Math-1050

Session 4

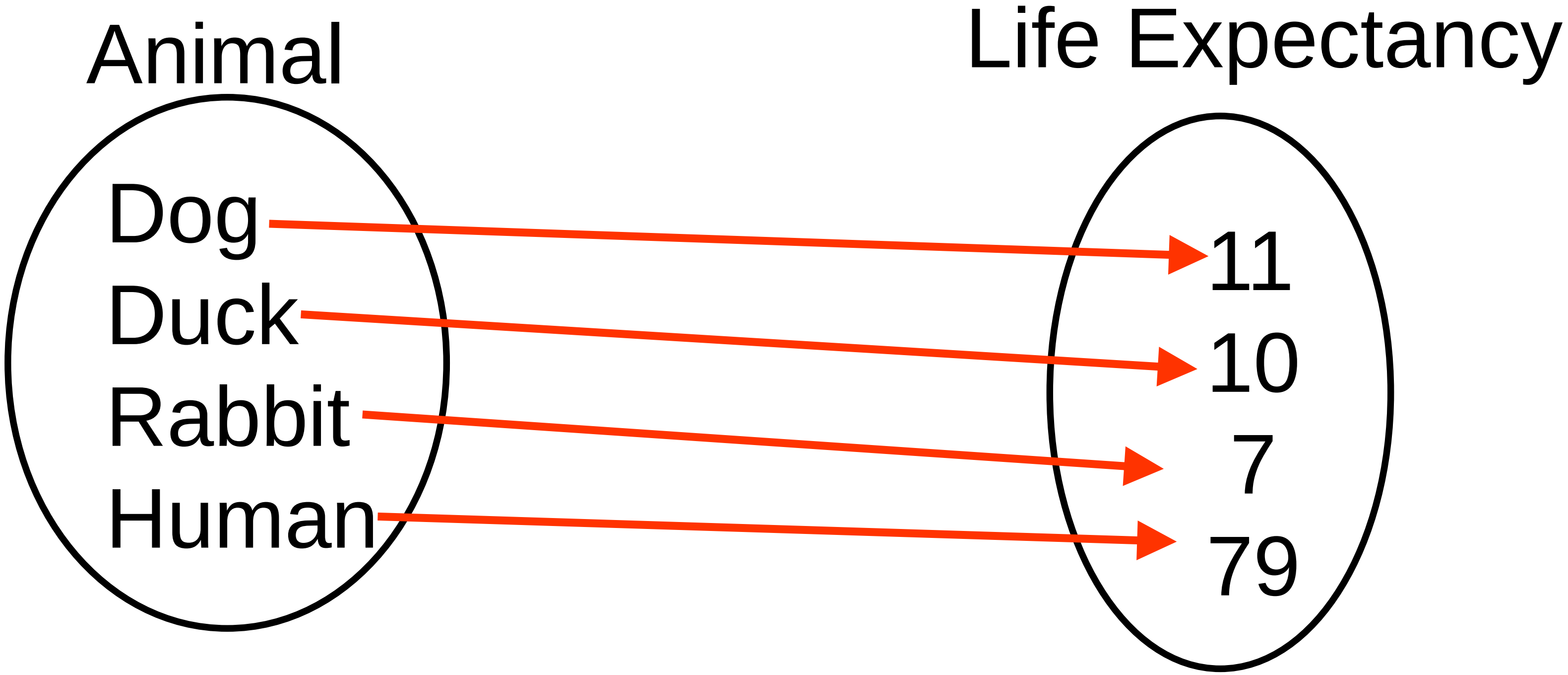
Relations and Functions

(Textbook Sections 3.1 and 3.2)

Relation: A "mapping" or pairing of input values to outputs.

