Math-1050

Session #33

Systems of *NON*-Linear Equations

Solution to an Equation in two variables: an ordered pair that makes the equation a true statement.

Solution to a system two equations in two variables: an ordered pair of real numbers that is a solution of both equation. (Where the graphs intersect.)

List the methods of solving systems of linear equations.

1. Graphing

2. Substitution

3. Elimination

4. Cramer’s Rule (uses determinants)

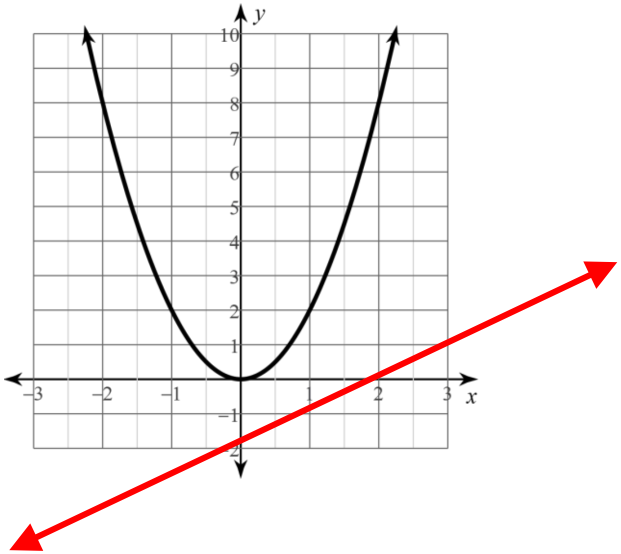
5. Row Operations (Gaussian Elimination)

6. Matrix Equation (uses inverse matrices)

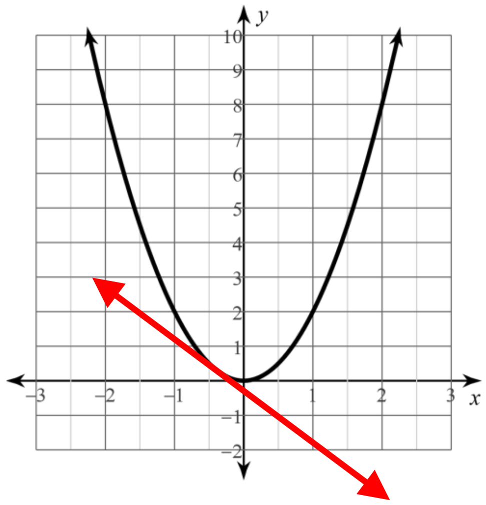
Which can be used to solve systems of ***non-linear*** equations.

There is no *general methodology*

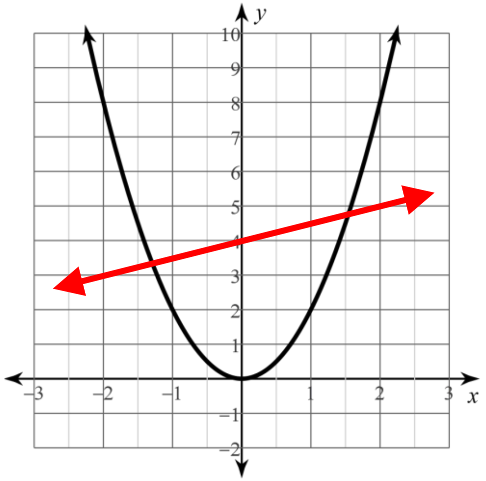
1. A parabola and a line



0 solutions



1 solution



2 solutions

Solve the System $y = 2x^2$ $y = 3x + 2$

Substitution: y = y

$$2x^2 = 3x + 2$$
$$2x^2 - 3x - 2 = 0$$
$$2x^2 - 4x + x - 2 = 0$$

-4 = -4 * 1

-3x = -4x + 1x

These are all of the terms in “the box”

	x	-2
2x	$2x^2$	-4x
1	x	-2

$$0 = (2x + 1)(x - 2)$$

$$x = -\frac{1}{2}$$

$$x = 2$$

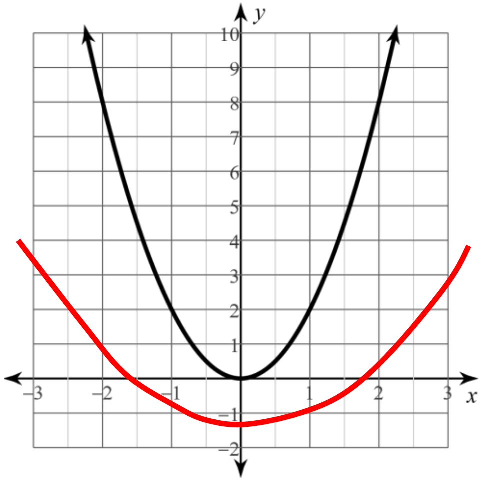
$$f\left(-\frac{1}{2}\right) = 3\left(-\frac{1}{2}\right) + 2$$

