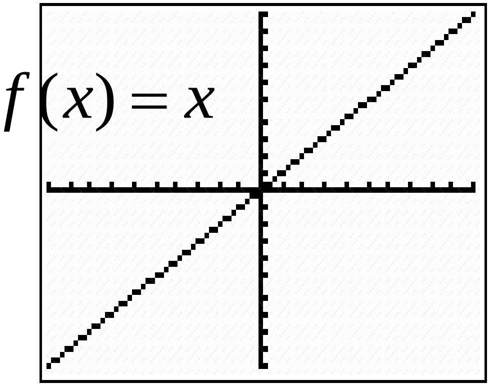
# Math-1050

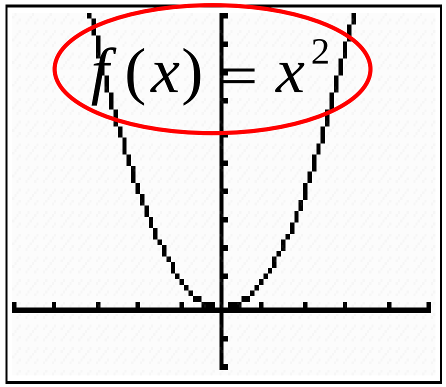
Session 5

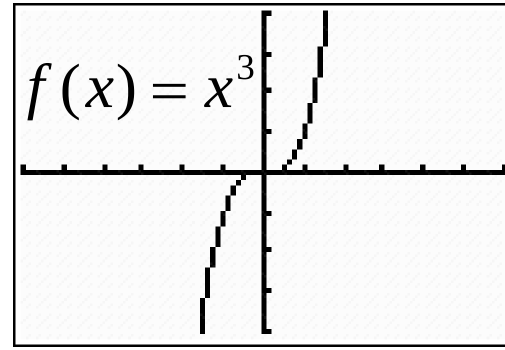
Properties of Functions

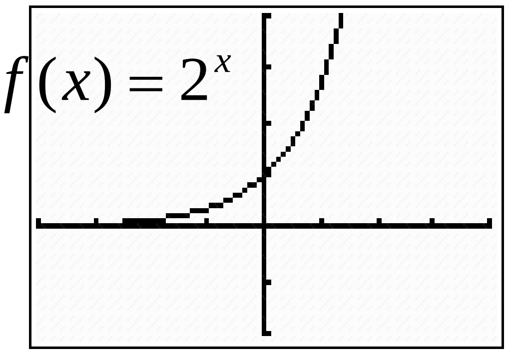
(Textbook Section 3.3)

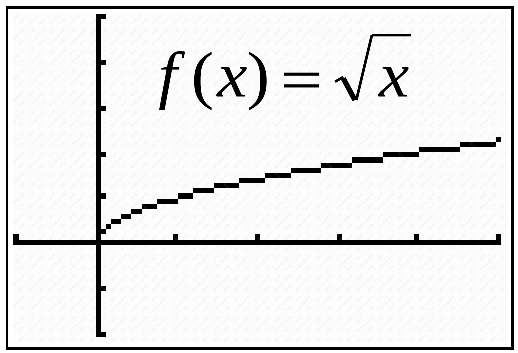
# Which functions are symmetric about the y-axis?

