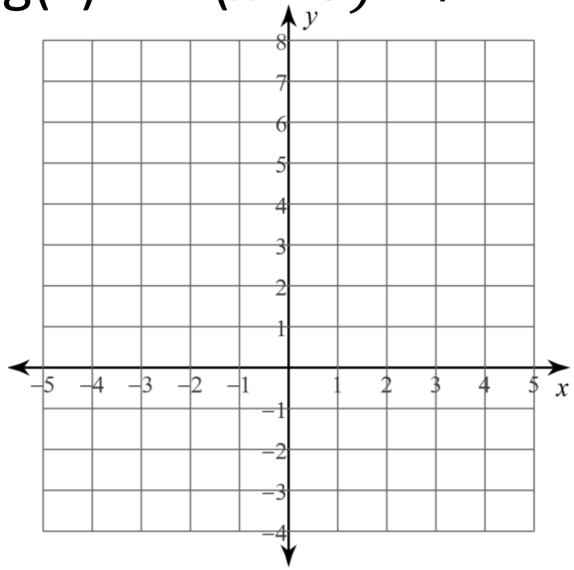
4-4: Building Quadratic Functions from Data

Interpret the transformation then graph the function

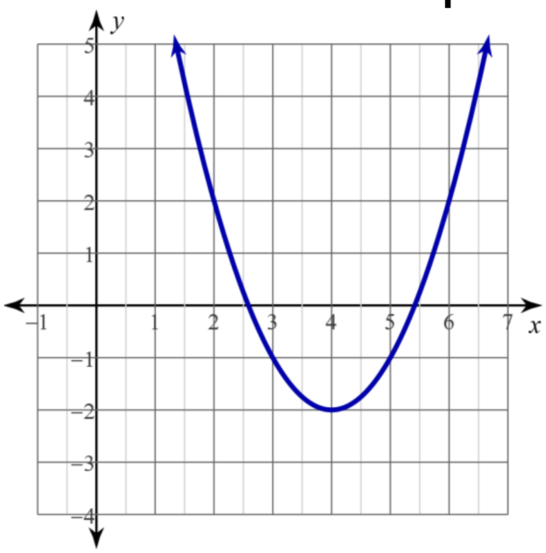
k(x) = $(x + 2)^2 - 3$

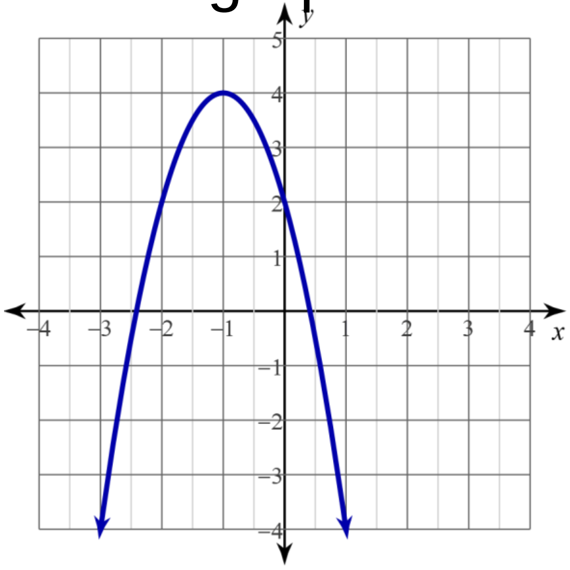


g(x) = $-2(x - 3)^2 + 4$



What is the equation that has been graphed?