$$\left(-\frac{1}{2}, \frac{1}{2}\right)$$

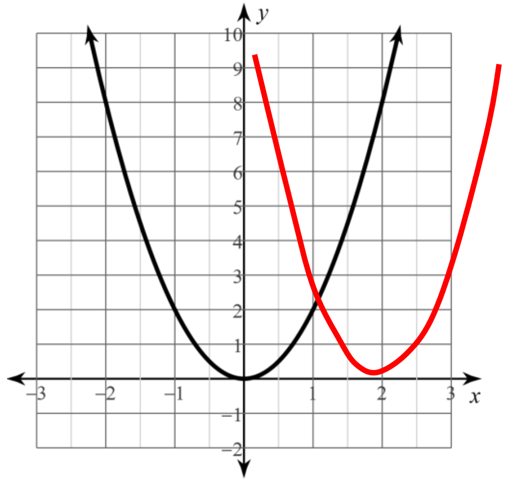
$$f(2) = 2(2)^2$$

$$(2, 8)$$

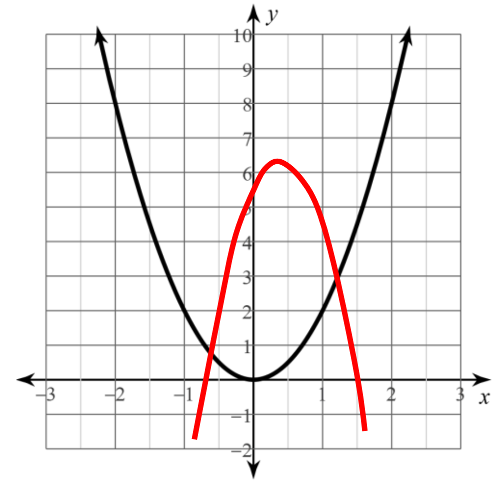
2. Two Parabolas



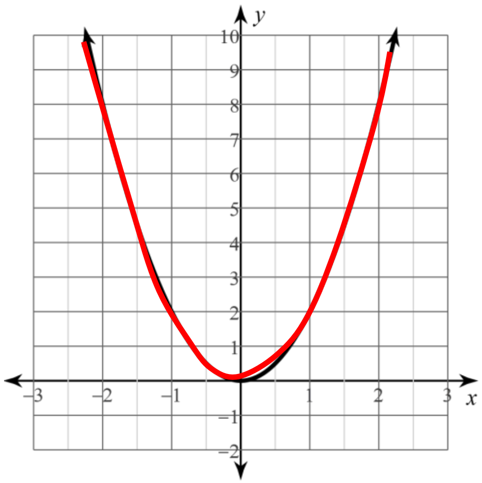
0 solutions



1 solution



2 solutions



<u>Same Parabola</u>: all points on the parabola are solutions (Infinitely many solutions)

Solve the system of equations.

