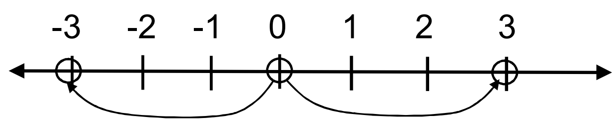
Math -1050: Lesson 1-6 (Absolute Value)

$|x| = 3$ The "*absolute value"* of a number is 3.

Means: "what number***s*** are a distance of *three units* from zero on the number line?"



The "zero" of the "argument" (the expression inside the absolute value symbol) is the "reference number.

$|x| = 3$ $\boxed{x = 3, -3}$

The "solution" (values of 'x' that make the equation "true") are a distance of "3" from the "reference" (or center) number.

The "zero" of the "argument" (the expression inside the absolute value symbol) is the "reference number.

$x - 5 = 0$

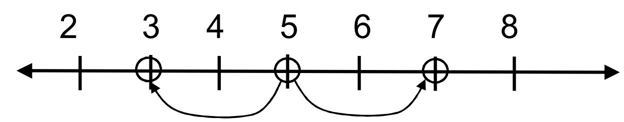
$x_{center} = 5$

$|x - 5| = 2$

$x = 5 \pm 2$

$\boxed{x = 7, 3}$

The "solution" (values of 'x' that make the equation "true") are a distance of "2" from the "reference" (or center) number.



This method of solving the equation explains the underlying reasons why the solution is x = 7, 3.

Another way to solve the equation.

$|x - 5| = 2$

Using the right side of the equation, we know that…

$|2| = 2$ $|-2| = 2$

By the substitution principle, ether:

$x - 5 = 2$ $x - 5 = -2$

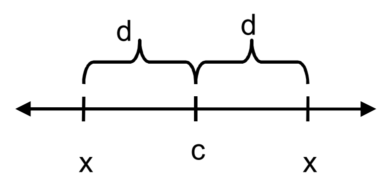
$\boxed{x = 7, 3}$

This method of solving the equation is often the easiest algebraically. It uses deductive logic to arrive at the solution.

Absolute Value: $|x - c| = d$ The number "d" is the distance between "x' and "c" on the number line.

$|x - (c)| = d$

The "zero" of the argument is the "center number" and "d" is the distance from the center number.



We need the "argument" to be a *linear expression* with a *lead coefficient of '1'* (same thing for synthetic division).

$|2x-7|=1$ The coefficient of 'x' is *in the way* making it difficulty to find the "distance"

Product of Absolute Value Expressions Property:

$$|a||b|=|ab|$$

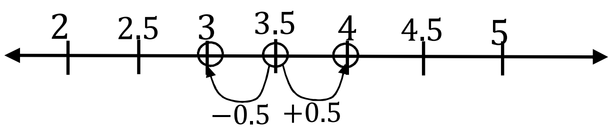
$|2x-7|=1$ Factor out the common factor using the property above.

$|2||x-3.5|=1$ Simplify

$2|x-3.5|=1$ Isolate the absolute value (divide by 2)

$|x-3.5|=0.5$ $x_{center}=3.5$ $distance=0.5$

$x=3.5\pm 0.5$ $x=3,4$



Text Book Theorem (pg. 135)

If a is a positive real number and if u is any algebraic expression, then

$$|u|=a \text{ is equivalent to } u=a \text{ or } u=-a \quad (1)$$

$\underbrace{|x-5|}_{u}=\underbrace{2}_{a}$ $\underbrace{x-5}_{u}=\underbrace{2}_{a}$ $\underbrace{x-5}_{u}=\underbrace{-2}_{-a}$

$x=7,3$

This method of solving the equation just uses a formula. Plug numbers into the formula in order to solve the equation.

It's your choice. Your can solve the equation by knowing "why" or solve the equation by knowing "what" (a formula).

$|x|=-5$ Means: "what numbers are a distance of *negative five* units from zero on the number line?"

What is the solution to the equation?

Is distance ever negative? $|x|=-5$ Has no solution.

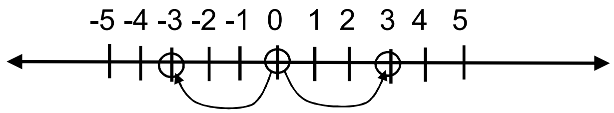
Use either (1) Obtain a linear argument (property)
or (2) the substitution principle
or (3) the textbook theorem

$|2x-1|=5$ $|-5|=5$ $|5|=5$

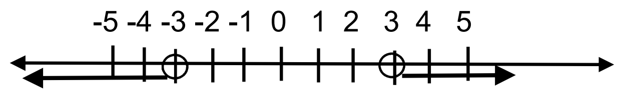
$2x-1=-5$	$2x-1=5$
+1 +1	+1 +1
$2x=-4$	$2x=6$
÷2 ÷2	÷2 ÷2
$x=-2$	$x=3$

$|x| > 3$ What numbers are greater than 3 units away from zero on the number line?

Find the numbers that are exactly 3 way from zero.



Shade all the numbers that are *further away* from 0 than -3 and +3



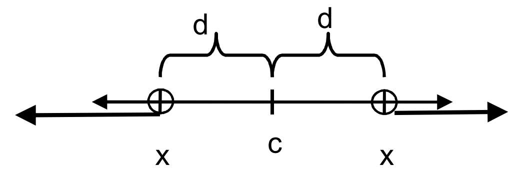
$|x| > 3 \rightarrow \quad x < -3 \;\; OR \;\; x > 3$

$x = (-\infty, -3) \cup (3, \infty)$

Absolute Value: $|x - c| > d$

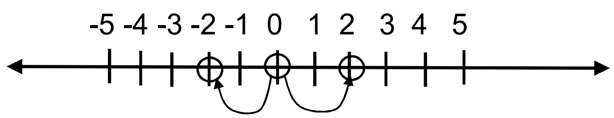
"What numbers are *greater than* "d" units away from the center number "c" on the number line?

$|x - (c)| > d$

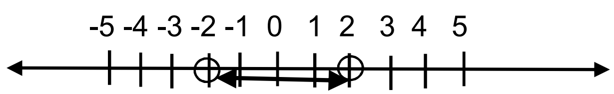


$|x| < 2$ What numbers are *less than* 2 units away from zero on the number line?

Find the numbers that are exactly 2 way from zero.



Shade all the numbers that are *closer* to 0 than -2 and +2



$|x| < 2 \rightarrow \quad x > -2 \;\; AND \;\; x < 2$

$-2 < x < 2$

$x = (-2, 2)$

Absolute Value: $|x - c| < d$

"What numbers are *less than* "d" units away from the center number "c" on the number line?

"c" is the "center number" and the distance from 'c' is less than "d" units