Vertex form $y = a(x-h)^2 + k$ $y = (x+3)^2 - 8$

1. Find the zeroes of the function.

Let y = 0 $0 = (x+3)^2 - 8$

Isolate the squared term

$8 = (x+3)^2$ “take square roots”

$\sqrt{8} = \sqrt{(x+3)^2}$

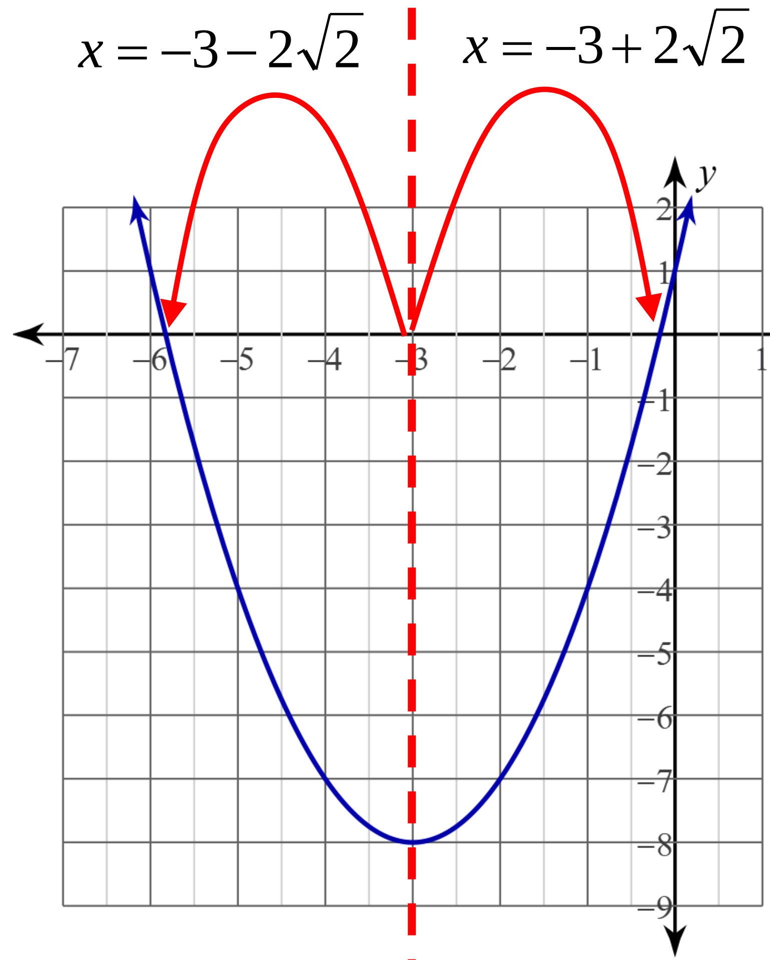
$\pm\sqrt{8} = x+3$ Simplify the radical

$\pm\sqrt{2*2*2} = x+3$

$\pm 2\sqrt{2} = x+3$ Solve for ‘x’