$$y = x^2 - 2x - 8$$

$$y = -x^2 + 4x - 8$$

Substitution: y = y

$$-x^2 + 4x - 8 = x^2 - 2x - 8$$

$$+x^2 \quad -4x \quad +8 \qquad +x^2 \quad -4x \quad +8$$

→ Non-standard quadratic equation

$$0 = 2x^2 - 6x$$

$$f(0) = -8$$

$$(0, -8)$$

$$0 = 2x^2 - 6x$$

$$\div 2 \quad \div 2 \quad \div 2$$

$$f(3) = (3)^2 - 2(3) - 8$$

$$0 = x^2 - 3x$$

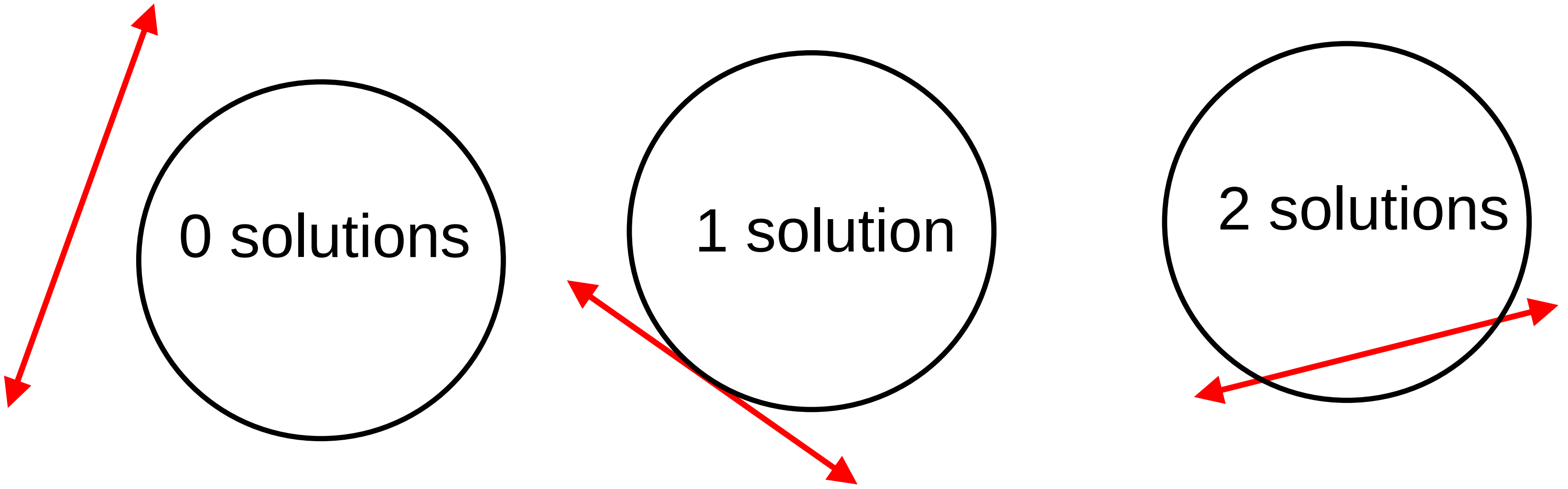
$$f(3) = -5$$

$$(3, -5)$$

$$0 = x(x - 3)$$

$$x = 0 \qquad x = 3$$

A circle and a line



Finding the equation of a circle:

What is the radius of the circle?

