Math-1050

Session #34

Sequences

What is a <u>sequence</u>?

<u>Sequence</u>: an ordered progression of numbers (a list)

<u>Finite sequence</u>: a sequence that has a final term, therefore the total terms in the sequence can be counted.

<u>example</u> 5, 10, 15, 20, 25, 30, 35

<u>Infinite sequence</u>: sequence that does not have a final term, so the number of terms is uncountable.

<u>Example</u>: 2, 4, 6, 8, 10, ...

5, 10, 15,…, 35 number pattern continues between 15 and 35

2, 5, 8,… number pattern continues after ‘8’ infinitely

How is a <u>sequence</u> a <u>relation</u> between <u>input</u> and <u>output</u>?

5, 10, 15, 20, 25

What are the input values?

The domain of a sequence is the "natural numbers"

k	a_k
1	-9
2	-6
3	-3
4	0
5	3
6	6
7	9
8	12
9	15

The input value refers to the relative position of the term in the sequence (1^{st}, 2^{nd}, 3^{rd}, etc.)

The range is the sequence itself (the set of all the individual numbers of the sequence).

a_k Represents a "generic" number in a sequence named "a" which has subscript "k" to identify the "kth" term of the sequence.

$a_3 = 5$ No French brackets and subscript '3' means the 3rd term of sequence 'a'.

Is the sequence given in the table finite or infinite?