$$x \to y$$

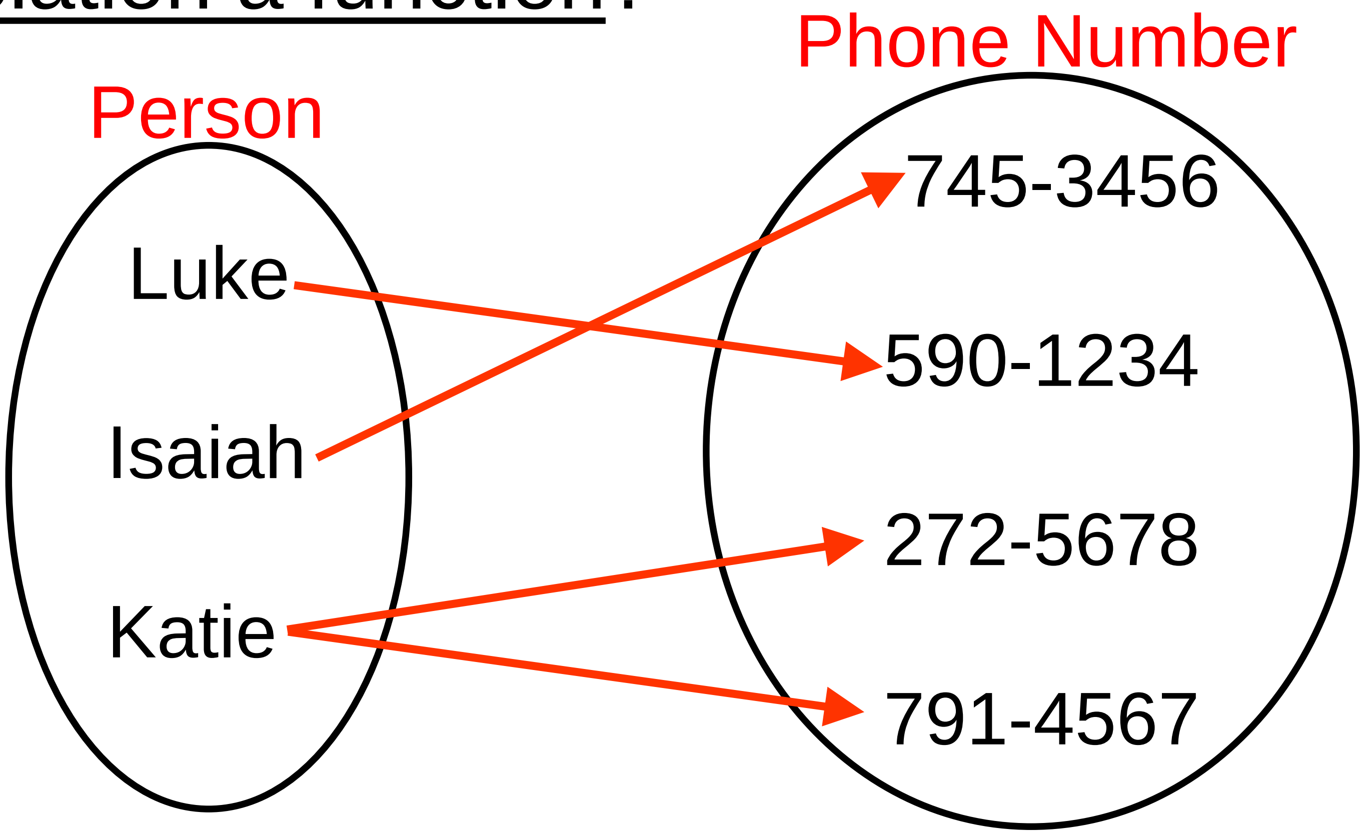
Function: A relation where each input has exactly one output.



Is this a relation or a function?

Both

Is the relation a function?

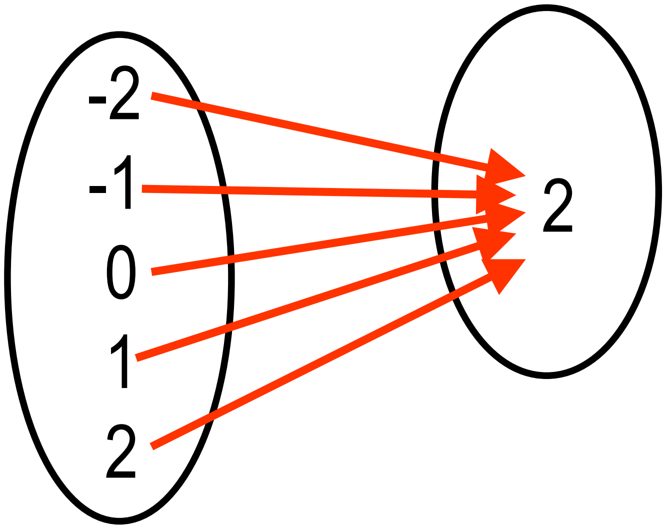


No: Katie has two different phone numbers

What is Luke's phone number?

