Math-1050
Session #25

6.6: Properties of Logs

(1) Product of Logs

(2) Log of a Power

(3) Log of a Quotient

$$f(x) = 5^{2x+4}$$ Find $$f^{-1}(x)$$

$$y = 5^{2x+4}$$ Replace f(x) with ‘y’

$$x = 5^{2y+4}$$ exchange ‘x’ and ‘y’

$$\log_5 x = 2y + 4$$ Log is the exponent (remember how to convert between the two?)

$$-4 + \log_5 x = 2y$$ Solve for ‘y’

$$\frac{-4 + \log_5 x}{2} = y$$

$$\boxed{y = -2 + \tfrac{1}{2}\log_5 x}$$

$$2^3 * 2^2 = 2^5$$

The product of powers → add the exponents

$$2^3 * 2^2 = 2^5$$

Logarithm: another way of writing the exponent

Convert each exponent above into a log:

$$\log_2 8 + \log_2 4 = \log_2 32$$

$$3 + 2 = 5$$

This is the logarithm equivalent of the multiply powers property of exponents.

Log of a Product Property

$$\log_2 15 = \log_2 (3 * 5)$$

$$\log_2 15 = \log_2 3 + \log_2 5$$

$$\log_b (RS) = \log_b R + \log_b S$$

log of a product = sum of the logs of the factors.

Expand the Logarithm: use properties of logs to rewrite a single log as an expression of separate logs.

$$\log_3 xy \qquad \log_3 x + \log_3 y$$

$$\log_3 45 \qquad \log_3 3 + \log_3 3 + \log_3 5$$

$$45 = 3 * 3 * 5 \qquad 2\log_3 3 + \log_3 5$$

Expand the Logarithm: use properties of logs to rewrite a single log as an expression of separate logs.

$$\boxed{\log(3xy^2)} = \log 3 + \log x + \log y^2$$

$$= \log 3 + \log x + \log y + \log y$$

$$= \log 3 + \log x + 2\log y$$

$$\boxed{\log_4 6} = \log_4 3 + \log_4 2$$

$$\boxed{\ln 2xyw} = \ln 2 + \ln x + \ln y + \ln w$$

Condense the Logarithm: apply properties of logarithms to rewrite the log expression as a single log.

$$\boxed{\log_2 7 + \log_2 5} \quad = \log_2 (7 * 5) \quad = \log_2 35$$

$$\boxed{\log 5 + \log x} \quad = \log 5x$$

$$\log_7 5 + \log_5 7$$

"unlike logs" → can't condense

“Condense the Log”

$$\log_5 2 + \log_5 7 \quad = \log_5 14$$

$$\log 9 + \log 4 \quad = \log 36$$

$$\log_5 6 + \log_8 4$$

“unlike logs” → can’t condense

"Expand the Log"

$$\log_2 9 = \log_2(3*3)$$

$$= \log_2 3 + \log_2 3$$

$$= 2\log_2 3$$

Notice something interesting

$$\log_2 9 = \log_2(3)^2 = 2\log_2 3$$

Log of a Power Property

"Expand the Product"

$$\log_3 16 \qquad = \log_3 (4 * 4)$$

$$= \log_3 4 + \log_3 4$$

$$= 2 \log_3 4$$

Notice something interesting

$$\log_3 16 \qquad = \log_3 (4)^2 = 2 \log_3 4$$

Log of a Power Property

“Expand the Product”

$$\log_5 10^2$$

Log of a product is the sum of the logs of the factors.

$$= \log_5 10 + \log_5 10$$

Combine “like terms”

$$= 2\log_5 10$$

$$\log_5 10^2 = 2\log_5 10$$

New property: “log of a power”

Use Log of a Power simplify

$$\log x^3 = 3\log x$$

$$\ln 8 = \ln 2^3 = 3\ln 2$$

$$\log \sqrt{x} = \log x^{1/2} = \frac{1}{2}\log x$$

Gotcha'

$$\log 3y^5 = 5\log 3y$$

$$\log 3y^5 = \log 3 + \log y^5$$

Which one?

$$5\log 3y = \log(3y)^5 = \log 3^5 y^5$$

Log of a Power

c $$\log_b R^c \rightarrow c\log_b R$$

A potential error is this:

$$\log_2 6x^3 = \cancel{3\log_2 6x}$$

What is the error ? ‘3’ is an exponent of the base ‘x’ not ‘6x’

Correct the error.

$$\log_2 6x^3 = \log_2 6 + \log_2 x^3$$

$$= \log_2 3 + \log_2 2 + 3\log_2 x$$

$$\frac{x^2}{y^3}$$

Using properties of exponents: rewrite this so the ‘y’ term is NOT in the denominator.