finite

n	1	2	…	5
b_n	2	4	…	10

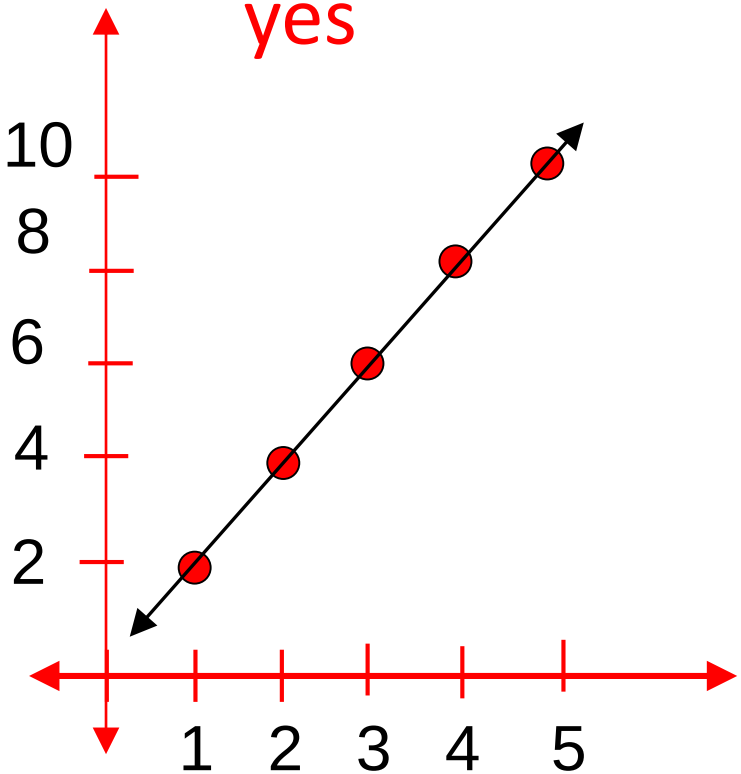
$$b_2 = ?$$

$$b_2 = 4$$

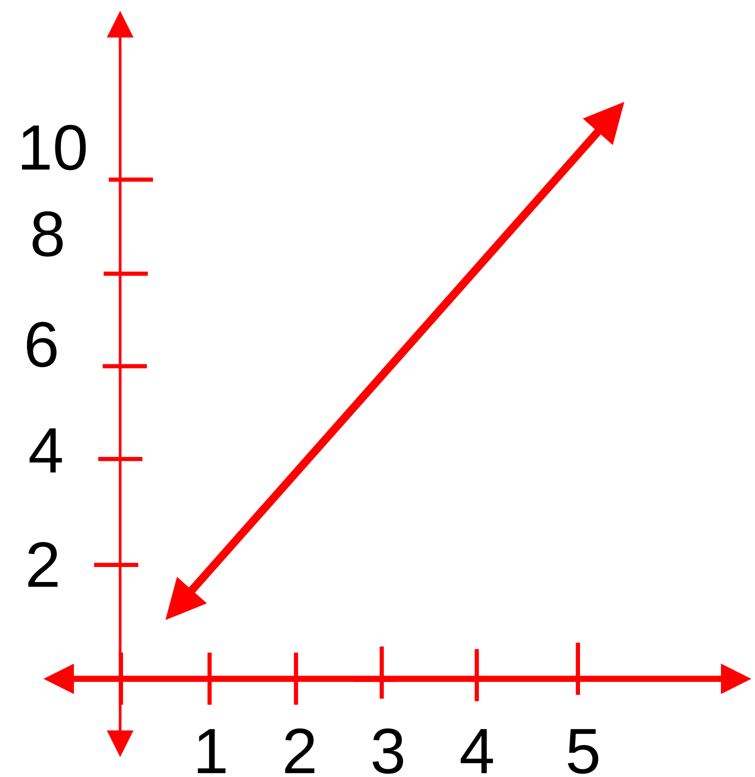
$\{b_n\} = \{2, 4, 6, \ldots\}$ Is a sequence a relation? yes

Graph the relation.

Is this sequence a linear relation? yes



Is this sequence a line?



<u>domain</u> = ? Discrete domain

D = {x = 1, 2, 3, 4, 5}

<u>Domain</u> = ?

D = (-∞,∞)