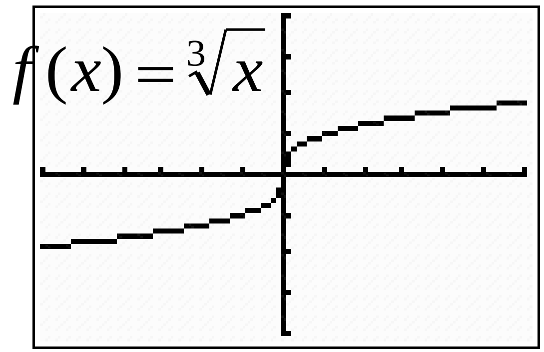


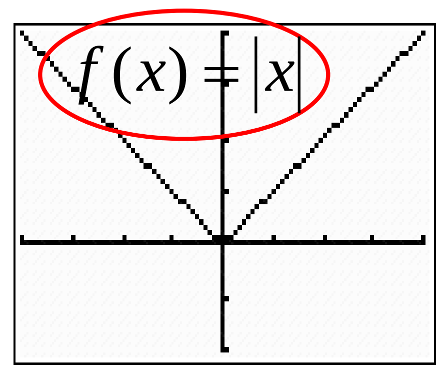



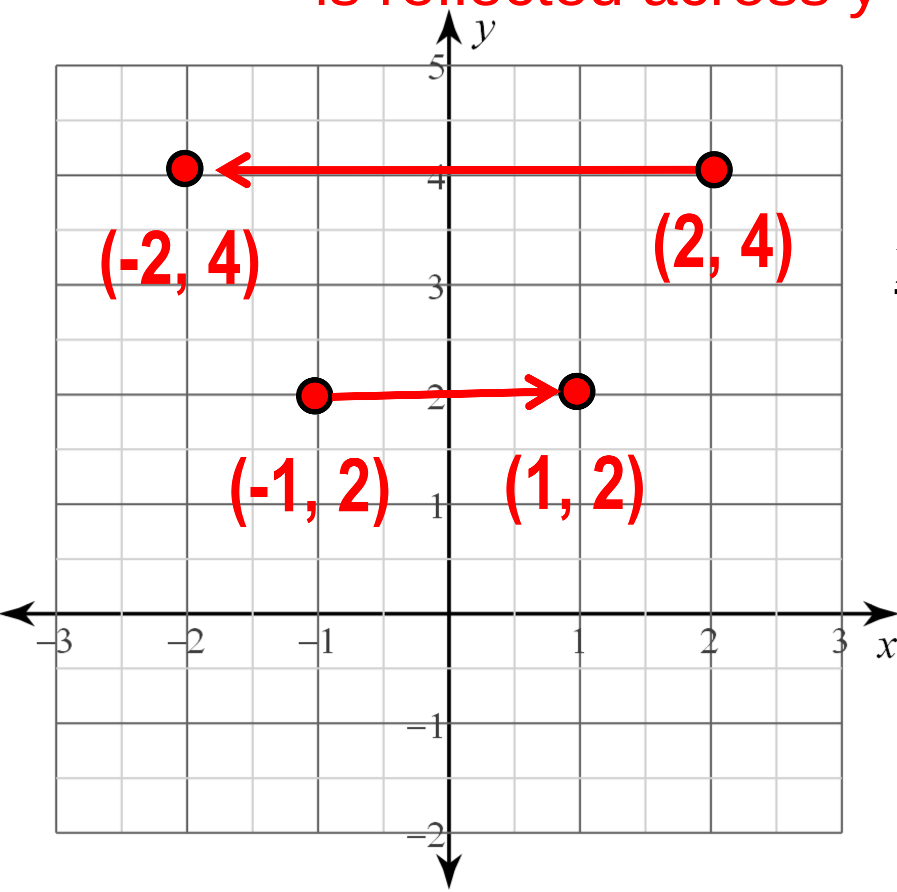

We call functions that are symmetric about the 'y'-axis,
<u>even functions</u>.

There are 6 ways to show a relation between input and output.

(1) x-y pairs

Reflect an x-y pair across the y-axis.

If the x-value of an x-y pair is multiplied by -1 → the x-y pair is reflected across y-axis.




If you have 5 points to reflect across the y-axis, you multiply the x-values of all the points by (-1).

There are 6 ways to show a relation between input and output.

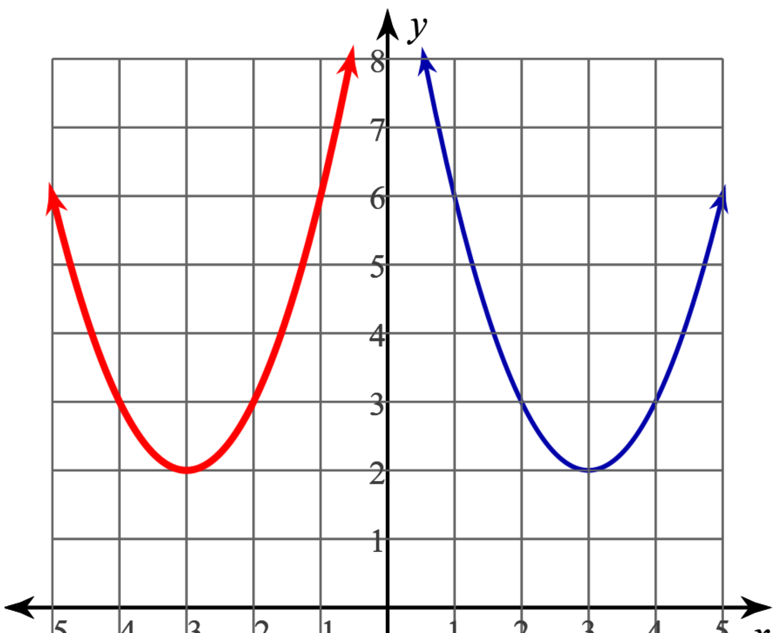
(2) equation

How do you change the equation to reflect the graph across the y-axis?

