Math-1050 Session 2 (Algebra Review #2)

-Synthetic Division

-finding quotients and remainders

-verifying if an expression is a factor of a polynomial

-Reducing Rational expressions to their lowest form (simply)

-Add/subtract Rational Expressions

-equal denominators

-denominators that are *additive inverses of each other*

-unequal denominators

-Simplify Complex Rational Expressions

- "n-th" roots (radicals)

- Converting between radical and rational exponent forms

→ *WOW!!*

Synthetic Division: A simpler way to divide *some* polynomials. You can use synthetic division when dividing by (when the *dividend* is) a ***linear divisor*** with a lead coefficient of ‘1’:

Linear Factor/divisor: an expression of the form: (ax + b) (remember: y = mx + b ??) where ‘a’ and ‘b’ are integers.

$$x-1\overline{)\,x^3-4x^2-15x+18}$$

$$1\,\overline{)\;1 \quad -4 \quad -15 \quad 18}$$

1

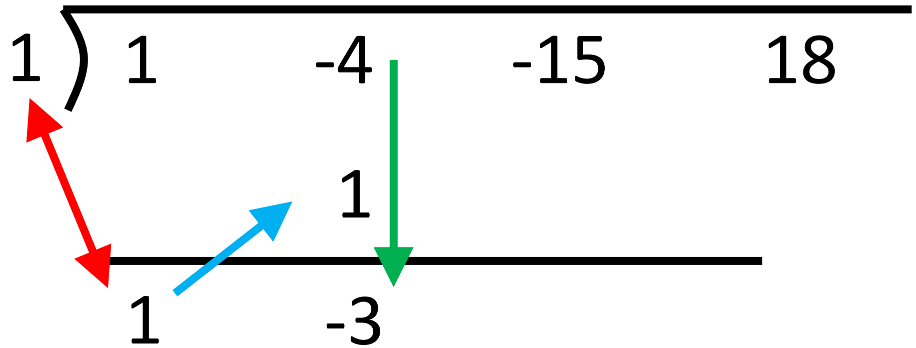
1st step: Write the polynomial with only its coefficients.

2nd step: Write the “zero” of the linear factor.

3rd step: add down

Is there an easier way to do this? Yes!

$$x-1\overline{\big)\, x^3-4x^2-15x+18}$$



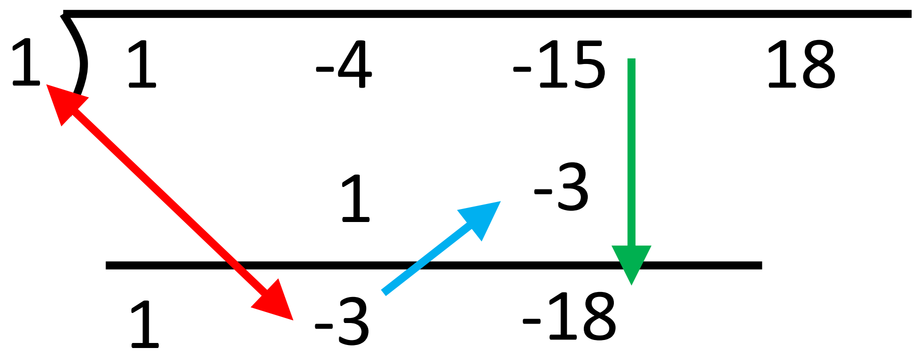
4th step: Multiply the “zero” by the lead coefficient.

5th step: Write the product under the next term to the right.

6th step: add the second column downward

Is there an easier way to do this? Yes!

$$x-1\overline{)\,x^3-4x^2-15x+18}$$



7th step: Multiply the “zero” by the second number

8th step: Write the product under the next term to the right.

9th step: add the next column downward

Is there an easier way to do this? Yes!

$$x-1 \overline{\big)\, x^3 - 4x^2 - 15x + 18}$$

1	1	-4	-15	18
		1	-3	-18
	1	-3	-18	0

10th step: Multiply the "zero" by the 3rd number

11th step: Write the product under the next term to the right

12th step: add the next column downward

$$\begin{array}{r} x^2 - 3x - 18 \\ x - 1 \overline{\smash{\big)}\, x^3 - 4x^2 - 15x + 18} \end{array}$$

1)	1	-4	-15	18
		1	-3	-18
	1	-3	-18	0

0 ← remainder

$$x^3 - 4x^2 - 15x + 18 \div (x - 1) \; = x^2 - 3x - 18$$

Because the remainder = 0, then (x – 1) is a factor AND x = 1 is a zero of the original polynomial!