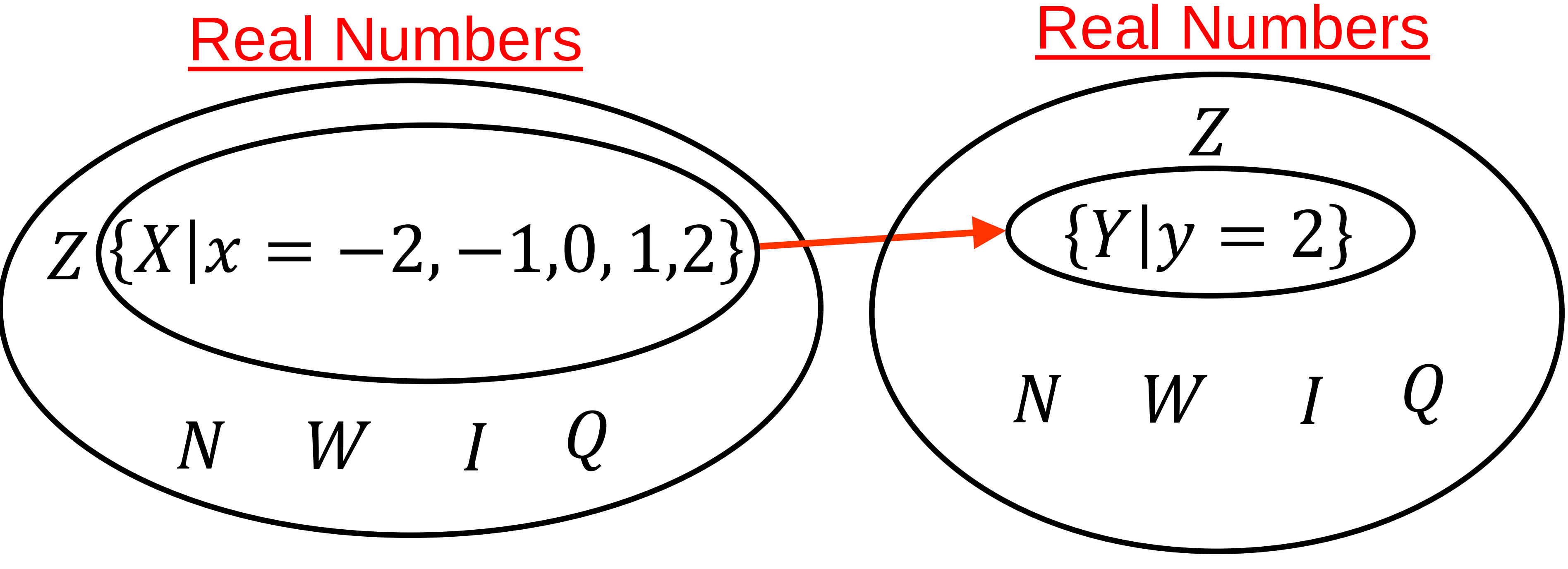
The phone number depends upon whom you are referring to→ Luke is the input, # is the output.

Secondary Math-2 and Math-3: “Mapping”



Conceptually, this is a very simplistic representation. These aren’t the only numbers that exist so it doesn’t really tell the whole story within the context of the entire real number system.

Textbook definition: Let *X* and *Y* be two non-empty sets. A **function** from *X* into *Y* is a relation that associates with each element of *X* exactly one element of *Y*.



The inputs are a sub-set of real numbers.
The output is a sub-set of the real numbers.

Math-3: Domain: the set made up of all of the input values that have corresponding output values.

Math-3: Range: the set made up of all of the corresponding output values.

Textbook: The set *X* is called the **domain** of the function. For each element of ‘x’ in *X*, the corresponding element in ‘y’ in *Y* is called the **value** of the function at ‘x’, or the **image** of ‘x’. The set of all images of the elements in the domain is called the **range** of the function. made up of all of the input values that have corresponding output values.

There are *at least* 6 ways to show a relation between input and output values.