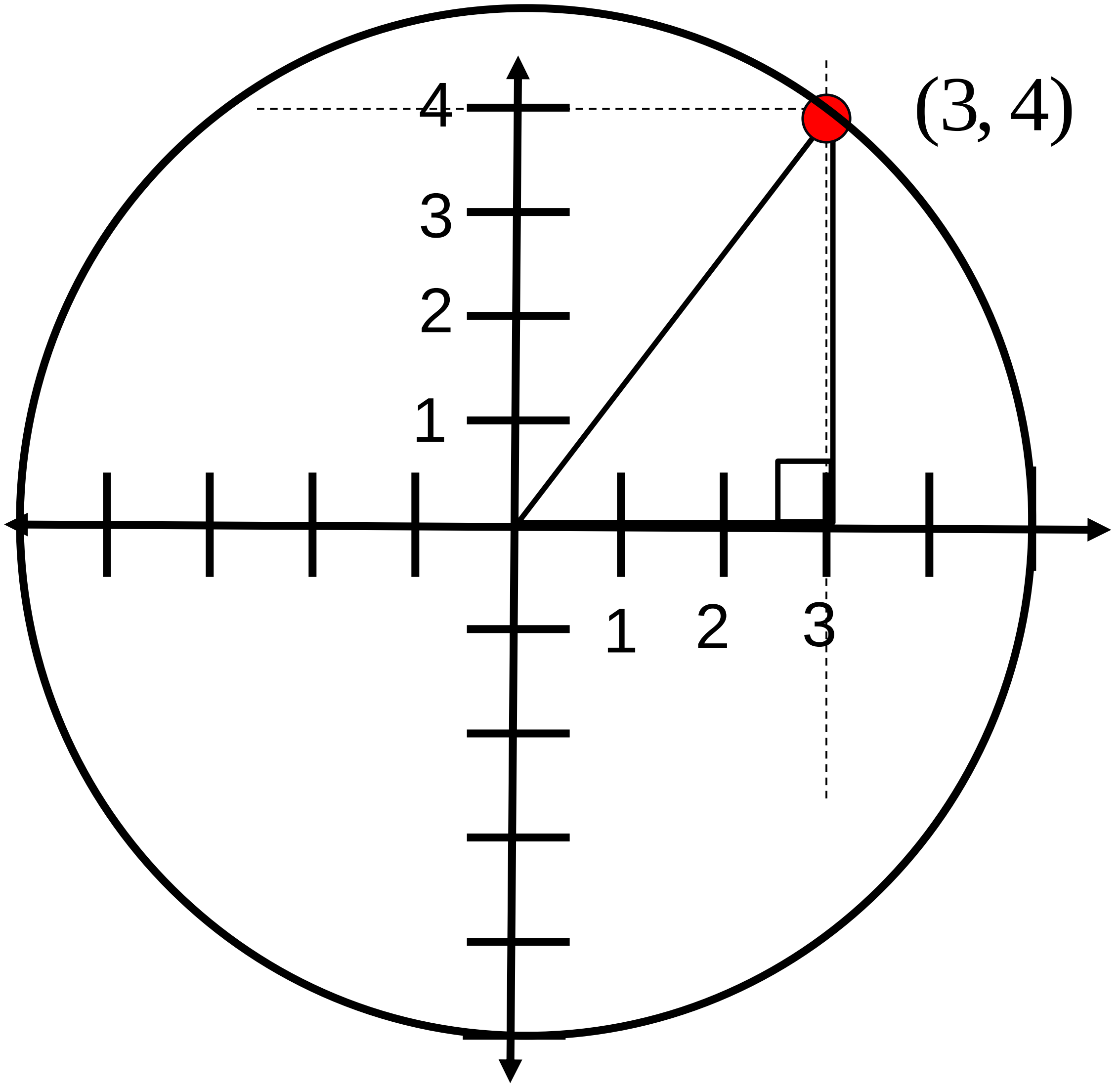
$$a^2 + b^2 = c^2$$

$$3^2 + 4^2 = r^2$$

$$9 + 16 = r^2$$

$$25 = r^2$$

$$r = 5$$

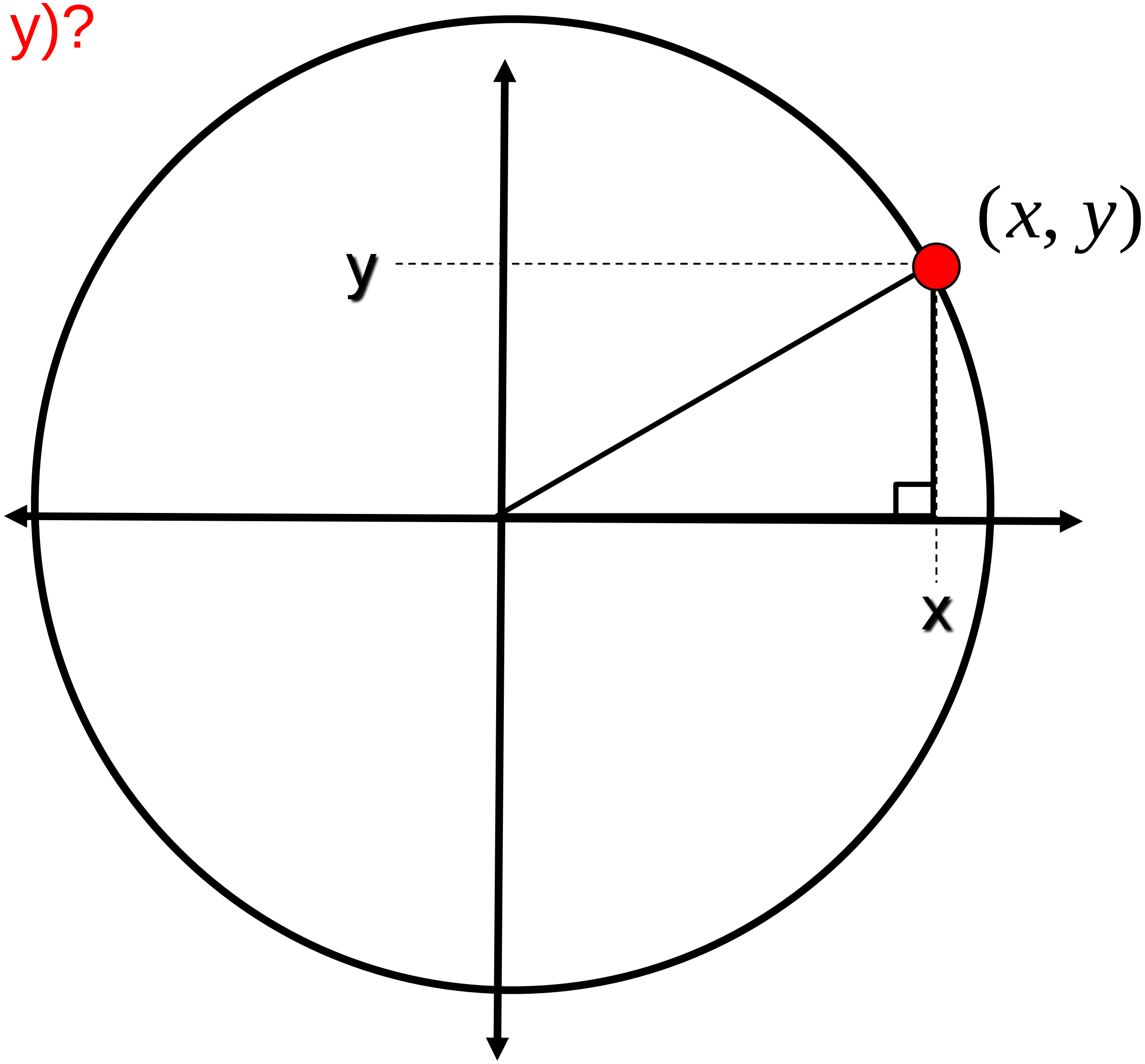


Now I will pick a random point on the circle.

What is the equation for the radius of the circle when the point is (x, y)?

$$x^2 + y^2 = r^2$$

This is the general equation for a circle centered at (0, 0)!!!