$x = -3 \pm 2\sqrt{2}$

x-coord of vertex



Intercept form Quadratic Equation

$$y = a(x-p)(x-q) \qquad y = (x+4)(x-2)$$

The y-value of an x-intercept always equals ___Zero___

$$0 = (x+4)(x-2)$$

$$0 = A * B$$

Zero Product Property: either

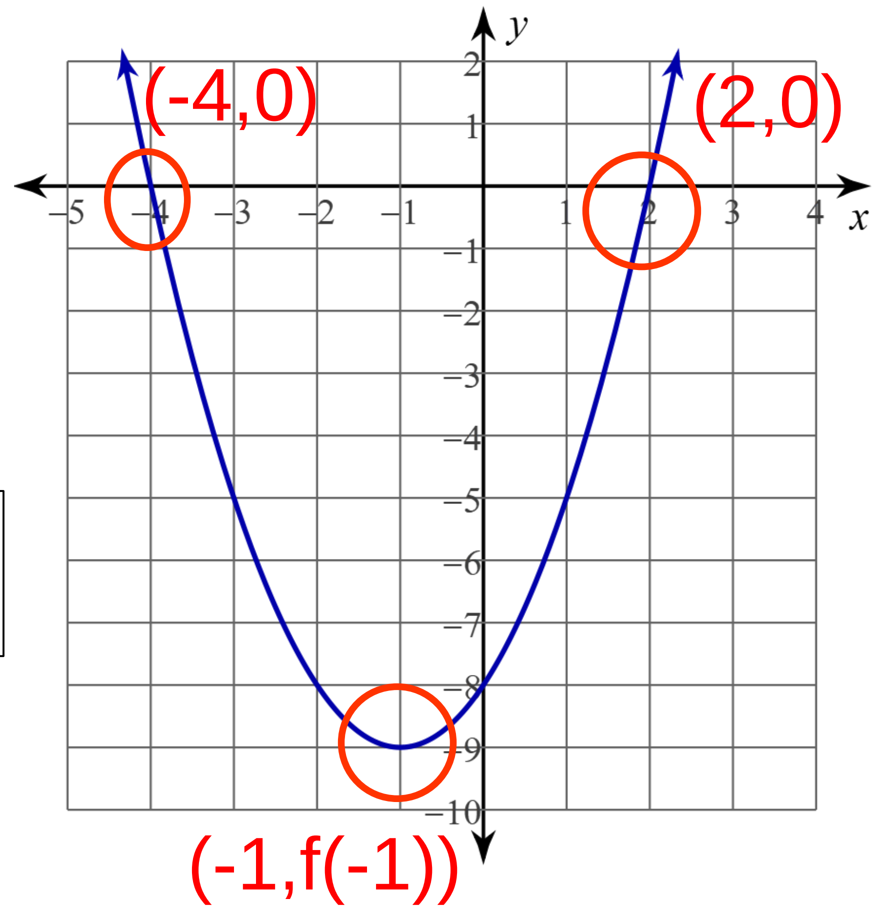
(x + 4) = 0 or (x – 2) = 0

x + 4 = 0 x – 2 = 0

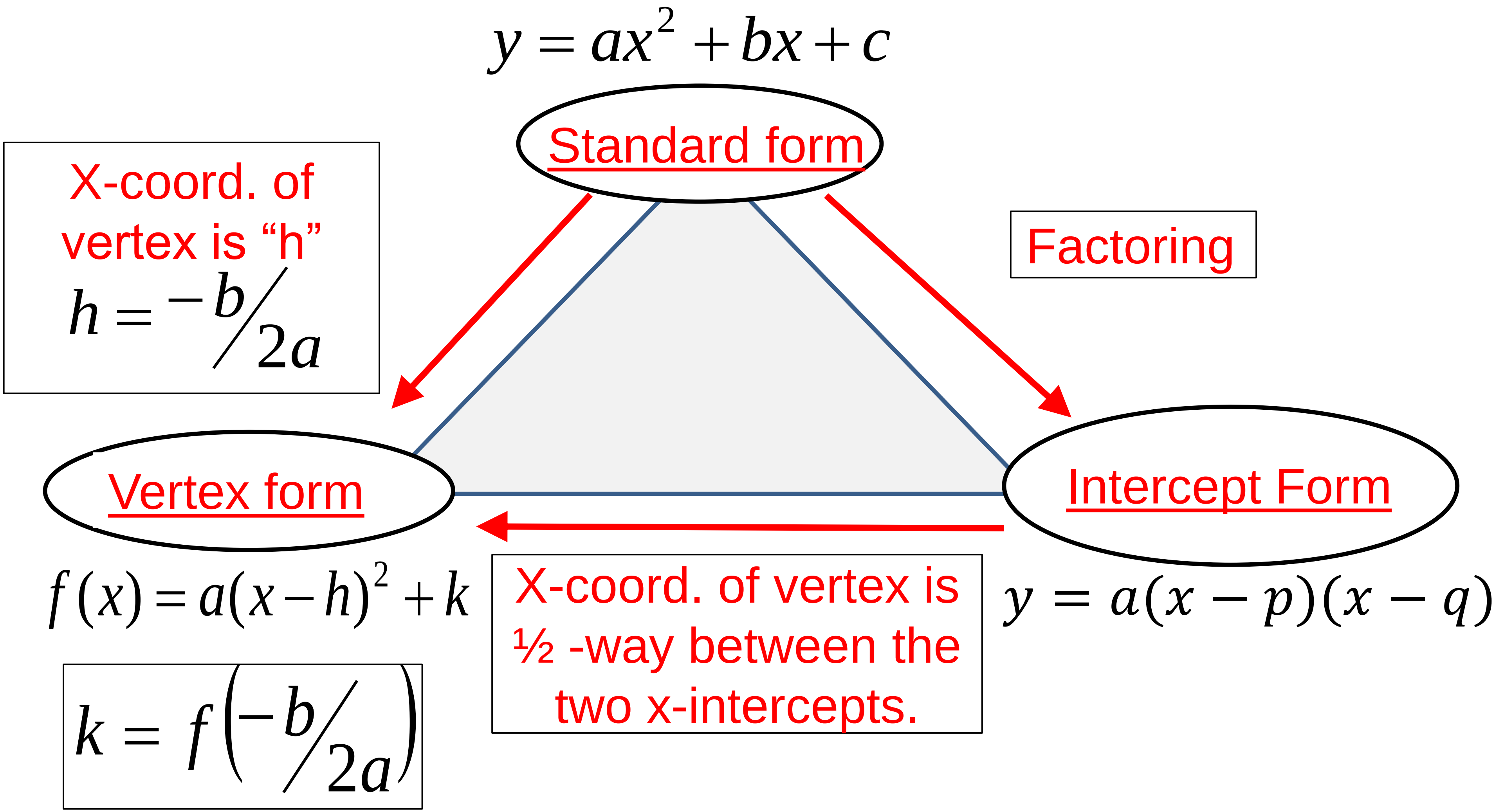
x = -4 x = +2

X-coord. of vertex is ½ -way between the two x-intercepts.

$$x_{vertex} = \frac{-4+2}{2} = -1$$