If all of the x-values of the x-y pairs in the relation are multiplied by (-1) the relation's graph will be reflect across the y-axis.

→ Replace 'x' in the equation with '-x' means you are multiplying every input value by (-1).

$$f(x) = (x-3)^2 + 2$$




$$f(-x) = ((-x)-3)^2 + 2$$

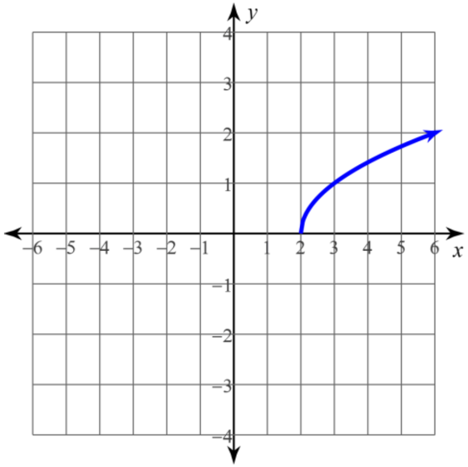
$$f(-x) = (-x-3)^2 + 2$$

$$f(-x) = [(-1)(x+3)]^2 + 2$$

$$f(-x) = (-1)^2(x+3)^2 + 2$$

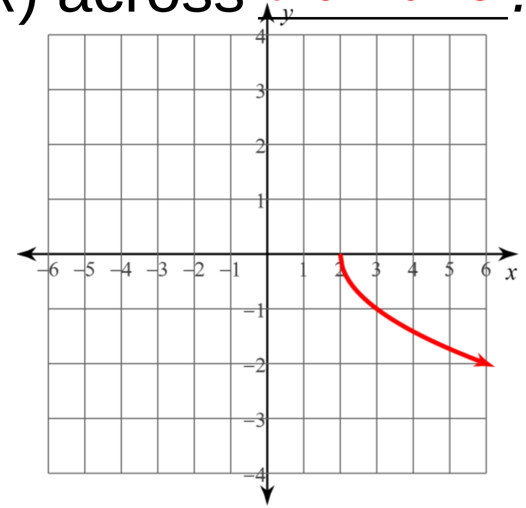
$$f(-x) = (x+3)^2 + 2$$

$$f(x) = \sqrt{x-2}$$



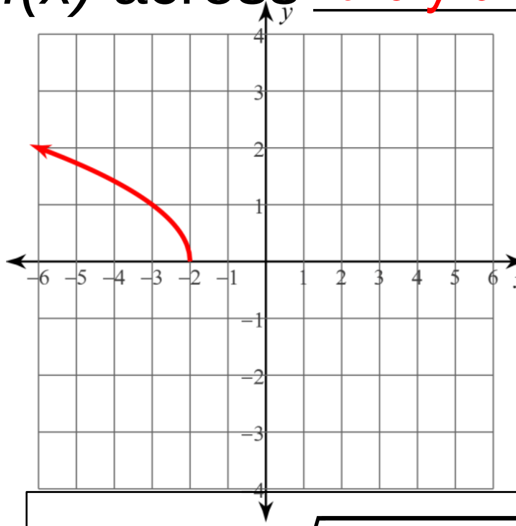
What does the square root equation look like…

*-f(x)* is a reflection of *f(x)* across the x-axis.



$$g(x) = -\sqrt{x-2}$$

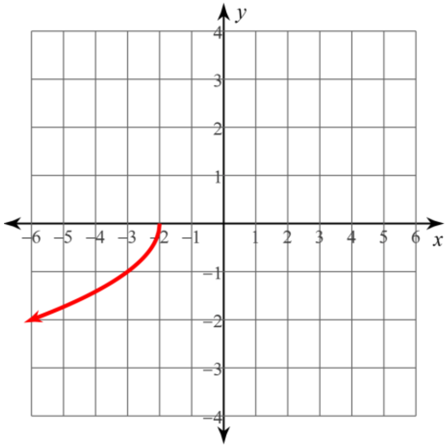
*f(-x)* is a reflection of *f(x)* across the y-axis.



$$g(x) = \sqrt{-x-2}$$

$$g(x) = \sqrt{-(x+2)}$$

*-f(-x)* is a reflection of *f(x)* across the origin.



