Math-1050
Session #24
The Logarithm Function

Finding the Inverse: exchange the locations of ‘x’ and ‘y’ in the equation then solve for ‘y’.

$$f(x) = (x-2)^2$$

$$y = (x-2)^2$$

$$x = (y-2)^2$$

$$\sqrt{x} = \sqrt{(y-2)^2}$$

$$\pm\sqrt{x} = y-2$$

$$\pm\sqrt{x}+2 = y$$

$$y = 2 \pm \sqrt{x}$$

Domain, Range, and Inverse Functions

Domain: The input values (that have corresponding outputs)

Range: The output values (that have corresponding inputs)

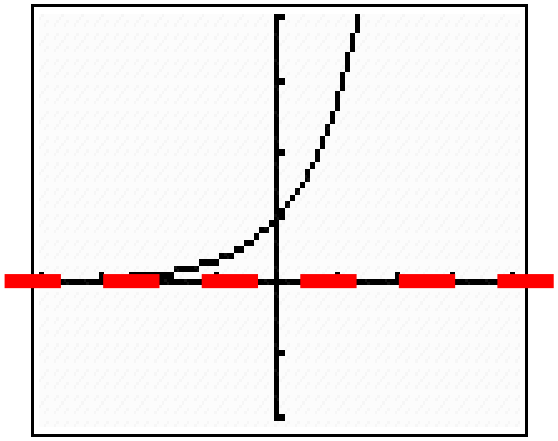
Inverse of a Function: A function resulting from an "exchange" of the inputs and outputs.

$f(x): Domain, Range$

$f^{-1}(x):$ Domain = range of f(x)

Range = domain of f(x)

Exponential Function



$f(x) = 10^x$

Domain = ? (-∞, ∞)

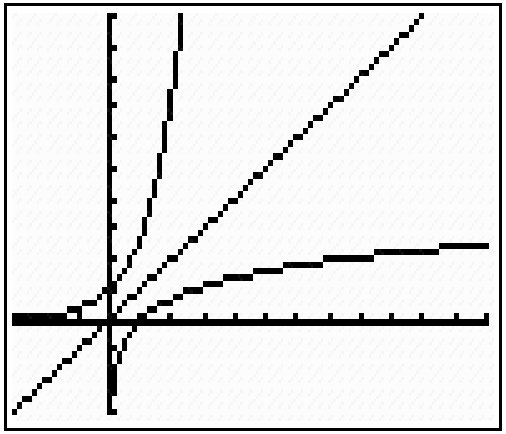
Range = ? (0, ∞)

Horizontal asymptote = ?

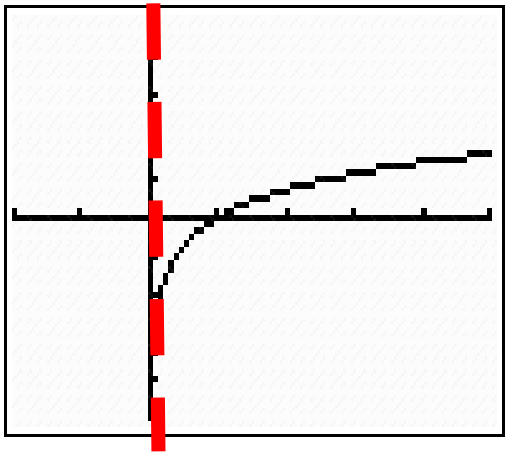
y = 0

y-intercept = ? (0, 1)

Inverse Functions



Logarithm Function



$f^{-1}(x) = log_{10}(x)$

Domain = ? (0, ∞)

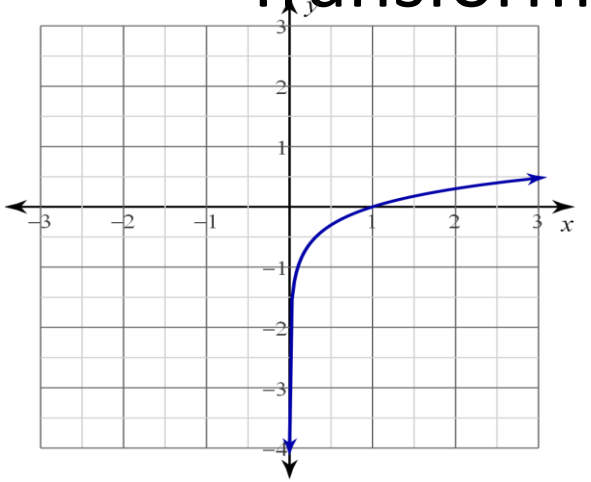
Range = ? (-∞, ∞)

Vertical asymptote = ?

x = 0

x-intercept = ? (1, 0)

Transformations of the Log Function



$f(x) = \log x$

Domain = ? (0, ∞)

Range = ? (-∞, ∞)

vertical asymptote = ?