Forms of the Quadratic Equation



Section 4.3 Focuses on the *Standard Form Quadratic*

Remember the Quadratic Formula?

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

I like this version.

What is the purpose of the formula?

The 'x' in the formula are the x-intercepts of the *standard form equation.* $y = ax^2 + bx + c$

$$x_{vertex} = \frac{-b}{2a}$$

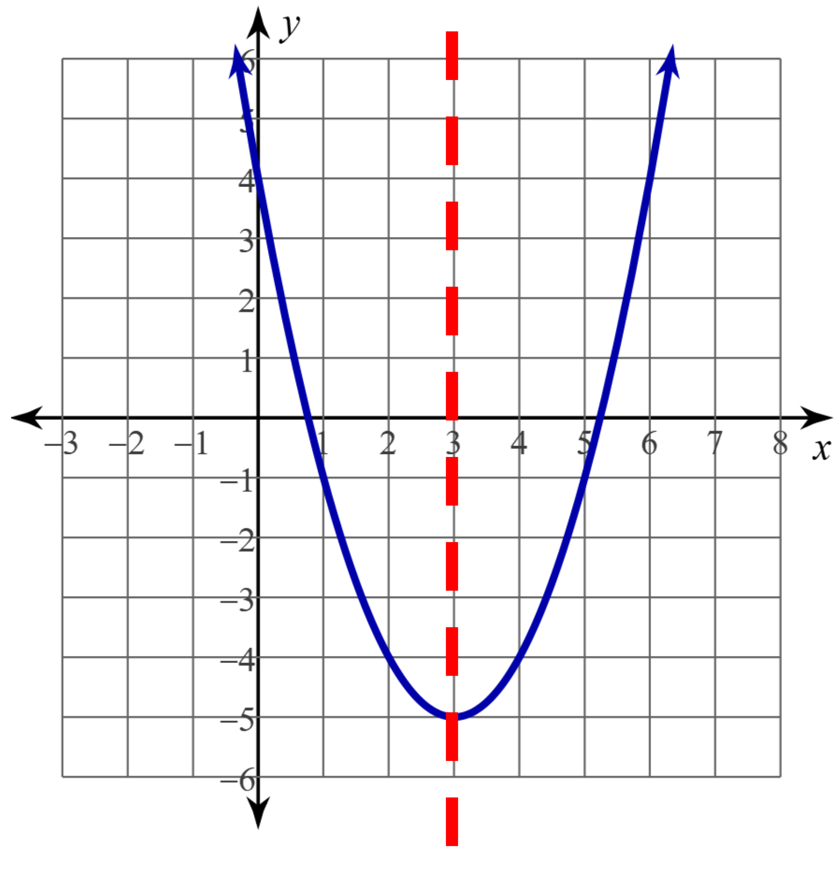
$$y_{vertex} = f\left(\frac{-b}{2a}\right)$$