An expression is a factor of another expression if and only if it divides evenly (has a remainder of zero).

Is (x + 3) a factor of f(x)?

$$f(x) = 3x^3 - x^2 - 20x + 30$$

-3	3	-1	-20	30
		-9	30	-30
	3	-10	10	0

0 ← remainder

Because the remainder = 0, then (x +3) is a factor AND x = -3 is a zero of the original polynomial!

Reducing Rational Expressions to their "lowest form" (simplification)

$$\frac{x+7}{7(x+9)}$$

Cannot use the *Inverse Property of Multiplication* on *addends*.

Addition and Subtraction mean:

Combine the terms into one term (if you can)

If you can't combine them (unlike terms) they still are connected to each other with addition.

Put terms being added/subtracted into parentheses *before* you try to simplify.

$$\frac{x+7}{7(x+9)} \rightarrow \frac{(x+7)}{7(x+9)}$$

$$\frac{2x-4}{4x+6} \rightarrow \frac{2x-4}{4x+6} \rightarrow \frac{2(x-2)}{2(x+3)}$$

You must FACTOR the fraction.

$$\frac{32}{44} \rightarrow \frac{4*8}{4*11}$$

$$\frac{x^2-4}{x^2-3x+2} \rightarrow \frac{(x-2)(x+2)}{(x-2)(x-1)}$$

Break it apart into the product of fractions.

$$\rightarrow \frac{4}{4}*\frac{8}{11}$$

$$\rightarrow \frac{(x-2)}{(x-2)}*\frac{(x+2)}{(x-1)}$$

Notice the fraction that equals ‘1’ ?

$$\rightarrow 1*\frac{8}{11} \qquad \rightarrow \frac{8}{11}$$

$$\rightarrow 1*\frac{(x+2)}{(x-1)} \qquad \rightarrow \frac{(x+2)}{(x-1)}$$

Multiplying Rational Expressions

Simplify before you multiply.

$$\frac{(x-1)}{2(x+3)} * \frac{x^2-9}{(x+1)} \rightarrow \frac{(x-1)}{2(x+3)} * \frac{(x+3)(x-3)}{(x+1)}$$

$$\rightarrow \frac{\cancel{(x+3)}}{\cancel{(x+3)}} * \frac{(x-1)(x-3)}{2(x+1)} \quad \boxed{\rightarrow \frac{(x-1)(x-3)}{2(x+1)}}$$

DON'T multiply the simplified version of the product, just leave it in factored form.

$$5 \div 5 = 1$$

$$5 * \frac{1}{5} = 1$$

Inverse Property of Multiplication

Division by a number
is the same as
multiplication by its reciprocal.

Divide Rational Expressions

$$\frac{x^2 + 2x - 35}{x^2 - 4x - 12} \div \frac{x^2 - 2x - 15}{x^2 + 9x + 14}$$

$$\rightarrow \frac{x^2 + 2x - 35}{x^2 - 4x - 12} * \frac{x^2 + 9x + 14}{x^2 - 2x - 15}$$

Simplify before you multiply.

$$\rightarrow \frac{(x+7)(x-5)}{(x-6)(x+2)} * \frac{(x+2)(x+7)}{(x-5)(x+3)}$$

$$\rightarrow \frac{(x+7)(x+7)}{(x-6)(x+3)}$$

Add/Subtract Rational Expressions with Equal Denominators

Which one is correct?

$$\frac{2x-7}{x^2+2} - \frac{x-4}{x^2+2}$$

$$\rightarrow \frac{2x-7-x-4}{x^2+2}$$

$$\rightarrow \frac{2x-7-x+4}{x^2+2}$$

Group expressions to avoid distributive property errors.

$$\frac{(2x-7)}{(x^2+2)} - \frac{(x-4)}{(x^2+2)} \rightarrow \frac{(2x-7)-(x-4)}{x^2+2} \rightarrow \frac{2x-7-x-(-4)}{x^2+2}$$

$$\rightarrow \frac{x-3}{x^2+2}$$

Add/Subtract Rational Expressions whose Denominators are Additive Inverses of each other.