What does the square root equation look like…

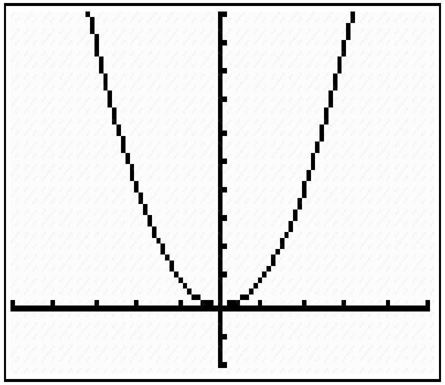
$$g(x) = -\sqrt{-x-2}$$

$$g(x) = -\sqrt{-(x+2)}$$

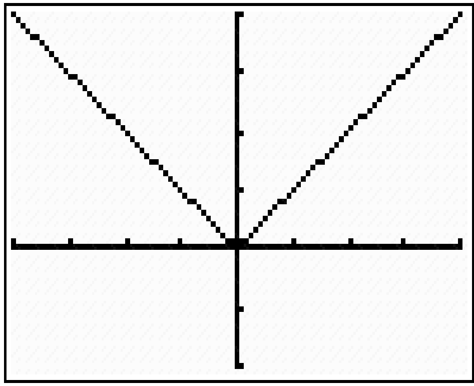
Even Function: if a function is reflected across the y-axis and it looks exactly like the un-reflected version of itself, the function is even.

Mathematically we say, "if $f(x) = f(-x)$ then the function is even. → The graph of *f(x)* looks exactly like the graph of *f(-x)*.

$$f(x) = x^2$$



$$f(-x) = (-x)^2$$
$$f(-x) = (-x)(-x)$$
$$f(-x) = x^2$$
$$f(x) = x^2$$



Since $f(x) = x^2$ and $f(x) = f(-x)$ for this function,

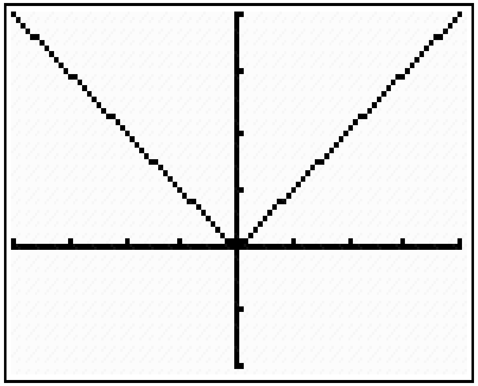
Therefore $f(x) = x^2$ is even.

$$f(x) = |x|$$

Even Function: if a function is reflected across the y-axis and it looks exactly like the un-reflected version of itself, the function is even.

Mathematically we say, “if $f(x) = f(-x)$ then the function is even. → The graph of *f(x)* looks exactly like the graph of *f(-x).*

$$f(x) = |x|$$



$$f(-x) = |-x|$$

$$f(-x) = |(-1) * x|$$

$$f(-x) = |(-1)| * |x|)$$

$$f(-x) = |x|$$

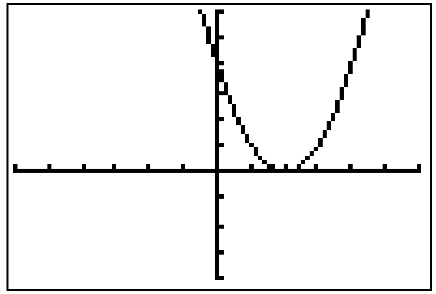
$$f(x) = |x|$$

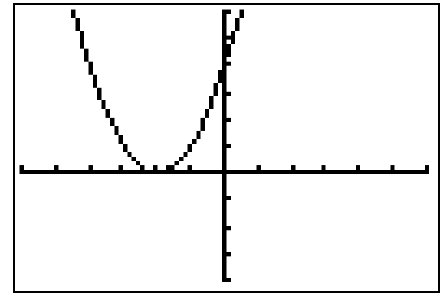
For the function $f(x) = |x|$, $f(x) = f(-x)$

Therefore $f(x) = |x|$ is even.

