Math-1050

Session #35

Arithmetic Sequences

Your turn: Write a linear equation that $y = mx + b$ contains the ordered pairs in the table below.

$\{a_k\} = \{2k + 3\}$

Slope= $m = \frac{\Delta y}{\Delta x} = 2$

$\Delta x = 1$

$y = 2x + b$

k	1	2	3	...
a_k	5	7	9	...

$\Delta y = 2$

How do you find 'b'?

Method #1: plug in an input—output pair.

$$(5) = 2(1) + b$$

$$3 = b$$

$$y = 2x + 3$$

Method 2: find the "y-intercept" by going leftwards in the table to the "zero-eth" term.

$$\{a_k\} = \{2k+3 : k = 1,2,3,\ldots\}$$

$$y = mx + b \qquad y = 2x + b \qquad y = 2x + 3$$

$\Delta x = 1$

k	0	1	2	3	4	5	6
a_k	3	5	7	9	11	13	15

$\Delta y = 2$

y-intercept = (0, b)

y-intercept = (0, 3)

Your turn: 1. Fill in the table.

2. Write a linear equation that contains the ordered pairs in the table below.

$$\{b_m\} = \{2(m-1)+5\}$$

$$y = mx + b \qquad y = 2x + b \qquad y = 2x + 3$$

$$\Delta x = 1$$

k	0	1	2	3	4	5	6
a_k	3	5	7	9	11	13	15

$$\Delta y = 2$$

$$\text{slope} = \frac{\Delta y}{\Delta x} = 2$$

y-intercept = (0, b)

y-intercept = (0, 3)

Whoa, they're the same sequence!!!

$$\{a_k\} = \{2k+3 : k = 1,2,3,\ldots\}$$

$$\{b_m\} = \{2(m-1)+5 : m = 1,2,3,\ldots\}$$

k	1	2	3	4	5	6
a_k	5	7	9	11	13	15
b_m	5	7	9	11	13	15

Is this true? $2(x-1)+5 = 2x+3$ yes

$$2x-2+5 = 2x+3$$

$$2x+3 = 2x+3$$

Whoa, they're the same sequence!!!

$\{a_k\} = \{2k+3 : k = 1,2,3,\ldots\}$ Emphasizes "0th" term

$\{b_m\} = \{2(m-1)+5 : m = 1,2,3,\ldots\}$ Emphasizes 1st term

x	0	1
a_x	3	
b_x		5

Arithmetic Sequence: a sequence where there is a "constant difference" between each of the adjacent terms.

$$\{a_k\} = \{2k+3 : k = 1,2,3,\ldots\} \qquad y = 2x+3$$

k	1	2	3	4	5	6
a_k	5	7	9	11	13	15

$$a_2 - a_1 = 2 \qquad a_4 - a_3 = 2$$

Arithmetic Sequence: each pair of adjacent terms has the same "common difference"

Note: the "common difference" becomes the slope of the linear equation passing through each ordered pair because the 1st difference of the input is always '1'.

$a_1 = a$

$a_2 = a_1 + d$

$a_3 = a_2 + d$ $a_3 = (a_1 + d) + d$ $a_3 = a_1 + 2d$

$a_4 = a_3 + d$ $a_4 = (a_1 + 2d) + \text{d}$ $a_4 = a_1 + 3d$

Explicitly Defined Arithmetic Sequence $\boxed{a_n = a_1 + d(n-1)}$

$\{a_k\}$ Where $a_k = d(k-1) + a_0$ (and k = 1,2,3,...)

The kth term of the sequence is the common difference times (k-1) plus the first term of the sequence, with k = 1, 2, 3,...

Explicitly define the following sequence of numbers.

n	a_n
1	-9
2	-6
3	-3
4	0
5	3
6	6
7	9
8	12
9	15

-9, -6, -3, 0, 3, 6, 9, 12, 15

$$\{a_n\} = \{3n - 12\}$$

$$\{a_n\} = \{3(n-1) - 9\}$$

Which one is easier?