Ordered Pairs: (-2, -1), (-1, 1), (0, 3)

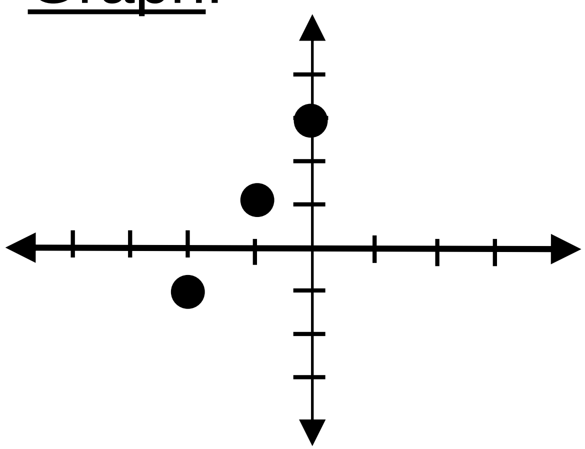
Data table:

x	-2	-1	0
y	-1	1	3

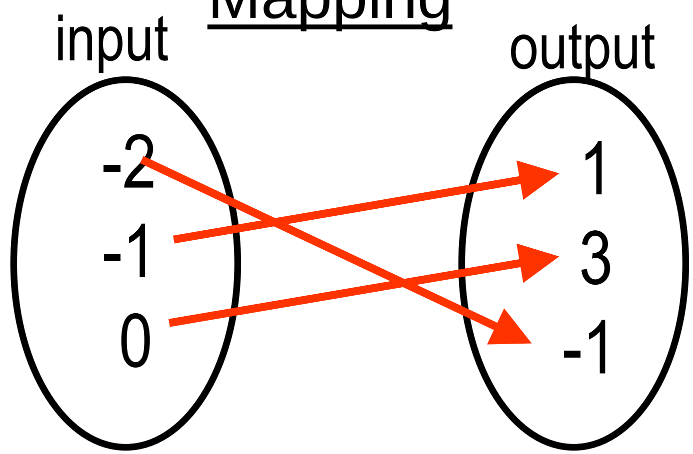
Equation: $y = x + 3$

Function notation: $f(0) = 3$

Graph:



Mapping



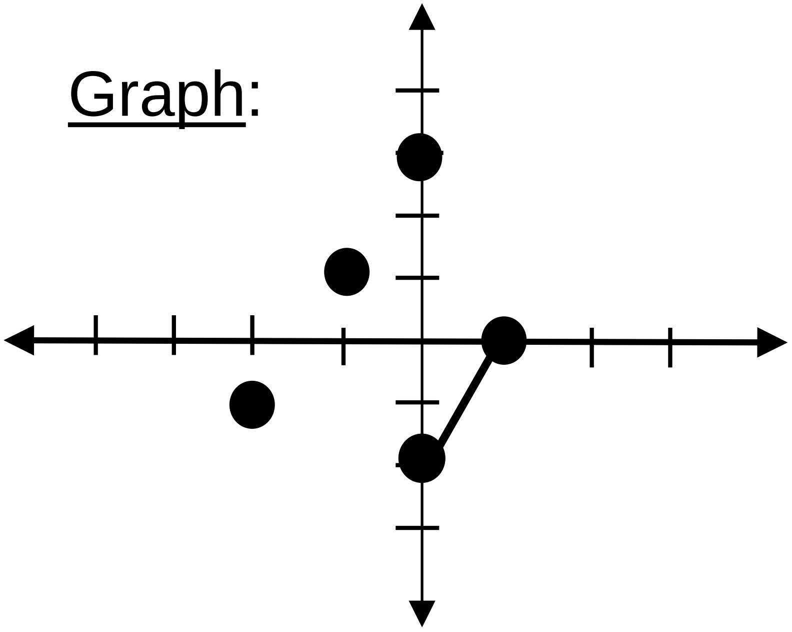
Which method *does not* specify the domain?