"Test" the following function to see if it is "even."

$$f(x) = (x-2)^2$$





$$g(x) = f(-x) = ((-x)-2)^2$$

Factor out -1 (still inside the square function)

$$= [(-1)(x+2)]^2$$

Exponent of a Product Property

$$= (-1)^2(x+2)^2$$

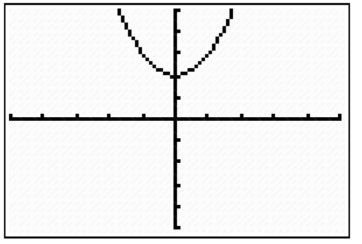
$$= (x+2)^2$$

$$f(-x) \neq f(x)$$

Not even since reflection across y-axis is not the same as the original equation (or the graph)

“Test” the following function to see if it is “even.”

$$f(x) = x^2 + 2$$



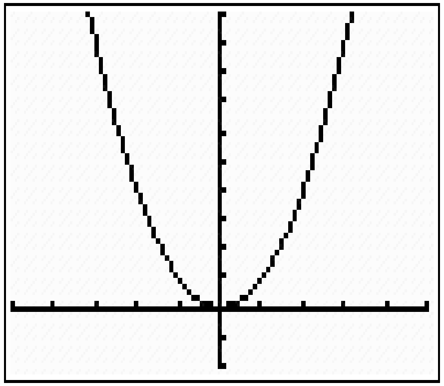
$$f(-x) = (-x)^2 + 2$$

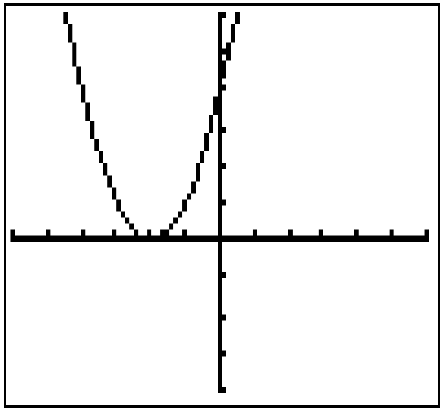
$$f(-x) = x^2 + 2$$

$$f(-x) = f(x)$$

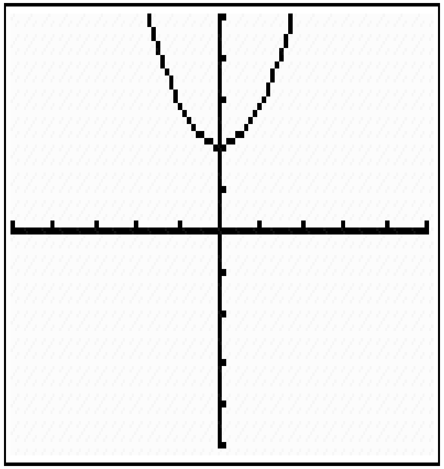
The original equation looks exactly like the equation that has been reflected across the y-axis → symmetric about the y-axis.

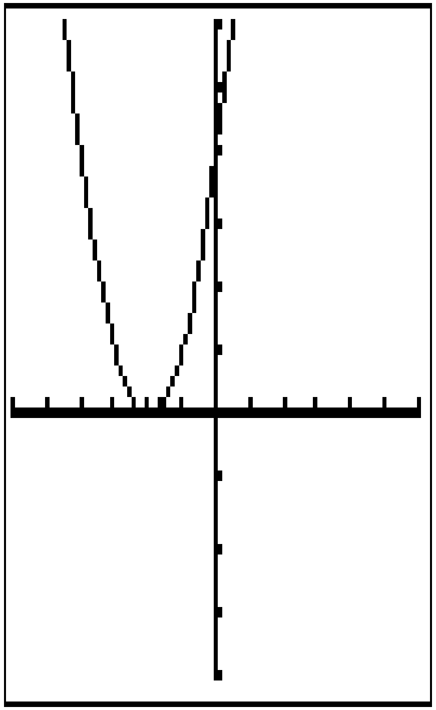
What transformations applied to an even function will cause to no longer be even?



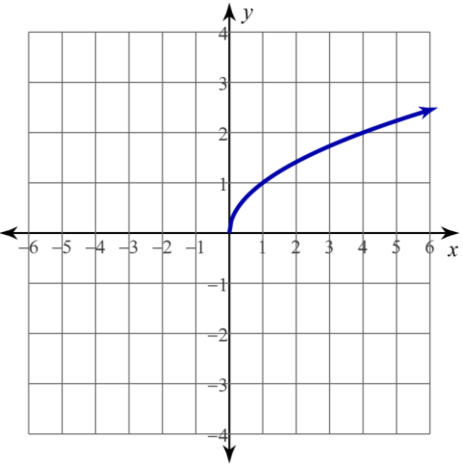


Left or right shifts.