What is the maximum/minimum function value?

If: a > 0 → parabola opens upward → y_{vertex} is a maximum function value.

Axis if Symmetry: The vertical line divides the parabola in to mirror images.

$$y = x^2 - 6x + 4$$



On the next slide we use the quadratic formula to find x-intercepts (of the standard form quadratic equation).

$$x_{axis\ of\ sym.} = {}^{-b}/_{2a}$$

$$y = ax^2 + bx + c$$

$$y = x^2 - 6x + 4$$

a = 1 b = -6 c = 4

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(\quad)}{2(\quad)} \pm \frac{\sqrt{(\quad)^2 - [4(\quad)(\quad)]}}{2(\quad)}$$

$$x = \frac{-(-6)}{2(1)} \pm \frac{\sqrt{(-6)^2 - [4(1)(4)]}}{2(1)}$$

$$x = \frac{6}{2} \pm \frac{\sqrt{36-16}}{2}$$

$$x = 3 \pm \frac{\sqrt{20}}{2}$$

$$x = 3 \pm \frac{\sqrt{4}\sqrt{5}}{2}$$

$$x = 3 \pm \frac{2\sqrt{5}}{2}$$

$$x = 3 \pm \sqrt{5}$$

If you don't replace *letters with parentheses* then *plug numbers into parentheses* you **WILL** make sign errors.

1) What is the axis of symmetry?

2) What is the vertex?

3) What are the x-intercepts?

$$y = x^2 - 6x + 4$$

$$x_{axis\ of\ sym.} = \frac{-b}{2a} = \frac{-(-6)}{2(1)}$$

$$x_{axis\ of\ sym.} = 3$$

$$x_{vertex} = 3$$

$$y_{vertex} = f\left(\frac{-b}{2a}\right) = (3)^2 - 6(3) + 4$$

$$y_{vertex} = -5$$

$$x_{intercepts} = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = 3 \pm \frac{\sqrt{(-6)^2 - 4(1)(4)}}{2(1)}$$

$$= 3 \pm \frac{\sqrt{36 - 16)}}{2} = 3 \pm \frac{\sqrt{20}}{2} = 3 \pm \frac{\sqrt{4}\sqrt{5}}{2}$$

$$= 3 \pm \sqrt{5}$$

What is the vertex form equation? $y = a(x-h)^2 + k$

$y = x^2 - 6x + 4$ $y = ax^2 + bx + c$ $a = VSF = 1$

$$x_{vertex} = \frac{-b}{2a} = \frac{-(-6)}{2(1)}$$

$x_{vertex} = 3$

$$y_{vertex} = f\left(\frac{-b}{2a}\right) = (3)^2 - 6(3) + 4$$

$y_{vertex} = -5$

$$y = a(x-h)^2 + k$$

$$y = (x-3)^2 - 5$$

1) What is the axis of symmetry?