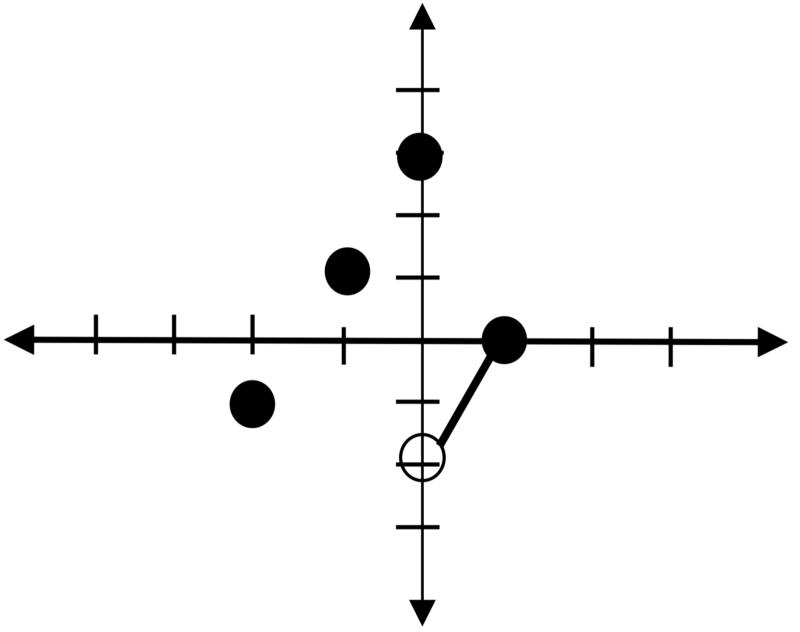
Does the Relation Represent a Function?

Ordered Pairs: (-2, -1), (-1, 1), (0, 3), (-2, 4)

No, input value '-2' has more than one *function value.*

Graph:





Yes.

No, input value '0' has more than one *function value.*

Does the Relation Represent a Function?

Equation: $f(x) = 2x + 1$

$$f(x+1) = 2(x+1) + 1$$

$$f(x+1) = 2x + 3$$

$$f(x+1) = 2x + 1 + 2$$

$$f(x+1) = f(x) + 2$$

If the input increases by '1', the output increases by '2'

$$f(x-1) = 2(x-1) + 1$$

$$f(x-1) = 2x - 1$$

$$f(x+1) = 2x + 1 - 2$$

$$f(x+1) = f(x) - 2$$

If the input decreases by '1', the output decreases by '2'

Each input will only have one function value.

Does the Relation Represent a Function?

Equation: $x^2 + y^2 = 1$ → (0, 1), (0, -1)

$$y^2 = 1 - x^2$$

$$y = \pm\sqrt{1 - x^2}$$

$$g(x) = \pm\sqrt{1 - x^2}$$

$$g(0) = \pm\sqrt{1 - (0)^2}$$

$$g(0) = \pm\sqrt{1}$$

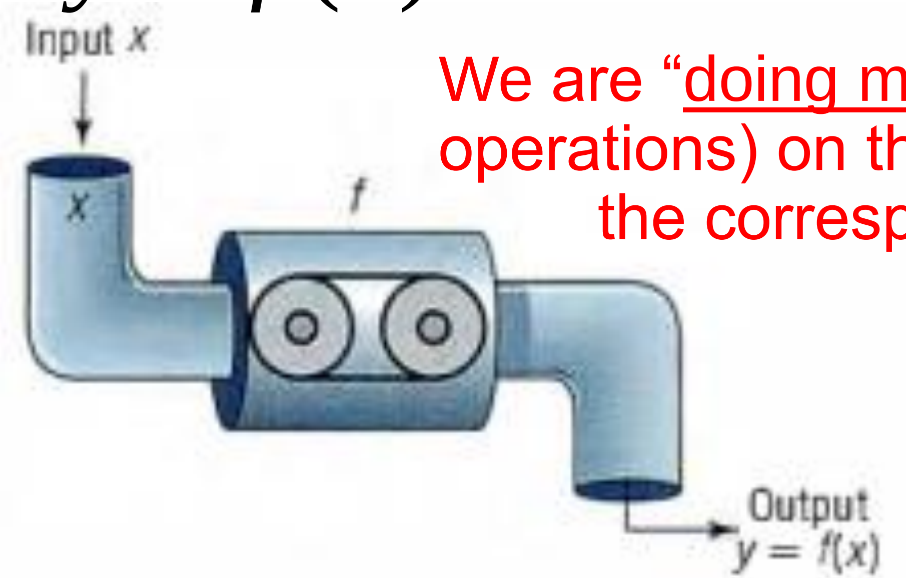
$$g(0) = \pm 1$$

Not a function: *at least* one input value has more than one function value.