$$x^2 y^{-3}$$

$$\log_3\left(\frac{5}{2}\right) = \log_3\left(5 * 2^{-1}\right)$$ Negative Exponent Property

$$= \log_3(5) + \log_3(2)^{-1}$$ Log of a Product Property

$$= \log_3 5 + (-1)\log_3 2$$ Log of a Power Property

$$= \log_3 5 - \log_3 2$$ Definition of Subtraction: (adding a negative is subtraction)

Log of a Quotient Property

$$\log_b\left(\frac{R}{S}\right) = \log_b R - \log_b S$$

$\log_3\left(\frac{5}{2}\right)$ "expand the quotient" $\log_3 5 - \log_3 2$

$\ln 8 - \ln 3$ "condense the quotient" $\ln\frac{8}{3}$

"Negative Log"→ denominator of the logarand

Expand the Quotient

$$\log\frac{4}{5} = \log 4 - \log 5 = \log 2 + \log 2 - \log 5$$

$$= 2\log 2 - \log 5$$

$$\ln\frac{3}{7} = \ln 3 - \ln 7$$

Condense the quotient

$$\log_4 5 - \log_4 2 = \log_4 \frac{5}{2}$$

$$\log_5 8 - \log_5 16 = \log_5 \frac{8}{16} = \log_5 \frac{1}{2}$$

Expand the Logarithm

$$\log\left(\frac{2x}{3y^5}\right) = \log 2x - \log 3y^5$$

The denominator is a product!

$$= \log 2x - (\log 3 + 5\log y)$$

Distributive property!

$$= \log 2x - \log 3 - 5\log y$$

$$= \log 2 + \log x - \log 3 - 5\log y$$

Logs of factors in the numerator will be positive.

Logs of factors in the denominator will be negative.

Expand the quotient

$$\ln \frac{zx^2}{5y^3} = \ln zx^2 - \ln 5y^3$$
$$= \ln z + \ln x^2 - (\ln 5 + \ln y^3)$$
$$= \ln z + 2\ln x - (\ln 5 + 3\ln y)$$
$$= \ln z + 2\ln x - \ln 5 - 3\ln y$$

$$= \log_4 \left(\frac{w^5}{x^7}\right)^2 = \log_4 \frac{w^{10}}{x^{14}}$$
$$= \log_4 w^{10} - \log_4 x^{14}$$
$$= 10\log_4 w - 14\log_4 x$$

$$\log_4 \frac{2\sqrt{x}}{4yz} = \log_4 2\sqrt{x} - \log_4 4yz$$

$$= \log_4 2 + \log_4 \sqrt{x} - \log_4 4 - \log_4 y - \log_4 z$$

$$= \log_4 2 + \frac{1}{2}\log_4 x - \log_4 4 - \log_4 y - \log_4 z$$

Change-of-Base Formula for Logarithms

$$\log_c a = \frac{\log_b a}{\log_b c}$$

Change to log base 10 or base 'e' (your calculator can do these).

Convert to base 10.

$$\log_4 5 = \frac{\log_{10} 5}{\log_{10} 4} = \frac{0.699}{0.6021} = 1.161$$

$$\log_4 5 = \frac{\ln 5}{\ln 4} = \frac{1.609}{1.386} = 1.161$$

$$log_{\sqrt{3}}\sqrt{6} = \frac{\log_{10}\sqrt{6}}{\log_{10}\sqrt{3}} = \frac{log\,6^{\frac{1}{2}}}{log\,3^{\frac{1}{2}}} = \frac{\frac{1}{2}\,log6}{\frac{1}{2}\,log\,3}$$

$$= \frac{log6}{log\,3} = log6 - log3$$

NO!!!!!