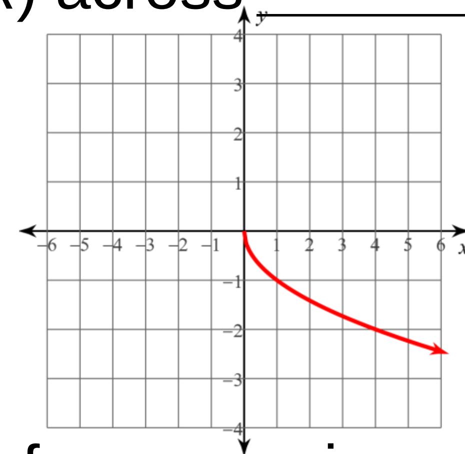


$f(x) = \sqrt{x}$




What is the equation…

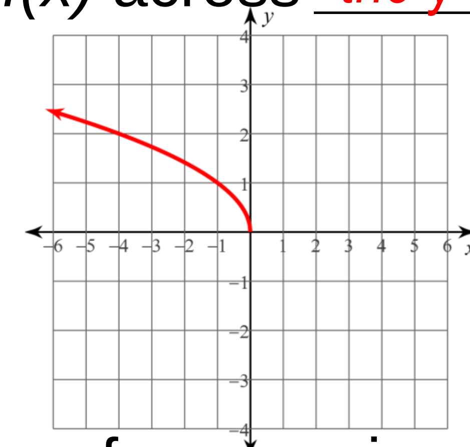
$-f(x)$ is a reflection of $f(x)$ across the x-axis.




for an x-axis reflection?

$g(x) = -\sqrt{x}$

$f(-x)$ is a reflection of $f(x)$ across the y-axis.




for a y-axis reflection?

$g(x) = \sqrt{-x}$

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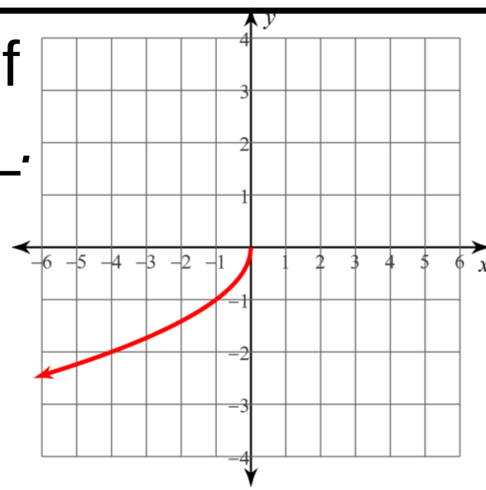
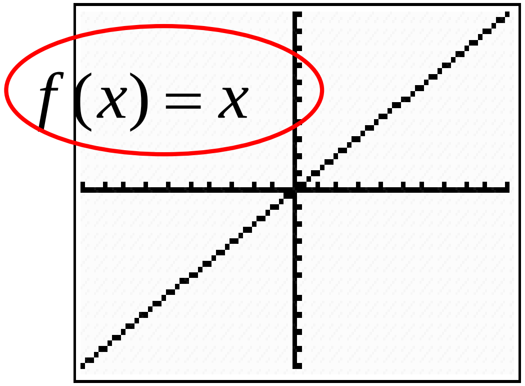
$-f(-x)$ is a reflection of $f(x)$ across the origin.

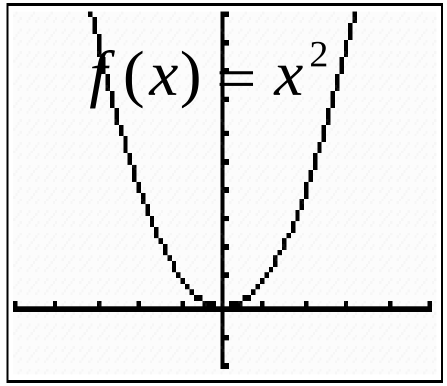


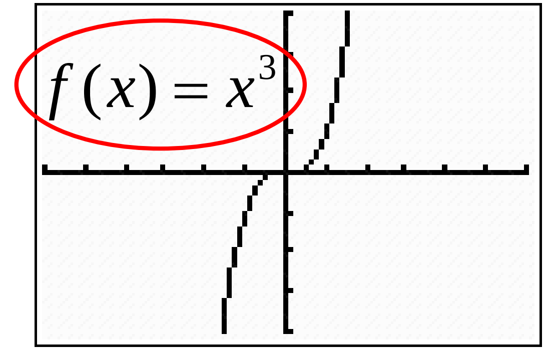

What the equation for a reflection across the origin?