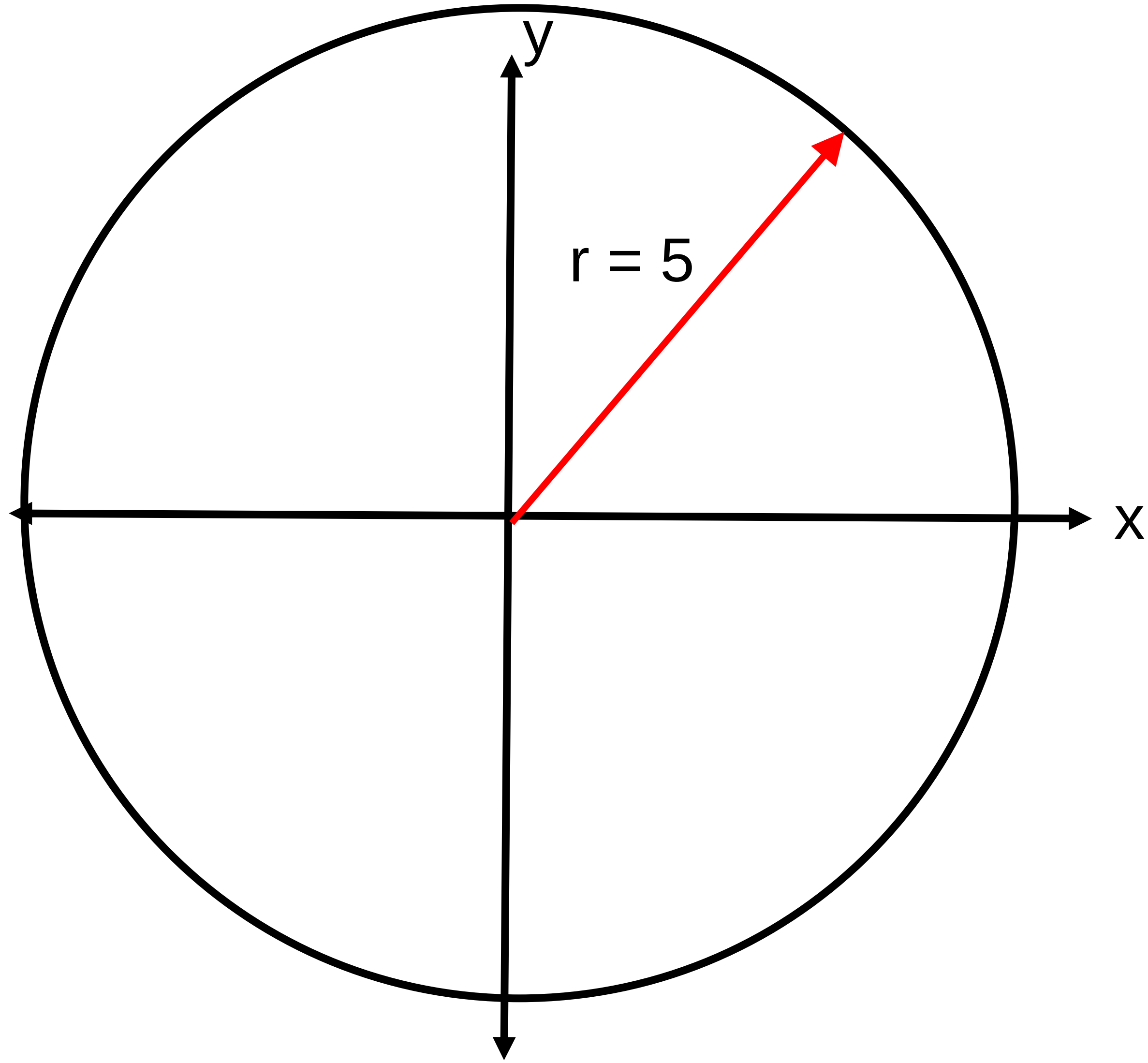


This is the equation of a circle centered at the origin whose radius is 5.

$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = 5^2$$

$$x^2 + y^2 = 25$$



Equations of Circles

$$r^2 = (x-h)^2 + (y-k)^2$$

'r' is the <u>radius</u> of the circle.

(h, k) is the <u>center</u> of the circle.

$$9 = (x-3)^2 + (y+4)^2$$

'r' = ? (h, k) = ?

r = 3 center is (3, -4)

$$3 = (x+2)^2 + (y-1)^2$$

'r' = ? (h, k) = ?

$r = \sqrt{3}$ center is (-2, 1)

A Line and a Circle

$$4 = x^2 + y^2$$

$$y = x + 3$$

Substitution: $y^2 = y^2$

$$y^2 = 4 - x^2$$

$$y^2 = (x+3)^2$$

$$4 - x^2 = (x+3)^2$$

→ Non-standard quadratic equation

$$4 - x^2 = x^2 + 6x + 9$$

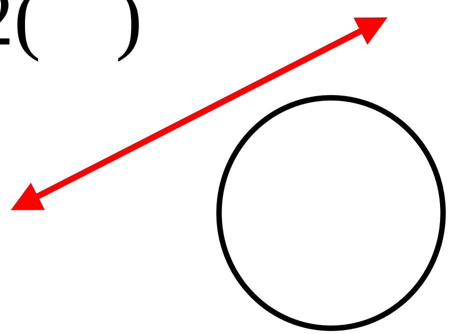
$$0 = 2x^2 + 6x + 5$$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(\ \ )}{2(\ \ )} \pm \frac{\sqrt{(\ \ )^2 - [4(\ \ )(\ \ )]}}{2(\ \ )}$$