$$\frac{1}{x-2}+\frac{3}{2-x}$$

Rewrite the denominator in "standard form"

$$\frac{1}{x-2}+\frac{3}{-x+2}$$

Use the *Identity Property of Multiplication* (multiply by "one" in the form of -1/-1)

$$\frac{1}{x-2}+\frac{3}{(-x+2)}*\frac{(-1)}{(-1)}$$

Simplify

$$\frac{1}{x-2}+\frac{-3}{x-2}$$

Add (or subtract)

$$\frac{-2}{x-2}$$

Add/Subtract Rational Expressions with Unequal Denominators

Use the Identity Property of Multiplication ("multiply an individual rational term by "one in the form of…") to obtain equal denominators.

$$\frac{3x+1}{2x} - \frac{1}{5}$$

$$\rightarrow \frac{5}{5} * \frac{(3x+1)}{2x} - \frac{1}{5} * \frac{2x}{2x}$$

$$\rightarrow \frac{5(3x+1)}{10x} - \frac{2x}{10x}$$

$$\rightarrow \frac{5(3x+1) - 2x}{10x}$$

$$\rightarrow \frac{15x + 5 - 2x}{10x}$$

$$\boxed{\rightarrow \frac{13x + 5}{10x}}$$

The textbook emphasizes obtaining the LCM (least common multiple) of the denominators rather than the "brute force" equal denominator.

Least Common Multiple Method

$$\frac{12}{x^2+5x-24}+\frac{3}{x-3}$$

Factor each denominator

$$\frac{12}{(x+8)(x-3)}+\frac{3}{(x-3)}$$

LCM of (x + 8)(x – 3) and (x – 3)?

$$(x+8)(x-3)$$

$$\frac{12}{(x+8)(x-3)}+\frac{3}{(x-3)}*\frac{x+8}{x+8}$$

Use Identity Prop. Of Mult.
(Multiply by "1" in the form of…)

$$\frac{12}{(x+8)(x-3)}+\frac{3(x+8)}{(x+8)(x-3)}$$

Add

$$\frac{12+3(x+8)}{(x+8)(x-3)}$$

Simplify

$$\frac{12+3x+24}{(x+8)(x-3)}$$

$$\frac{3x+36}{(x+8)(x-3)}$$

$$\frac{3(x+12)}{(x+8)(x-3)}$$

Simplify a Complex Rational Expression

$$\frac{1/2}{5/6}$$

Division: Division by a number is the same thing as…

Multiplication by the reciprocal of the number.

$$\rightarrow \frac{1}{2} \div \frac{5}{6}$$

$$\rightarrow \frac{1}{2} * \frac{6}{5}$$

Simplify before you multiply.

$$\rightarrow \frac{1 * \cancel{2} * 3}{\cancel{2} * 5}$$

$$\rightarrow \frac{3}{5}$$

$$\frac{\frac{2}{3}+\frac{x}{x+2}}{\frac{5x+4}{x+3}}$$

Combine Numerator into a single fraction

$$\rightarrow \left(\frac{(x+2)}{(x+2)} * \frac{2}{3} + \frac{3}{3} * \frac{x}{(x+2)}\right) \div \frac{5x+4}{x+3}$$

$$\rightarrow \left(\frac{2x+4}{3(x+2)} + \frac{3x}{3(x+2)}\right) \div \frac{5x+4}{x+3}$$

$$\rightarrow \frac{(5x+4)}{3(x+2)} \div \frac{(5x+4)}{(x+3)}$$

Notice the grouping

Convert to multiplication

$$\rightarrow \frac{\cancel{(5x+4)}}{3(x+2)} * \frac{(x+3)}{\cancel{(5x+4)}}$$

Simplify

$$\rightarrow \frac{(x+3)}{3(x+2)}$$

Properties of Radicals

Product of Radicals

$$\sqrt[n]{a} * \sqrt[n]{b} \leftrightarrow \sqrt[n]{ab}$$

$$3\sqrt{8} * 5\sqrt{3} \rightarrow 3 * 5 * \sqrt{8} * \sqrt{3} \rightarrow 3 * 5 * \sqrt{4} * \sqrt{2} * \sqrt{3}$$

$$\rightarrow 3 * 5 * 2 * \sqrt{2} * \sqrt{3} \rightarrow 30 * \sqrt{6} \rightarrow 30\sqrt{6}$$

$$-2\sqrt{56x^3y} \rightarrow -2 * \sqrt{x^2} * \sqrt{8 * 7xy}$$

$$\rightarrow -2 * |x| * \sqrt{4} * \sqrt{2 * 7xy} \qquad \rightarrow -4|x|\sqrt{14xy}$$

We'll talk about this a little later.