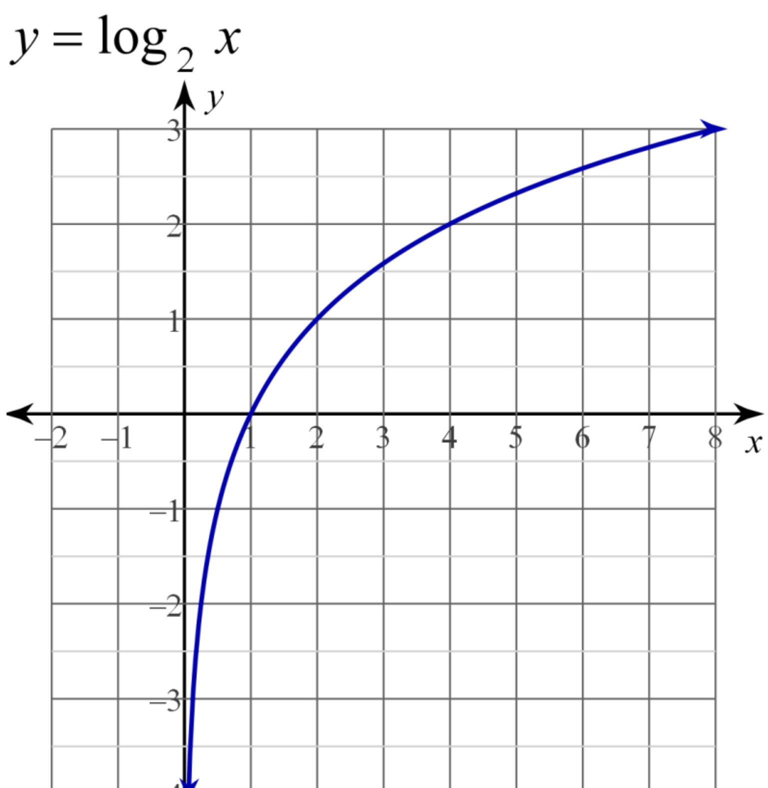
$$\frac{\log 6}{log3} \neq \log\left(\frac{6}{3}\right)$$

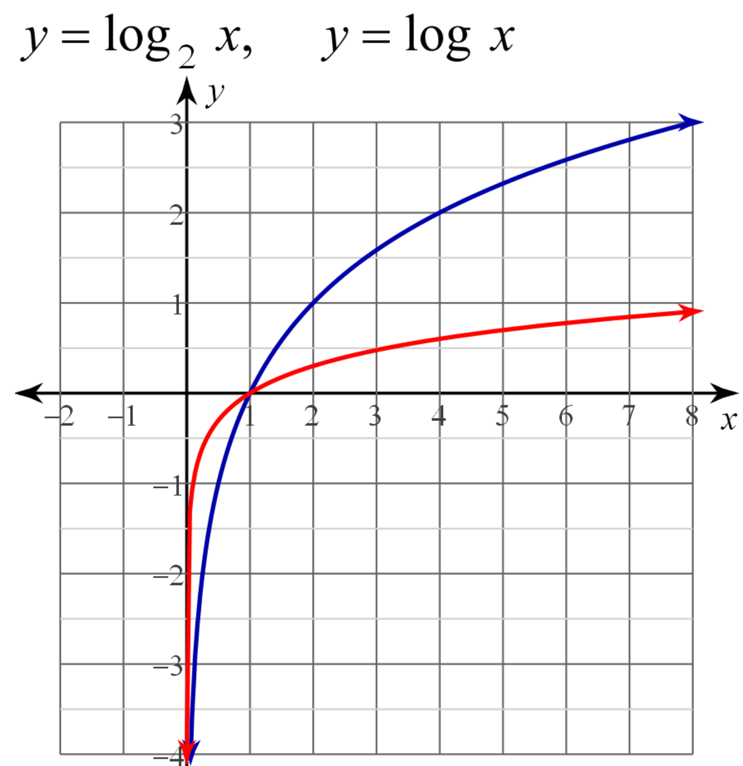
All non-base 10 logs are vertical stretches of the base 10 log.

$$y = \log_2 x$$

change of base formula.

$$y = \frac{\log x}{\log 2} = \frac{\log x}{0.301} = 3.32 * \log x$$





Simplify

$$\log_2 2$$

$$\log_2 2 = x$$

$$2^x = 2$$

$$x = 1$$

Using Change of base:

$$\log_2 2 = \frac{\log 2}{\log 2} = 1$$

Simplify: $\log_4 16$ "4 raised to what power equals 16?"

$$\log_4 4^2 \qquad 2\log_4 4 \qquad 2(1) = 2$$

$$\log_2 \sqrt{2}$$

"2 raised to what power equals the square root of 2?"

$$\log_2 2^{1/2} \qquad \tfrac{1}{2}\log_2 2 \qquad \tfrac{1}{2}(1) \qquad \tfrac{1}{2}$$

Simplify: $5\log_3 27$

$5\log_3 3^3$ $\quad (3*5)\log_3 3$ $\quad 15$

$6\log_2(16) - 4$

$6\log_2(2^4) - 4$

$(4*6)\log_2(2) - 4$

$(24)(1) - 4$

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Simplify:

$$8\log_5(125)+3 \quad = 27$$

$$2\log_9(81)-5 \quad = -1$$

$$-6\log_3(\sqrt[2]{3})+4 \quad = 1$$