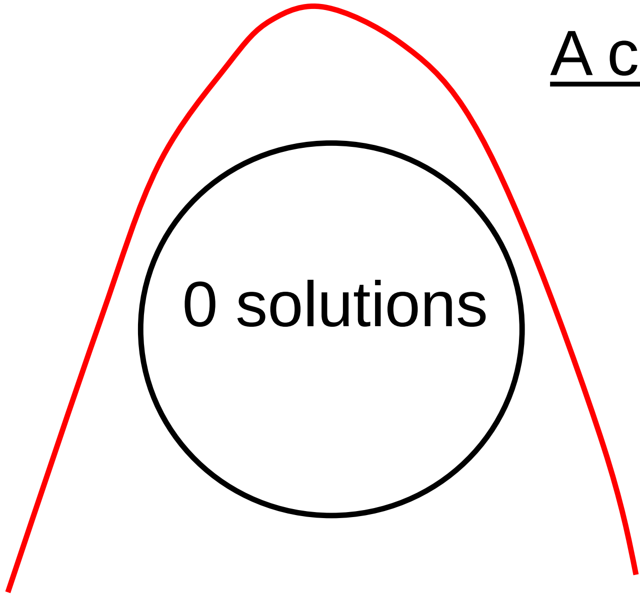
$$x = \frac{-(6)}{2(2)} \pm \frac{\sqrt{(6)^2 - [4(2)(5)]}}{2(1)}$$

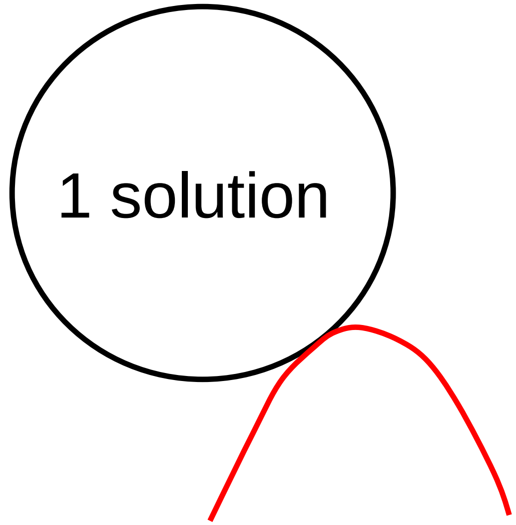
$$x = -3/2 \pm \frac{\sqrt{36 - 40}}{2}$$

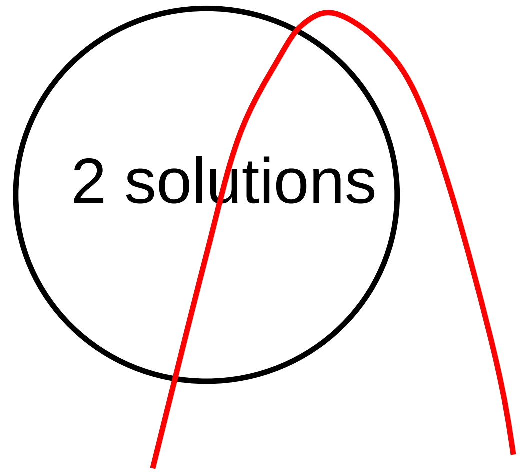


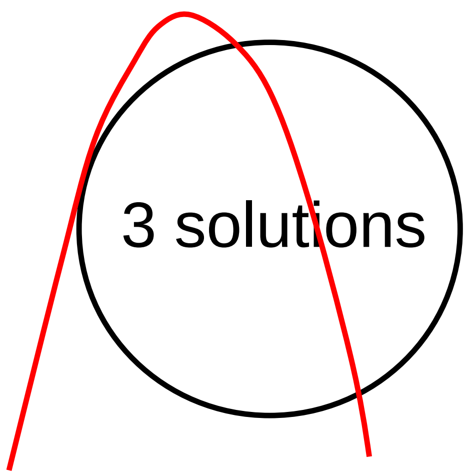
→ Imaginary zeroes→ no solution

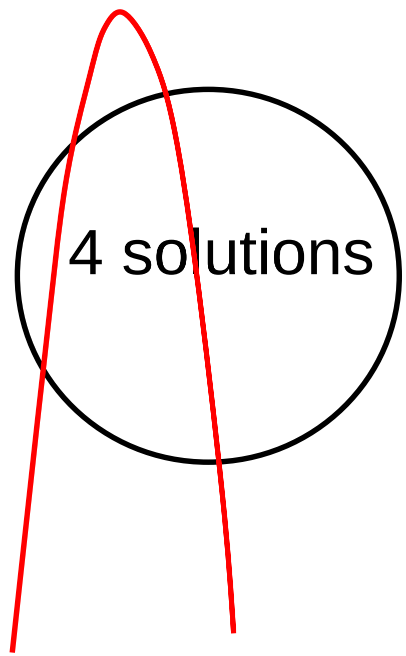
A circle and a parabola











A Parabola and a Circle

Substitution: $x^2 = x^2$

$$10 = x^2 + y^2 \quad y = x^2 - 7$$

$$x^2 = 10 - y^2$$

$$x^2 = y + 7$$

$$10 - y^2 = y + 7$$

$$y^2 + y - 3 = 0$$

→ Non-standard quadratic equation

$$y = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-(\ \ )}{2(\ \ )} \pm \frac{\sqrt{(\ \ )^2 - [4(\ \ )(\ \ )]}}{2(\ \ )}$$