$y = f(x)$

Function Notation

When we say "y is a function of x" we mean:



We are "doing math" (performing mathematical operations) on the input value 'x' to determine the corresponding output value 'y'.

Therefore, the output value 'y' is *dependent upon* what the input value 'x' is.

→ 'y' is the dependent variable and 'x' is the independent variable

Which of the following equations is *"'y' is a function of x"?*

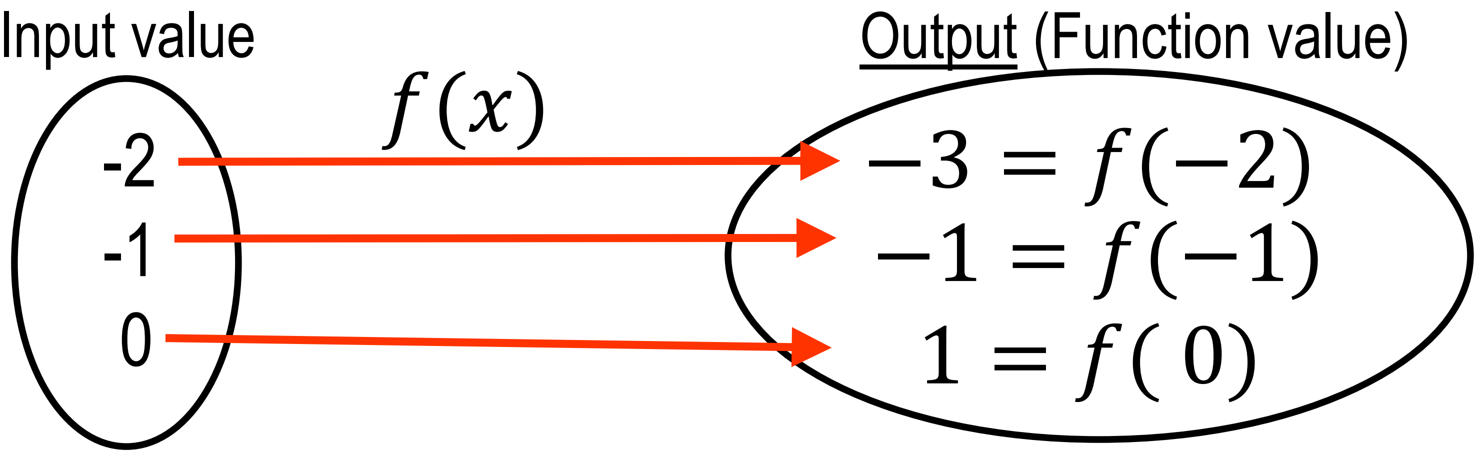
$x = f(y)$ $\quad f(x, y) = a$ $\quad y = f(x)$

$x = \frac{1}{2}y - 3$ $\quad 2x - y = -6$ $\quad y = 2x + 6$

Implicitly Defined Function:

"doing math to both 'x' and 'y'.

Finding a Function Value $f(x) = 2x + 1$



Find the Function Value

$$f(x) + f(3)$$

Using the substitution principle: $(2x + 1) + (2(3) + 1)$

Simplify: $2x + 8$

Therefore: $\boxed{f(x) + f(3) = 2x + 8}$

$$f(x) = 4x^2 + 6x$$

Find: $$\frac{f(x+h) - f(x)}{h}$$

By substitution: $$\frac{[4(x+h)^2 + 6(x+h)] - [4x^2 + 6x]}{h}$$

simplify: $$\frac{[4(x^2 + 2xh + h^2) + 6x + 6h)] - [4x^2 + 6x]}{h}$$

$$\frac{4(x^2 + 2xh + h^2) + 6x + 6h - 4x^2 - 6x}{h}$$

$$\frac{4x^2 + 8xh + 4h^2 + 6x + 6h - 4x^2 - 6x}{h}$$

$$\frac{\cancel{4x^2} + 8xh + 4h^2 + \cancel{6x} + 6h - \cancel{4x^2} - \cancel{6x}}{h}$$

$f(x) = 4x^2 + 6x$

Find: $\dfrac{f(x+h) - f(x)}{h}$

simplify: $\dfrac{\cancel{4x^2} + 8xh + 4h^2 + \cancel{6x} + 6h - \cancel{4x^2} - \cancel{6x}}{h}$

common factor 'h' $\dfrac{8xh + 4h^2 + 6h}{h}$

simplify $\dfrac{\cancel{h}(8x + 4h + 6)}{\cancel{h}}$

$8x + 4h + 6$

Important Facts about Functions

(1) For each ‘x’ in a domain of a function *‘f’*, there is exactly one image *f(x)* in the range; however, an element in the range can result from more than one ‘x’ in the domain.