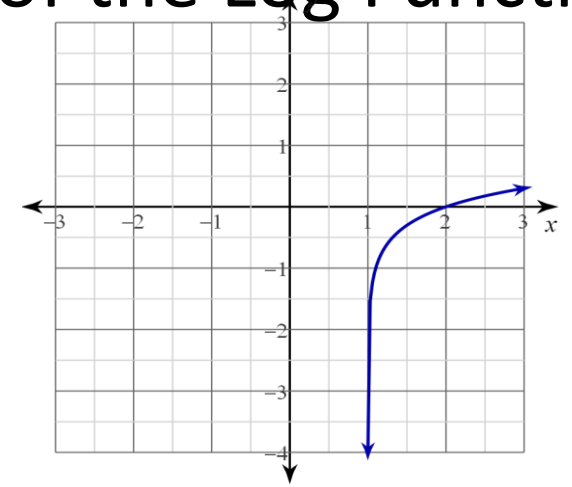
x = 0

X-intercept = ?

x = 1

Where increasing = ?

(0, ∞)



$g(x) = \log(x-1)$

Right 1 shift

Domain = ? (1, ∞)

Range = ? (-∞, ∞)

asymptote = ? x = 1

Logarand

$$g(x) = 3\log(2x-1)+5$$

Logarand

Vertical Asymptote: The value of ‘x’ that makes the logarand equal to zero (the “zero” of the logarand).

Vertical asymptote = ?

$2x - 1 = 0$

$x = ½$

Evaluating Logs on your calculator

$\log 8 = ?$

Push buttons:

[log] [8)] [=] 0.903089987

$\ln 10 = ?$

Push buttons:

[ln] [10)] [=] 2.302585093

$\log 0 = ?$ error Why?

Only input values x = (0, ∞) have corresponding outputs.

$\log(-3) = ?$ error Why?

-3 is not in the “domain” of the function.