$$y = \frac{-(1)}{2(1)} \pm \frac{\sqrt{(1)^2 - [4(1)(-3)]}}{2(1)}$$

$$y = -\frac{1}{2} \pm \frac{\sqrt{1 + 12}}{2}$$

$$y = -\frac{1}{2} \pm \frac{\sqrt{13}}{2}$$

→ Now we need to find ‘x’ values

A Parabola and a Circle

→ Now we need to find ‘x’ values

$$10 = x^2 + y^2$$

$$y = x^2 - 7$$

Substitution: y = y

$$y = -\frac{1}{2} \pm \frac{\sqrt{13}}{2}$$

$$x^2 = 7 - \frac{1}{2} - \frac{\sqrt{13}}{2}$$

$$x^2 = \frac{13}{2} - \frac{\sqrt{13}}{2}$$

$$x = \pm\sqrt{\frac{13}{2} - \frac{\sqrt{13}}{2}}$$

$$-\frac{1}{2} + \frac{\sqrt{13}}{2} = x^2 - 7$$

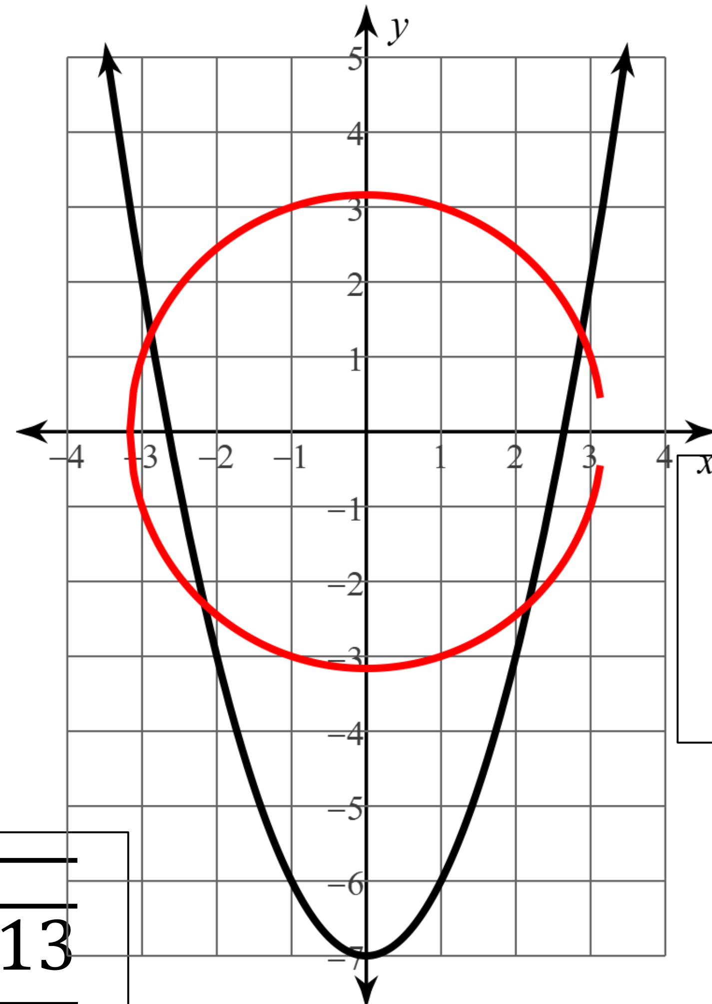
$$x^2 = 7 - \frac{1}{2} + \frac{\sqrt{13}}{2}$$