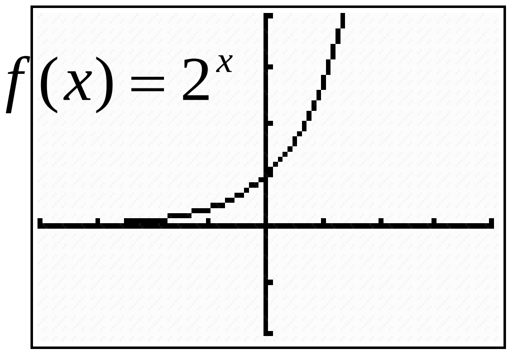
$g(x) = -\sqrt{-x}$

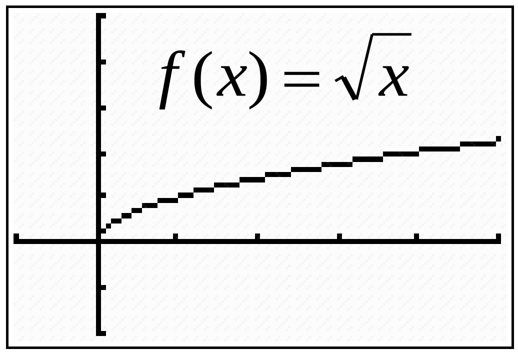
# Which functions are symmetric about the origin?

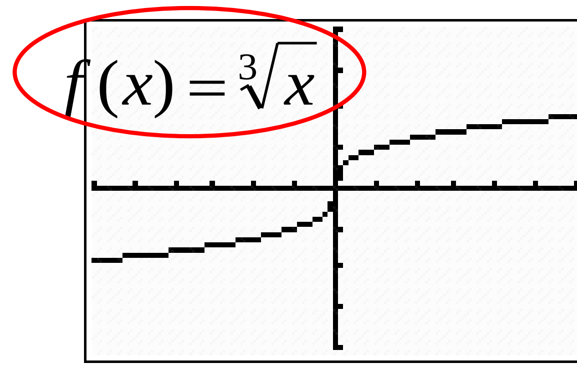


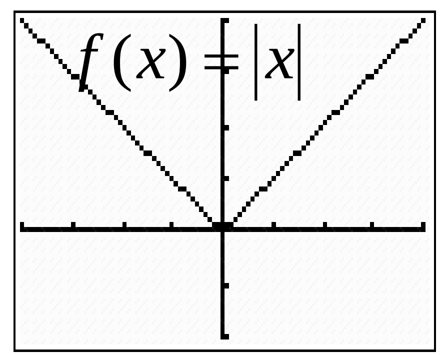



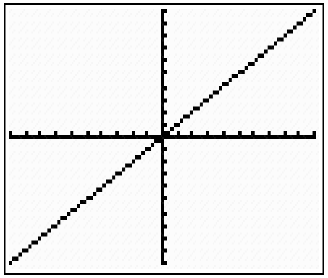




We call functions that are symmetric about the origin, odd functions.

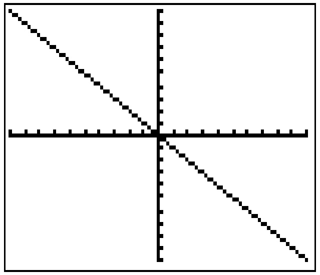
<u>"Odd" functions</u>: When the graph is reflected across the origin, it looks exactly like the original graph. Sequentially reflecting a graph across the x=axis and then the y-axis is a reflection across the origin.



$$f(x) = x$$

$$g(x) = -f(x)$$

x-y pairs:
multiply y-values by (-1)

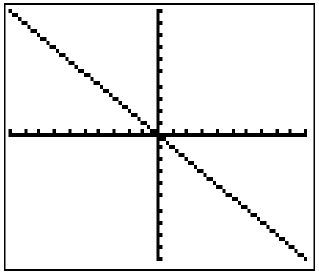


$$g(x) = -x$$

Describe algebraically how the parent function should be transformed in order to reflect it across the y-axis.

$$k(x) = f(-x)$$

x-y pairs:
multiply x-values by (-1)



$$g(x) = (-x) = -x$$

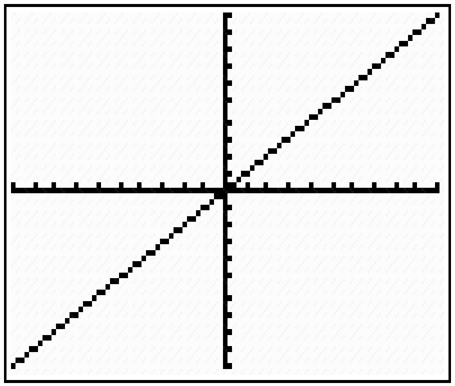
What do you notice about the two reflections?