2) What is the vertex?

3) What are the x-intercepts?

$$y = 2x^2 - 4x + 5$$

$$x_{axis\ of\ sym.} = \frac{-b}{2a} = \frac{-(-4)}{2(2)}$$

$$x_{axis\ of\ sym.} = 1$$

$$x_{vertex} = 1$$

$$y_{vertex} = f\left(\frac{-b}{2a}\right) = 2(1)^2 - 4(1) + 5$$

$$y_{vertex} = 3$$

$$x_{intercepts} = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = 1 \pm \frac{\sqrt{(-4)^2 - 4(2)(5)}}{2(2)}$$

$$= 1 \pm \frac{\sqrt{16 - 40)}}{2} = 1 \pm \frac{\sqrt{-24}}{2} = 1 \pm \frac{\sqrt{4}\sqrt{-1}\sqrt{6}}{2}$$

$$= 1 \pm i\sqrt{6}$$

What is the vertex form equation? $y = a(x-h)^2 + k$

$y = 2x^2 - 4x + 5$ $\quad y = ax^2 + bx + c$

$$a = VSF = 2$$

$$x_{vertex} = \frac{-b}{2a} = \frac{-(-4)}{2(2)}$$

$$x_{vertex} = 1$$

$$y_{vertex} = f\left(\frac{-b}{2a}\right) = 2(1)^2 - 4(1) + 5$$

$$y_{vertex} = 3$$

$$y = a(x-h)^2 + k$$

$$y = 2(x-1)^2 + 3$$

What is a *function value*? It is an ***output value*** of the function.

What is a the *maximum function value* for the following:

(1) Graph: It is the ***y-value*** of a point that is a "peak."

(2) x-y pairs or table of values: It is the maximum ***y-value*** in the collection of points.

(3) equation: Depends upon the function family:

$f(x) = -(x+1)^2 + 3$ ***y-value*** of the vertex

$g(x) = -|x| - 5$ ***y-value*** of the vertex

$k(x) = -3\sqrt{x-2} + 1$ ***y-value*** of the endpoint

The *maximum function value* in the ***range*** is the maximum function value.

$\boldsymbol{Range: y = (-\infty, 5]}$

Finding the equation of a parabola given one point and the vertex.

Vertex: (3, 5), point (4, 7)

(1) Start with the "general" vertex form equation.

$$y = a(x-h)^2 + k$$

(2) Substitute the vertex into the equation: $y = a(x-3)^2 + 5$

(3) Substitute the point into the equation: $7 = a(4-3)^2 + 5$

(4) Solve for the vertical stretch factor "a" $7 = a(4-3)^2 + 5$

-2 -2

$2 = a(1)^2$

$a = VSF = 2$

(5) Write the equation.

$$y = 2(x-3)^2 + 5$$

Section 4.4: Quadratic Modeling

*Revenue ('**R**')*: the amount of money obtained from the sale of an item (sale's price) ('***p***').

The revenue depends upon how many items ('***x***') are sold.

Write an equation to relate these two quantities. $R = px$

$$If \quad p\uparrow \quad and \quad x\uparrow \quad then \quad R \uparrow$$

$$If \quad p\downarrow \quad and \quad x\downarrow \quad then \quad R \downarrow$$

$$If \quad p\uparrow \quad and \quad x\downarrow \quad does \quad R\uparrow \quad or \quad R\downarrow \quad ?????$$

$$If \quad p\downarrow and \quad x\uparrow does \quad R\uparrow \quad or \quad R\downarrow \quad ?????$$

The Law of Demand: price ('***p***') and number of items sold ***('x')*** are related.

Texas Instrument Calculators $x = 21{,}000 - 150p$

$$R = px$$

(1) Write revenue as a function of price.

(2) What price gives maximum revenue?

(3) What revenue results from charging the prince found in #2?

(4) How many units “x” will be sold at this price?