Properties of Radicals

Quotient of Radicals

$$\sqrt[n]{\frac{a}{b}} \leftrightarrow \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$$\frac{\sqrt{12}}{\sqrt{2}} \rightarrow \frac{\sqrt{2} * \sqrt{6}}{\sqrt{2}} \rightarrow \frac{\cancel{\sqrt{2}} * \sqrt{6}}{\cancel{\sqrt{2}}}$$

Inverse Property of Multiplication

$$\rightarrow \sqrt{6}$$

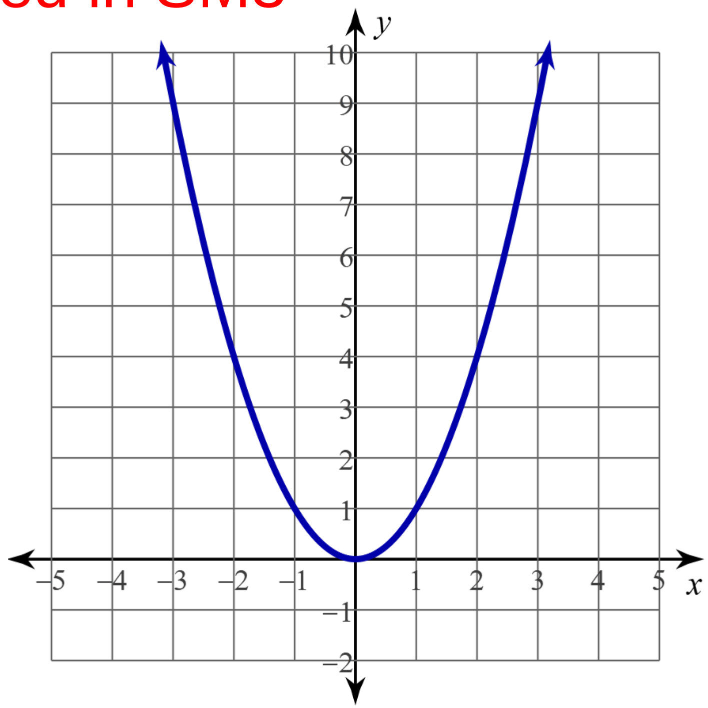
$$\frac{\sqrt{48x^3}}{\sqrt{16x}} \rightarrow \frac{\cancel{\sqrt{16x}} * \sqrt{3x^2}}{\cancel{\sqrt{16x}}}$$

Inverse Property of Multiplication

$$\rightarrow \sqrt{3x^2} \quad \rightarrow |x|\sqrt{3}$$

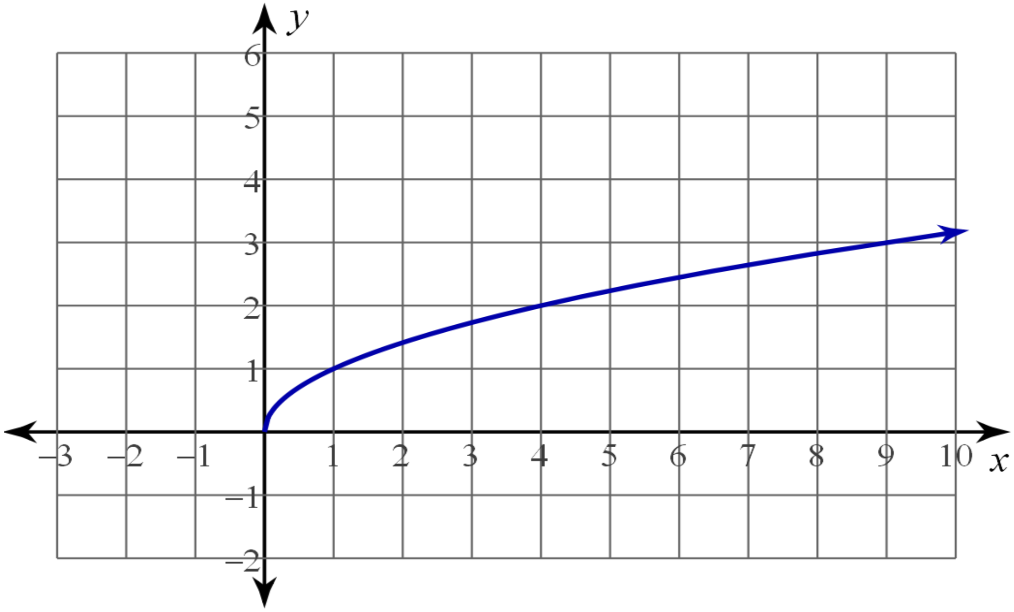
Why the *absolute value* of ‘x’?