Transformations of the Log Function

$$f(x) = \log x$$

$$g(x) = 2\log(x+1) - 3$$

VSF = 2

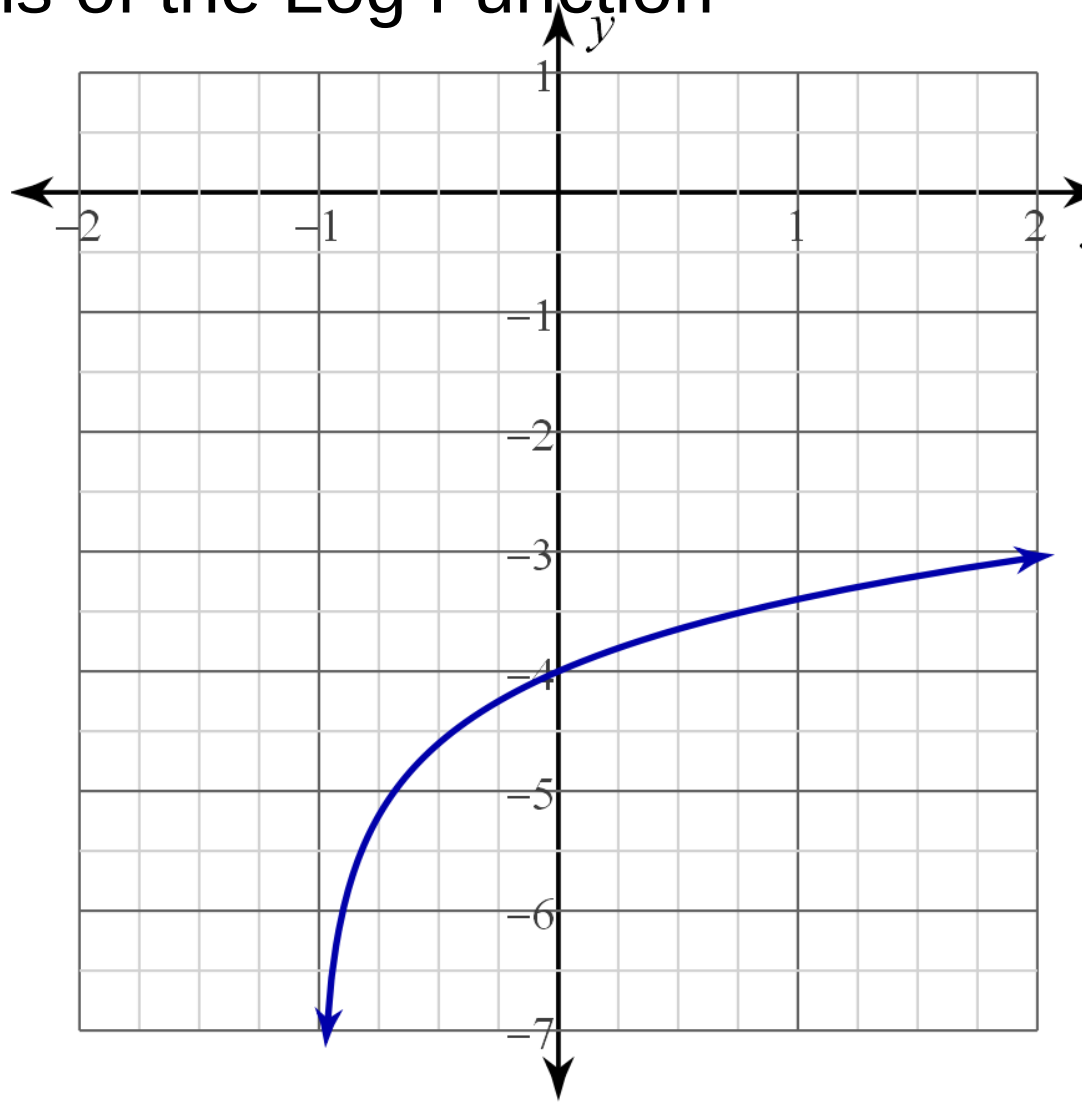
left 1 translation

Down 3 translation

<u>Domain</u> = ? x = (-1, ∞)

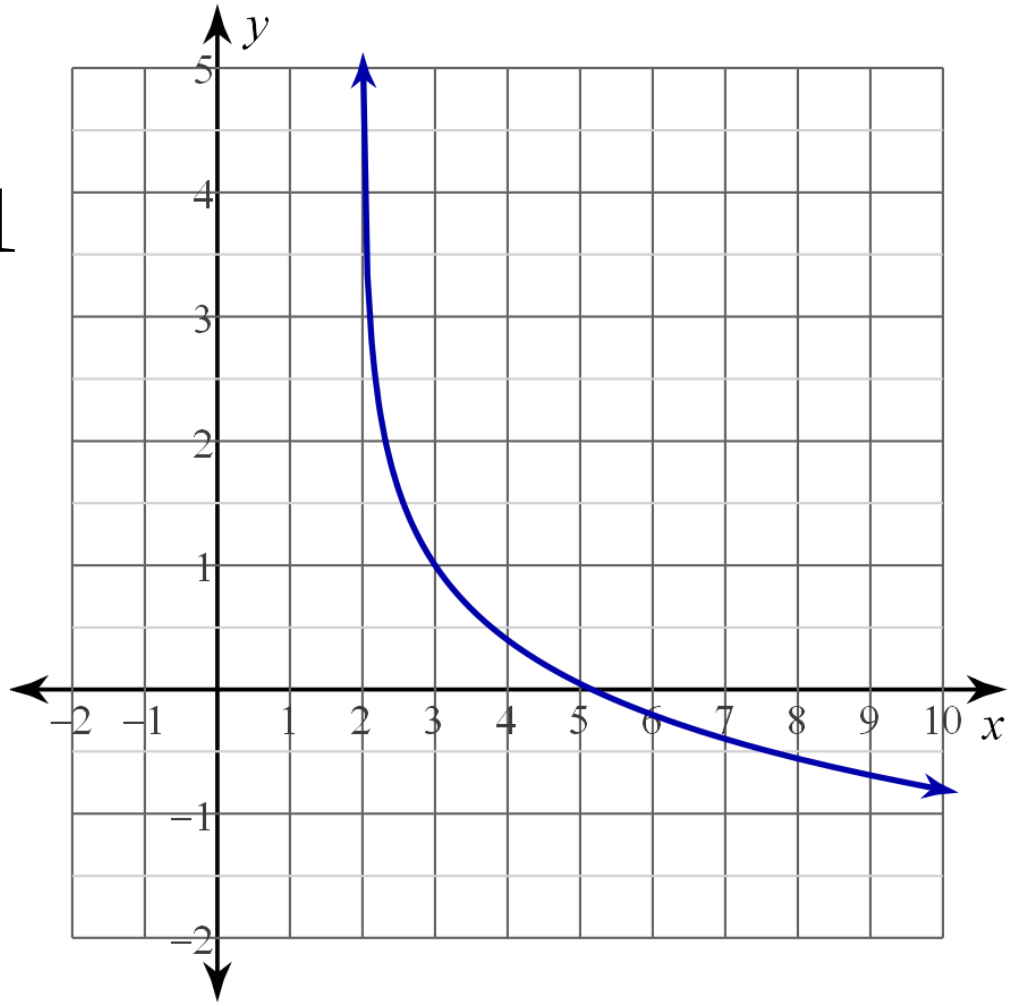
<u>Range</u> = ? (-∞, ∞)

<u>Asymptote</u> = ? x = -1



$$f(x) = \log x$$

$$g(x) = -3\log(x-2)+1$$



Reflected → x-axis
VSF = 3
Right 2 translation
Up 1 translation

Domain = ? x = (2, ∞)

Range = ? (-∞, ∞)

Asymptote = ? x = 2

NOT exponential (has a vertical asymptote, does NOT have a horizontal asymptote.