continuous domain

Discrete set of numbers: a countable set of numbers

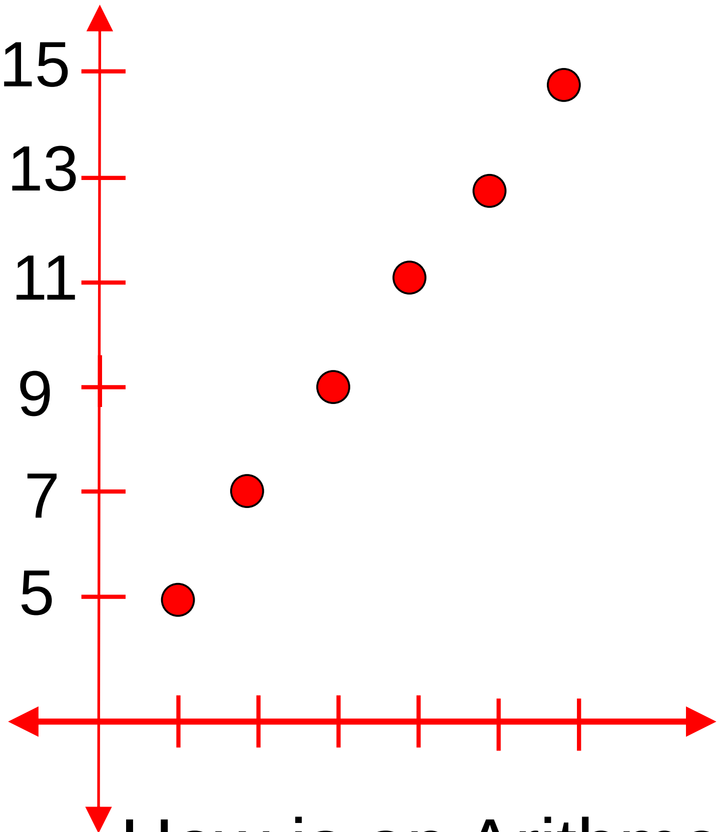
Example: {1, 2, 3, 4, 5}

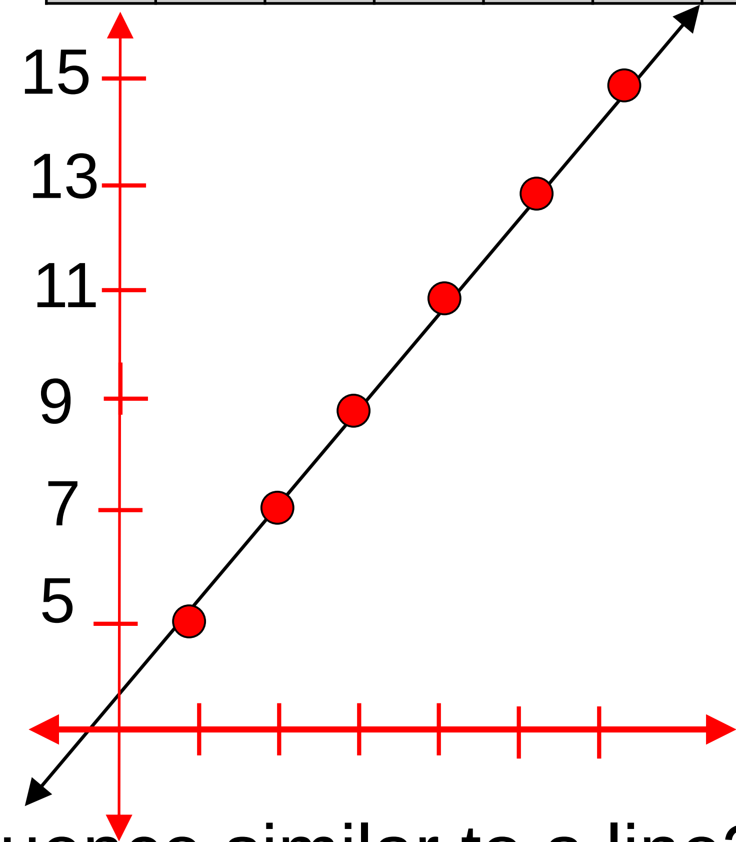
Continuous set of numbers: an infinite set of numbers that includes every number in an interval of numbers (with no "gaps"). Example (-5, 3]

Arithmetic Sequence: a sequence that is a linear relation.

$$\{a_k\} = \{5, 7, 9, 11, 13, 15\}$$

k	1	2	3	4	5	6
a_k	5	7	9	11	13	15





How is an Arithmetic sequence similar to a line?

Constant slope (or 1st difference is constant)

How is an Arithmetic sequence different from a line?

Discrete domain vs. continuous domain

$$\{a_k\} = \{2k+3 : k = 1,2,3,...\}$$

Defines a “rule” so that you can find the “kth” term

We call this method of defining the sequence “set-builder” notation.

Spoken: “the sequence ‘a’ is defined as 2k +3 with ‘k’ taking on the values 1, 2, 3, and so forth”

Explicitly Defined Sequence: a formula that you can use to directly find any term in the sequence without having to find the preceding terms.

$$a_1 = (-1)^{1+1}\left(\frac{2}{1}\right)$$

$$a_1 = 2$$

$$\{a_k\} = \left\{(-1)^{k+1}\left(\frac{2}{k}\right)\right\}$$

$$a_2 = (-1)^{1+2}\left(\frac{2}{2}\right)$$

Write down the 1st six terms of the following sequence.

k	1	2	3	4	5	6
a_k	2	-1	$\frac{2}{3}$	$-\frac{1}{2}$	$\frac{2}{5}$	$-\frac{1}{3}$

Examples Sequences

Running total: if your car payment is $200/month, a running total would look like:

Month	1	2	3	4
Total ($)	200	400	600	800

1. Define this sequence explicitly and name it "C".

$$C_m = 200m \qquad (m = 1,2,3,4)$$

Recursively Defined Sequences have two parts.

$$b_1 = 4$$

Specifies the initial element of the sequence.

$$b_n = b_{n-1} + 2$$

Gives a way to show the relation between adjacent elements of the sequence.