$$x^2 = \frac{13}{2} + \frac{\sqrt{13}}{2}$$

$$x = \pm\sqrt{\frac{13}{2} + \frac{\sqrt{13}}{2}}$$

A Parabola and a Circle

$$10 = x^2 + y^2 \qquad y = x^2 - 7$$



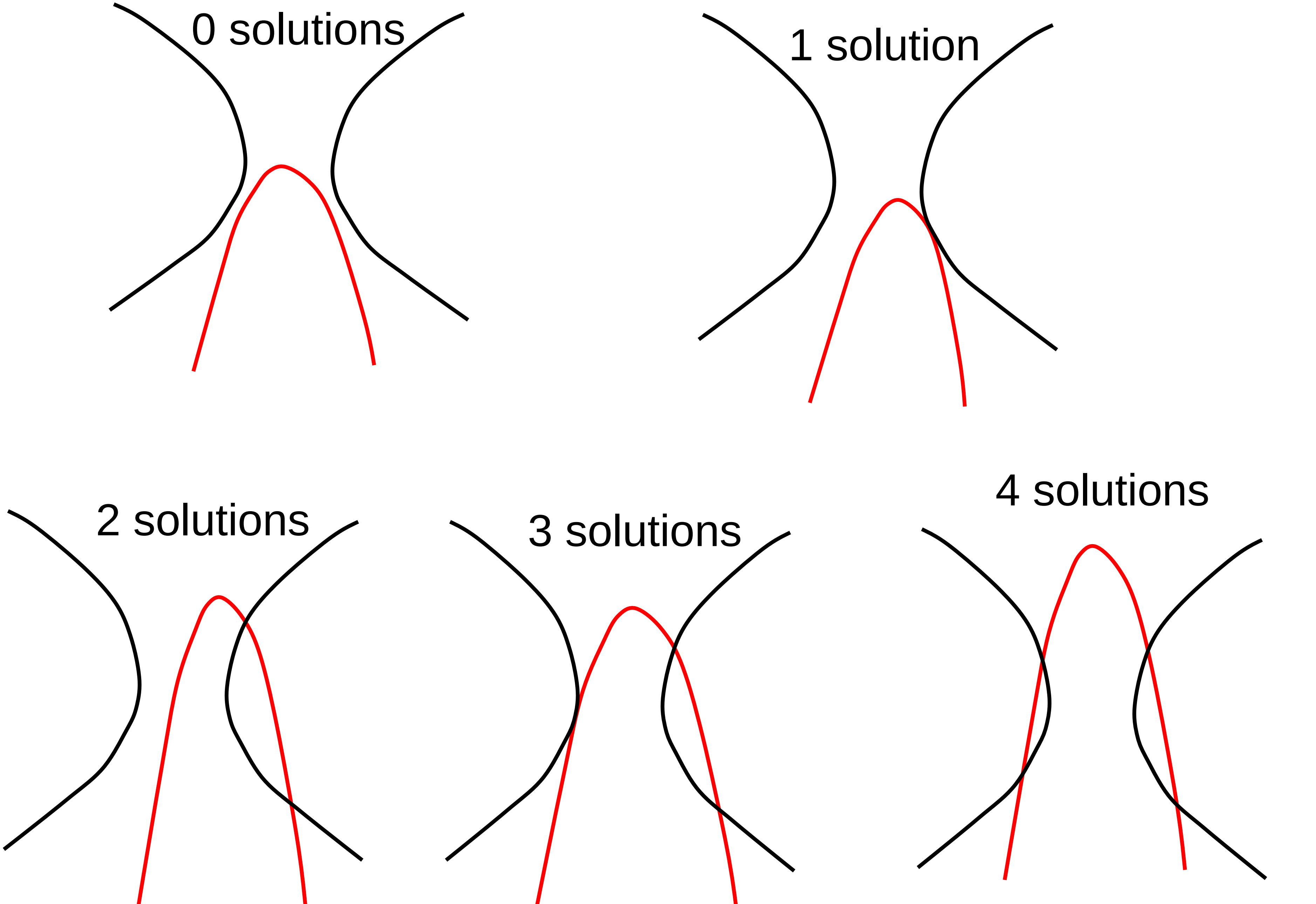
$$y = -\frac{1}{2} + \frac{\sqrt{13}}{2}$$

$$x = \pm\sqrt{\frac{13}{2} + \frac{\sqrt{13}}{2}}$$

$$y = -\frac{1}{2} - \frac{\sqrt{13}}{2}$$

$$x = \pm\sqrt{\frac{13}{2} - \frac{\sqrt{13}}{2}}$$

A hyperbola and a parabola



Substitution: $x^2 = x^2$

Hyperbola and Parabola

$x^2 = y^2 + 4$ $\qquad$ $4 = x^2 - y^2$ $\qquad$ $y = x^2 - 4$

$x^2 = y + 4$

$y^2 + 4 = y + 4$ → Non-standard quadratic equation

$y^2 - y = 0$

$y(y - 1) = 0$

$y = 0, 1$ → Now we need to find ‘x’ values

$y = x^2 - 4$ $\qquad$ $0 = x^2 - 4$ $\qquad$ $x = 2, -2$

$y = x^2 - 4$ $\qquad$ $1 = x^2 - 4$ $\qquad$ $x = \sqrt{5}, -\sqrt{5}$

A Hyperbola and a Parabola

$4 = x^2 - y^2$ $\quad x = 2, -2$

$y = x^2 - 4$ $\quad x = \sqrt{5}, -\sqrt{5}$

$y = \sqrt{5}^2 - 4$ $\quad y = (-\sqrt{5})^2 - 4$

$y = 1$ $\quad y = 1$

$(\sqrt{5}, 1)$ $\quad (-\sqrt{5}, 1)$

$y = (2)^2 - 4$ $\quad y = (-2)^2 - 4$

$y = 0$ $\quad y = 0$

$(2, 0)$ $\quad (-2, 0)$