Background: Functions we learned in SM3



Squaring Function $f(x) = x^2$

Square Root Function $g(x) = \sqrt{x}$

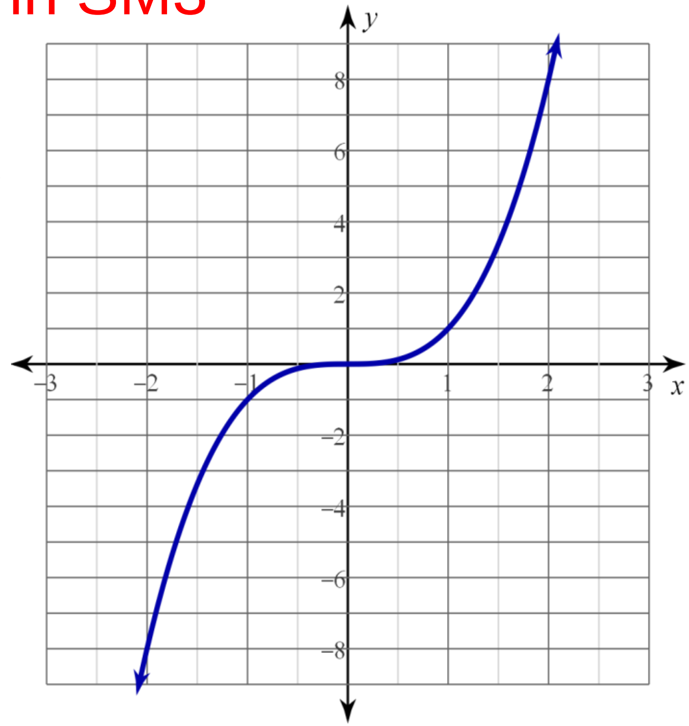


The square root of a number is always positive (called the "principal square root")

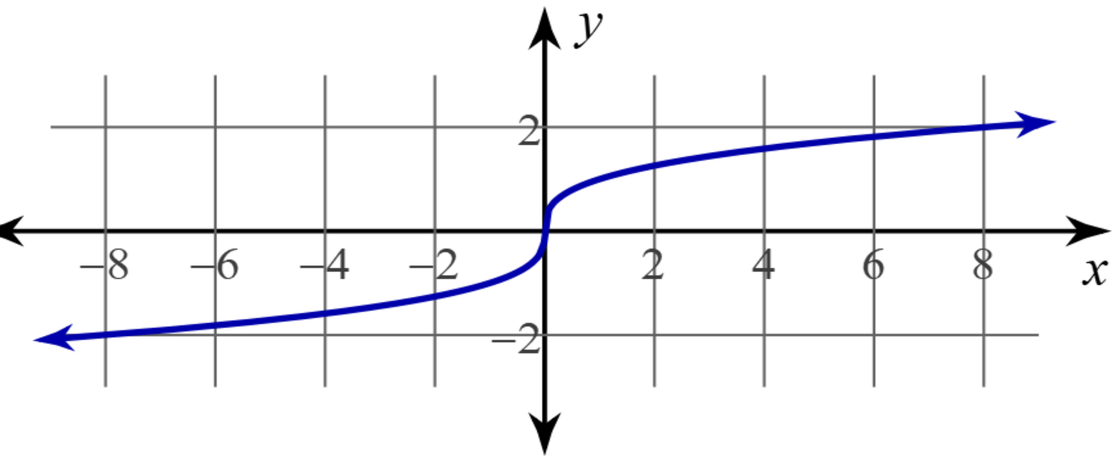
$$3 = \sqrt{x}$$

<u>Background</u>: Functions we learned in SM3

<u>The Cube Function</u> $f(x) = x^3$



<u>Cubed Root function:</u> $g(x) = \sqrt[3]{x}$



The cubed root of a number can be either positive <u>*OR*</u> negative.