(2) ‘*f’* is the symbol that we use to denote the function. It is symbolic of the equation that we use to get from an ‘x’ in the domain to *f(x)* in the range.

(3) If $y = f(x)$, then ‘x’ is called the independent variable or ***argument*** of *‘f’*, and ‘y’ is called the dependent variable or the value of *‘f’* (function value) at ‘x’.

Finding the Domain of a Function

An equation in the form *y = f(x)* is the only symbolic form for a relation that *does not* specify what the domain and range are.

Which of the families of functions represented by their *parent functions* do not have the entire real number system ("*all real numbers"*) as their domain?

Linear
$$f(x) = x$$

Square Root
$$f(x) = \sqrt{x}$$

Logarithm
$$f(x) = log_b x$$

Square
$$f(x) = x^2$$

Cube Root
$$f(x) = \sqrt[3]{x}$$

Rational
$$f(x) = \frac{g(x)}{h(x)}$$

Cube
$$f(x) = x^3$$

Exponential
$$f(x) = b^x$$

Square Root

$f(x) = \sqrt{x}$ ← The *radicand* (expression under the radical) *cannot be negative*.

Square roots of negative numbers are *not* in the real number syst.

We say: $x \geq 0$ where 'x' represents the radicand.

Find the Domain of $f(x) = 2 - 5\sqrt{3 - 4x}$

The *radicand cannot be negative*.

$3 - 4x \geq 0$ Solve for 'x'

$3 \geq 4x$

$\frac{3}{4} \geq x$ $x \leq \frac{3}{4}$

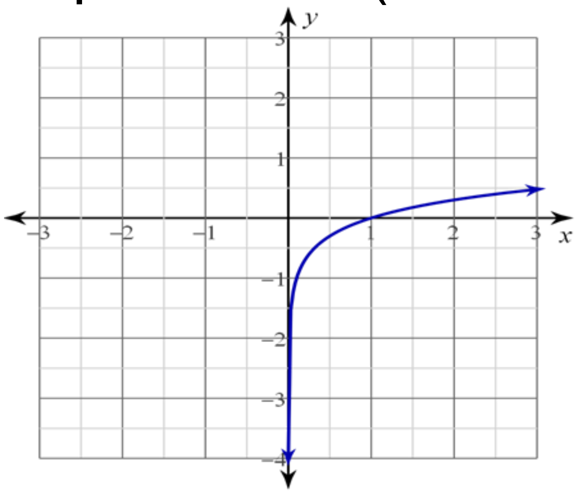
$$D: \left\{ x \middle| x \leq \frac{3}{4} \right\}$$

Logarithm

$$f(x) = log_2(x)$$

The *logarand* (expression to the right of the base '2') *must be positive*.

The graph below shows that when x ≤ 0, there are no corresponding output values (function values, images of 'x' in the range).



$$f(x) = \log x$$

We say: $x > 0$ where 'x' represents the logarand.

Find the Domain of

$$f(x) = 3 - 5\log_2(2x - 7)$$

The logarand must be positive. $2x - 7 > 0$

Solve for 'x' $2x > 7$

$$x > \frac{7}{2}$$

$$D: \left\{x \middle| x > \frac{7}{2}\right\}$$

Rational

$$f(x) = \frac{g(x)}{h(x)}$$

The *denominator cannot be zero*.

Any number divided by '0' will not result in a real number.

Essentially, the result is ∞ (infinity) which is not a number.

We say: $x \neq 0$ where 'x' represents the denominator.

Find the Domain of $f(x) = \dfrac{x^2 - 3x + 5}{x^2 - 9}$

The *denominator cannot be zero*.

$x^2 - 9 \neq 0$ (Property of Inequality is the same as the Property of Equality.)