The second one since you don't have to go "backwards" to find the "zero^th" term.

Recursively Defined Sequences have two parts.

$$b_1 = 4$$

Specifies the initial element of the sequence.

$$b_n = b_{n-1} + 2$$

Gives a way to show the relation between adjacent elements of the sequence.

Recursively Defined Sequences

$b_1 = 4$ $\qquad$ $b_n = b_{n-1} + 2$ $\qquad$ For all n > 1

$b_2 = b_1 + 2$ $\qquad$ $b_3 = b_2 + 2$

Is this an arithmetic sequence?

Yes, because a “common difference” is always added to the previous term to find the next term.

What is the common difference? d = 2

What is explicit formula for the sequence?

$a_k = d(k-1) + a_1$ $\qquad$ $a_k = 2(k-1) + 4$

Two ways to write a formula to define the numbers in a sequence.

Explicitly Defined Sequence

Recursively Defined Sequence

These two methods can be used for all sequences, not just arithmetic sequences.

Recursively Defined Arithmetic Sequence:

$$b_1 = number \qquad b_n = b_{n-1} + d$$

For all n > 1

d = common difference

Explicitly Defined Arithmetic Sequence:

$$b_n = d(n-1) + b_1$$

For all n ≥ 1

Find the recursive formula.

-9, -6, -3, 0, 3, 6, 9, 12, 15

$b_1 = -9$ $\qquad b_n = b_{n-1} + 3$ $\qquad$ For $1 < n \leq 9$

Find the Explicit formula

$$b_n = \{3(n - 1) - 9 : \text{for n} = 1,2,3,\ldots,9\}$$

$$a_4 = 3 \qquad a_{20} = 35$$

a) Find the common difference

b) Find the 1st term

c) Find the Recursive formula

d) Find the Explicit formula

$$a_n = d(n-1) + a_1$$

$$a_{20} = d(20-1) + a_1$$

$$a_4 = d(4-1) + a_1$$

$$\boxed{35 = d(19) + a_1}$$

$$\boxed{3 = d(3) + a_1}$$

Two equations, 2 unknowns → solve the system of equations

$$\left.\begin{aligned} 3 - d(3) &= a_1 \\ 35 - d(19) &= a_1 \end{aligned}\right\}$$

$$3 - 3d = 35 - 18d$$

$$15d = 30 \qquad \boxed{d = 2}$$

Substitution → $\boxed{3 = d(3) + a_1}$

$$3 = (2)(3) + a_1$$

$$\boxed{a_1 = -3}$$

$$a_4 = 3 \quad a_{20} = 35$$

a) Find the common difference

b) Find the 1st term

c) Find the Recursive formula

d) Find the Explicit formula

$$a_n = d(n-1) + a_1$$

$$\boxed{d = 2} \quad \boxed{a_1 = -3}$$

<u>Recursive formula:</u> $a_1 = -3 \quad a_n = a_{n-1} + 2$

<u>Explicit formula:</u> $a_n = 2(n-1) - 3$

Finding the Sum of the 1st "n" terms of an Arithmetic Sequence

$$Sum_n = S_n = a_1 + a_2 + a_3 + \cdots + a_n$$

$$\sum_{k=1}^{n} a_k = \sum_{k=1}^{n} [d(n-1) + a_1]$$

$$= \frac{n}{2}[2a_1 + d(n-1)]$$

$$= \frac{n}{2}[a_1 + a_n]$$

Find the Sum of the 1st "n" terms of the Arithmetic Sequence

$$\{a_n\} = \{3n - 12\}$$

$$\sum_{k=1}^{n} a_k = \boxed{= \frac{n}{2}[a_1 + a_n]} = \frac{n}{2}[-9 + (3n - 12)]$$

$$= \frac{n}{2}(3n - 21)$$