$$2 = \sqrt[3]{8}$$

$$-2 = \sqrt[3]{-8}$$

Properties of Radicals

Even and Odd Index numbers

$\sqrt[n]{a^n} = |a| \quad \text{(if n is even)}$

The *even* root of a number is always positive.

This is easy to see if the numbers are integers.

$\sqrt[4]{(-2)^4} = |-2| = 2 \qquad (-2)^4 = 16 \qquad \sqrt[4]{16} = 2$

But when we simplify radicals with variables in them, we must specify the result to be positive. $\sqrt[4]{x^4} = |x| \neq x$

Because ‘x’ is not necessarily a positive number (and ‘-x’ is not necessarily a negative number).

$\sqrt[n]{a^n} = a \quad \text{(if n is odd)}$

Since ‘a’ is either a positive or a negative number when it is listed as just a variable.

Properties of Radicals

Converting Radicals to Rational Exponents

$a^{1/n} \leftrightarrow * \sqrt[n]{a}$ $\qquad$ $a^{m/n} \leftrightarrow * \sqrt[n]{a^m} \leftrightarrow (\sqrt[n]{a})^m$

$5(y)^{1/3} \rightarrow 5\sqrt[3]{y}$ $\qquad$ $2\sqrt[5]{x^3y^2} \rightarrow 2x^{3/5}y^{2/5}$

In our textbook, simplifying rational exponents means to covert to simplified radicals.

$125^{2/3} \rightarrow (\sqrt[3]{125})^2 \rightarrow (5)^2 \rightarrow 25$

Rationalizing the denominator: using mathematical properties to change an irrational number (or imaginary) in the denominator into a rational number.

We take advantage of the idea:

$$\sqrt{2}*\sqrt{2} = \sqrt{2*2} = \sqrt{4} = 2$$

$$\sqrt{3}*\sqrt{3} = \sqrt{3*3} = \sqrt{9} = 3$$

$$\frac{1}{\sqrt{2}}*\frac{\sqrt{2}}{\sqrt{2}} \rightarrow \frac{\sqrt{2}}{2}$$

Identity Property of Multiplication

multiplying by ‘1’ doesn’t change the number.

$$\frac{25}{\sqrt{15}}*\frac{\sqrt{15}}{\sqrt{15}} \rightarrow \frac{25\sqrt{15}}{15} \rightarrow \frac{5*5**\sqrt{15}}{5*3} \rightarrow \frac{5\sqrt{15}}{3}$$

What about higher *index numbers*?

$$\frac{1}{\sqrt[3]{x}} * \frac{\sqrt[3]{x*x}}{\sqrt[3]{x*x}} \to \frac{\sqrt[3]{x*x}}{\sqrt[3]{x*x*x}} \to \frac{\sqrt[3]{x^2}}{x}$$

How many more ‘x’s are needed in the denominator radicand?

Remember: the cubed root of x-cubed equals x. $\sqrt[3]{x^3} = x$

We need two more x’s under the denominator radical.

Using the multiply powers property we don’t have to write out all the individual x’s.

$$\frac{1}{\sqrt[3]{x}} * \frac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}} \to \frac{\sqrt[3]{x^2}}{x}$$

Rationalizing a *binomial* radical Denominator

$$\frac{1}{\sqrt{3}-5}$$

Multiply by the *conjugate* of the denominator (using the Identity Property of multiplication)

$$\frac{1}{(\sqrt{3}-5)} * \frac{(\sqrt{3}+5)}{(\sqrt{3}+5)}$$

Multiply by "one in the form of…"

$$\rightarrow \frac{\sqrt{3}+5}{(9-25)} \quad \rightarrow \frac{\sqrt{3}+5}{-14} * \frac{-1}{-1} \quad \boxed{\rightarrow \frac{-\sqrt{3}-5}{14}}$$