$$f(-x) = -f(x)$$

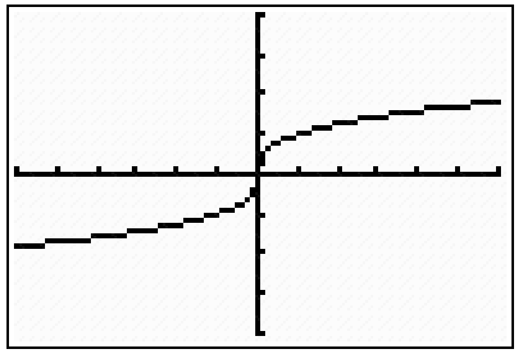
Mathematically we say "if $f(-x) = -f(x)$ then *f(x)* is an <u>odd function.</u>"

→ The graph of -*f(x)* looks exactly like the graph of *f(-x).*

$f(x) = x$



$g(x) = \sqrt[3]{x}$



$g(x) = -\sqrt[3]{x}$

$g(-x) = -g(x)$

*g(x)* is an <u>odd function</u>

$f(-x) = -x$

$-f(x) = -x$

$f(-x) = -f(x)$

*f(x)* is an <u>odd function</u>

$g(-x) = \sqrt[3]{-x}$

$g(-x) = \sqrt[3]{(-1)(x)}$

$g(-x) = \sqrt[3]{(-1)} * \sqrt[3]{x}$

$g(-x) = -\sqrt[3]{x}$

“Test” the following function to see if it is “even.”

$$f(x) = 4(x-2)^3 + 1$$

$$f(-x) = 4((-x)-2)^3 + 1$$

$$= 4(-x-2)^3 + 1$$ Factor out -1 (still inside the parentheses)

$$= 4[(-1)(x-2)]^3 + 1$$ Exponent of a Product Property

$$= 4(-1)^3(x-2)^3 + 1$$

$$f(-x) = -4(x-2)^3 + 1$$

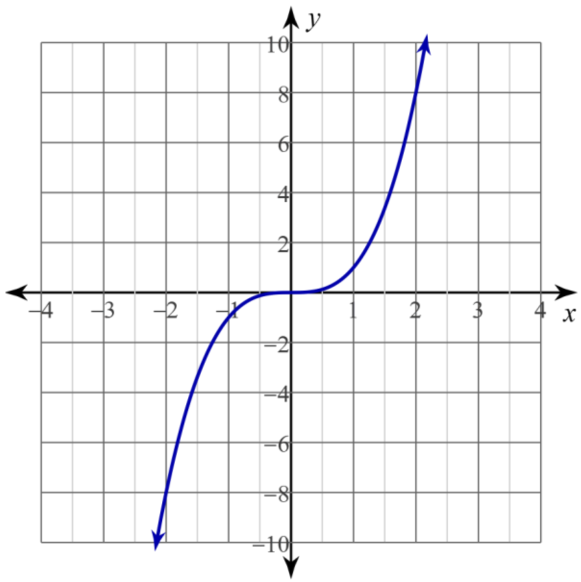
$$-f(x) = (-1)[4(x-2)^3 + 1]$$

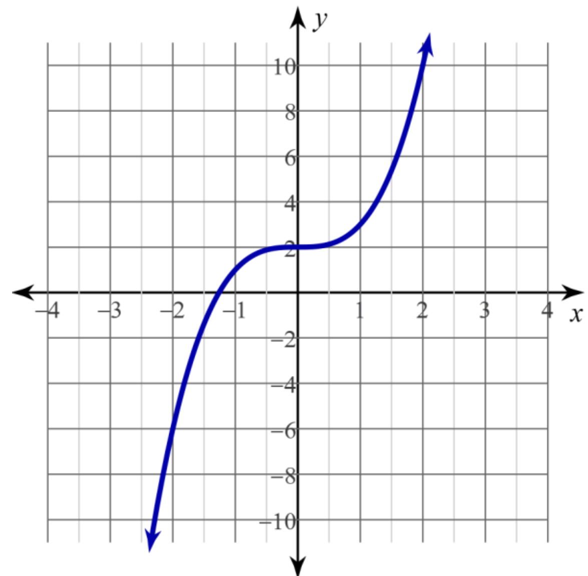
$$f(-x) = -4(x-2)^3 - 1$$

$$f(-x) \neq f(x)$$

Not odd since reflection across y-axis is not the same as a reflection across the x-axis.

# What transformations applied to an odd function will cause to no longer be odd?




Left or right shifts,
Up or down shifts.