Recursively Defined Sequences

$b_1 = 4$ $\qquad b_n = b_{n-1} + 2$

Find the 1st 5-terms of the sequence.

$b_2 = b_1 + 2$ $\qquad b_3 = b_2 + 2$

In order to find the 3rd term, we need to find out what the 2nd term is:

$b_2 = b_1 + 2$ $\qquad b_2 = 4 + 2 = 6$ $\qquad b_3 = 6 + 2 = 8$

4, 6, 8, 10, 12,…

Define the following sequence of numbers recursively.

-9, -6, -3, 0, 3, 6, 9, 12, 15

$b_1 = -9$ $\quad b_n = b_{n-1} + 3$ $\quad$ For $1 < n \leq 9$

Explicitly defined:

$$b_n = \{3(n-1) - 9 : \text{for n} = 1,2,3,\ldots,9\}$$

$$\{a_k\} = \{\ 1, 3, 5, 7, 9\ \} \qquad \sum_{k=1}^{5} a_k = 1 + 3 + 5 + 7 + 9$$

If we add together all the terms of the sequence we get something called a ***series.***

$$\sum_{k=1}^{5} a_k = 26$$

Summation Notation uses Greek letter (capital) sigma

Carl Friedrich Gauss (1777 – 1855)

As a young student (7-10 years old) his class was asked to add the first 100 numbers,

1 + 2 + 3 + …+ 98 + 99 + 100 = ?

You have one minute → what's the answer?

Karl immediately gave the answer: 5050.

How did he do it so fast?

1	+	2	+	3	+ …+	48	+	49	+	50
100	+	99	+	98	+ …	53	+	52	+	51
101		101		101		101		101		101

= 50(101) = 5050

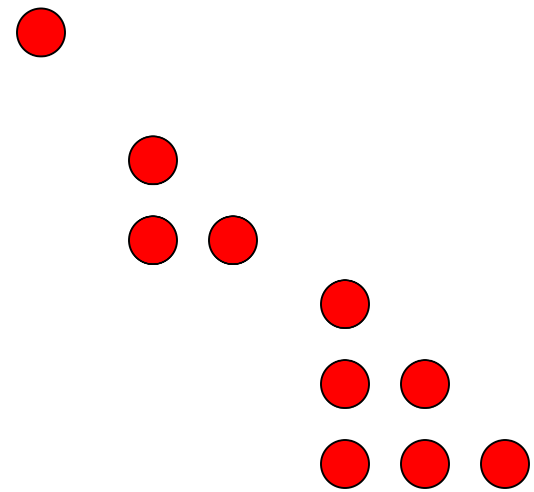
Triangular Numbers

Why are they called "triangular numbers"?