$x^2 \neq 9$ $\quad x \neq 3, -3$ $\quad D: \{x | x \neq 3, x \neq -3\}$

Combining Functions with Arithmetic Operations

$(f + g)(x) = f(x) + g(x)$ Adding Functions

$(f - g)(x) = f(x) - g(x)$ Subtracting Functions

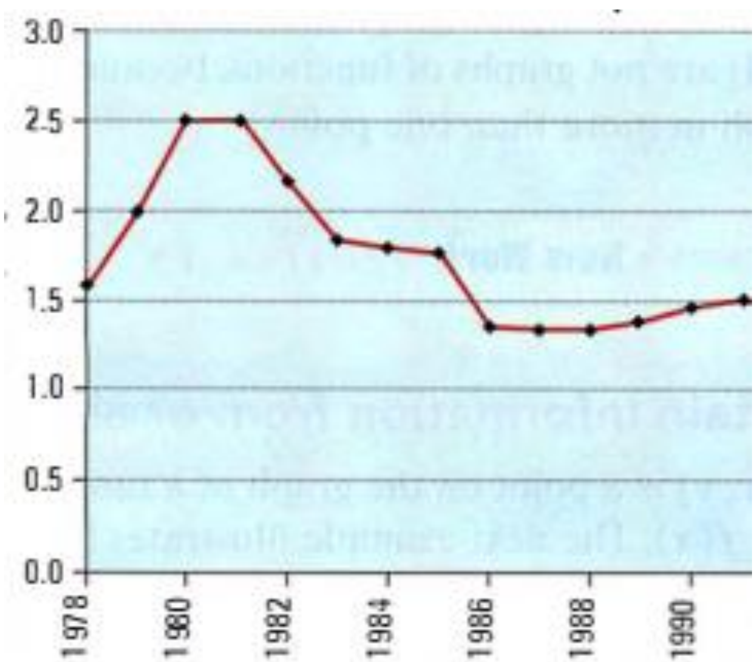
$(f \cdot g)(x) = f(x) * g(x)$ Multiplying Functions

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$ Dividing Functions

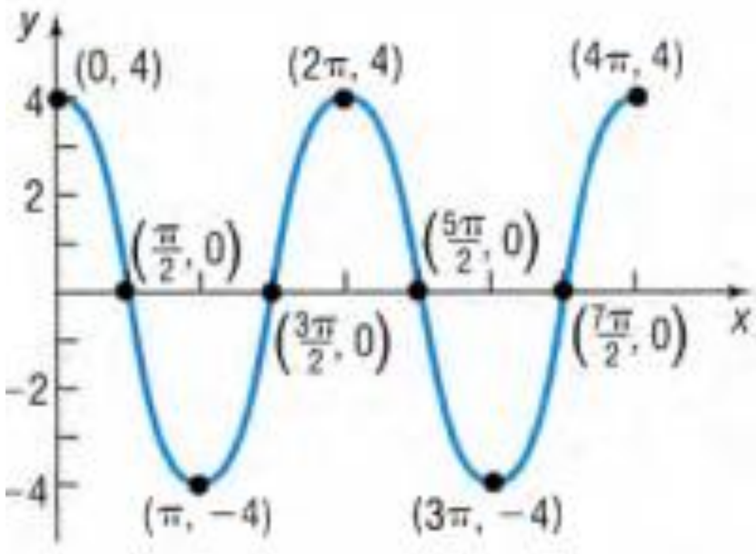
Now we go to textbook section 3.1

Year	Price	Year	Price
1978	1.5651	1985	1.7540
1979	1.9829	1986	1.3459
1980	2.4929	1987	1.3274
1981	2.4977	1988	1.3111
1982	2.1795	1989	1.3589
1983	1.8782	1990	1.4656
1984	1.8310	1991	1.4973

Both forms of the function, represent the same thing: The price of gasoline as a function of calendar year.



Which representation helps you to more easily see the trend in prices?



Function?

Domain = ?

Range = ?

List the intercepts.

$f(0), f\left(\frac{3\pi}{2}\right)$, and $f(3\pi)$?

How many intersections does the function have with the line *y* = *2* ?

For $x = 4, f(x) = ?$

Where is the function positive? → For $f(x) > 0, x = ?$