1

1 + 2 = 3

1 + 2 + 3 = 6

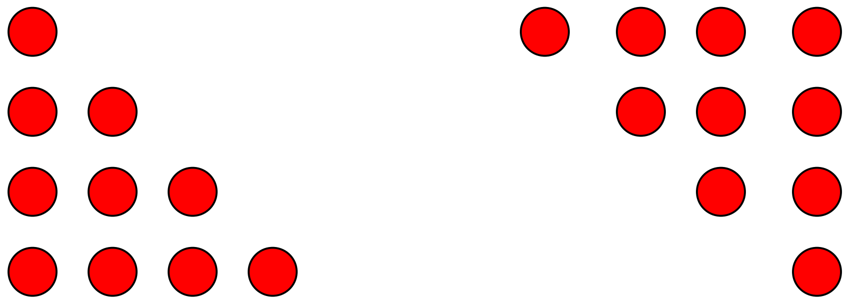


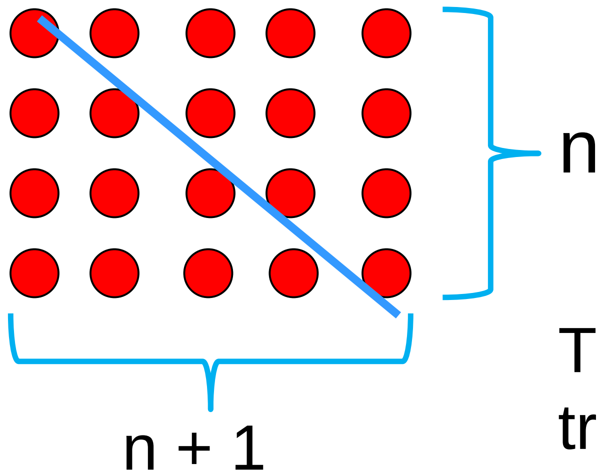
How can we calculate the 100th triangular number?

Triangular Numbers

$$T_N = \frac{N(N+1)}{2}$$

$$T_{100} = \frac{100(100+1)}{2} = 5050$$





Total number in the rectangle = n(n + 1)

The rectangle is one trianglular number added to itself.

This is Gauss's number!

The sum of the first ‘n’ integers is a triangular number.

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

(Where ‘c’ is a *constant*)

$$\sum_{k=1}^{n} c = \underbrace{c + c + c + c + c + \cdots + c}_{n}$$

$$\sum_{k=1}^{n} c = cn$$

The sum of the first ‘n’ “perfect square integers”

$$\sum_{k=1}^{n} k^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2 = 1 + 4 + 9 + \cdots + n^2$$

$$= \frac{n(n+1)(2n+1)}{6}$$

The sum of the first ‘n’ “perfect square integers”

$$\sum_{k=1}^{n} k^3 = 1^3 + 2^3 + 3^3 + \cdots + n^3 = 1 + 8 + 27 + \cdots + n^3$$

$$= \left(\frac{n(n+1)}{2}\right)^2$$

Properties of Series

#1 $$\sum_{k=1}^{n}(ca_k) = ca_1 + ca_2 + \cdots + ca_n = c\sum_{k=1}^{n} a_k$$

#2 $$\sum_{k=1}^{n}(a_k + b_k) = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k$$

#3 $$\sum_{k=1}^{n}(a_k - b_k) = \sum_{k=1}^{n} a_k - \sum_{k=1}^{n} b_k$$

#4 $$\sum_{k=j+1}^{n} a_k = \sum_{k=1}^{n} a_k - \sum_{k=1}^{j} b_k$$ Where: $0 < j < n$

Using Properties of Series

$$\sum_{k=1}^{6}(2k) = 2\sum_{k=1}^{6} a_k = \frac{2(\text{n})(\text{n}+1)}{2} = \frac{2(6)(6+1)}{2}$$

#1

$= 42$

$$\sum_{k=1}^{7}(k^2+3k+2) = \sum_{k=1}^{7} k^2 + \sum_{k=1}^{7} 3k+2$$

#2

#1

$$= \sum_{k=1}^{7} k^2 + 3\sum_{k=1}^{7} k + 2$$

$$= \frac{n(n+1)(2n+1)}{6} + \frac{3(n)(n+1)}{2} + 2\text{n}$$

$$= \frac{7(7+1)[2(7)+1]}{6} + \frac{3(7)(7+1)}{2} + 2(7)$$

$= 5138$

Using Properties of Series

$$\sum_{k=4}^{10}(2k^3) \quad \boxed{\#1} \quad = 2\sum_{k=4}^{10}k^3 \quad \boxed{\#4} \quad = 2\left(\sum_{k=1}^{10}k^3 - \sum_{k=1}^{3}k^3\right)$$

$$= 2\left(\frac{10(10+1)}{2}\right)^2 - 2\left(\frac{3(3+1)}{2}\right)^2$$

$$= 2(55)^2 - 2(6)^2 \quad \boxed{= 5978}$$