What is a logarithm?

A logarithm is another way of writing an exponent.

$$2^x = 8$$

x is the exponent

$$\log_2 8 = x$$

Log = exponent

Both of these equations are saying the same thing:
"2 raised to what power is 8?"

Exponential Form

$2^3 = 8$

base

Logarithm Form

Log$\square$=$\square$

$\square$

$\log_2 8 = 3$

base

"base 2 raised to the 3rd is 8"

"log base 2 of 8 is 3"

$3^x = 9$

What exponent of 3 equals 9?

$\log_3 9 = x$

What exponent of 3 equals 9?

x = 2

Convert to logarithm form

Log$\square_{\square}$ = $\square$

What is the solution?

x = 2 $5^x = 25$ $\log_5 25 = x$

x = 3 $4^x = 64$ $\log_4 64 = x$

x = ??? $b^x = y$ $\log_b y = x$

x = 2 $9^x = 81$ $\log_9 81 = x$

x = 3 $10^x = 1000$ $\log_{10} 1000 = x$

Convert to exponential form

$\log_{\square}\square = \square$

What is the solution?

x = 2 $\log_{10} 100 = x$ $10^x = 100$

x = 3 $\log_3 27 = x$ $3^x = 27$

x = 0 $\log_9 1 = x$ $9^x = 1$

x = 16 $\log_4 x = 2$ $4^2 = x$

x = 32 $\log_2 x = 5$ $2^5 = x$

What exponent of the base equals the logarand?

Finding the Inverse $f(x) = 3^x$

$f^{-1}(x) = ?$ Shift 'x' and 'y'

$x = 3^y$ "Undo the Exponential" (Convert to a log)

"A log is an exponent"

$y = \log_3 x$ $f^{-1}(x) = \log_3 x$

$g(x) = \log_4(x-2) + 1$ $g^{-1}(x) = ?$

$x = \log_4(y-2) + 1$ "Isolate the log"

$x - 1 = \log_4(y-2)$ "undo the log" (Convert to exponential)

"A log is an exponent"

$y - 2 = (4)^{x-1}$

$g^{-1}(x) = (4)^{x-1} + 2$

Common Logarithm: has a base of 10.

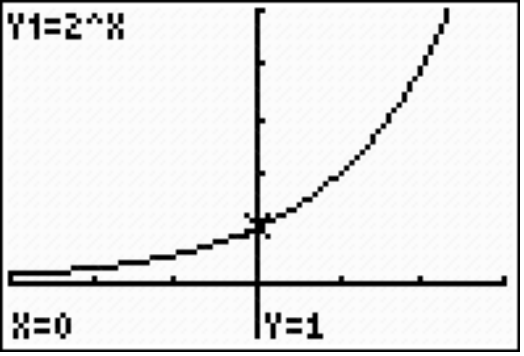
$$\log_{10} 100 = x$$

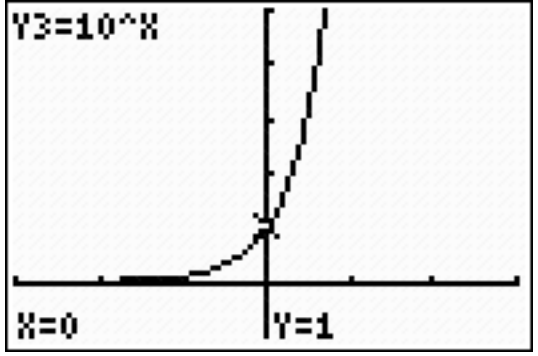
We usually write it in this form: $\log 100 = x$

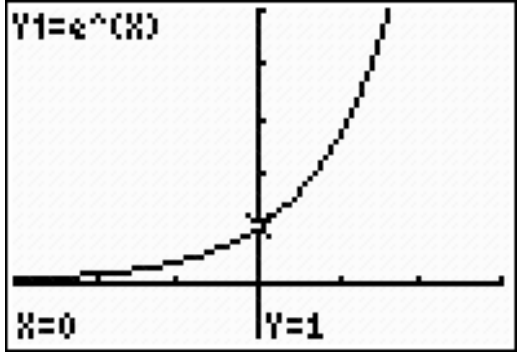
Natural Logarithm: has a base of e.

$$\log_e 2.718 = 1$$

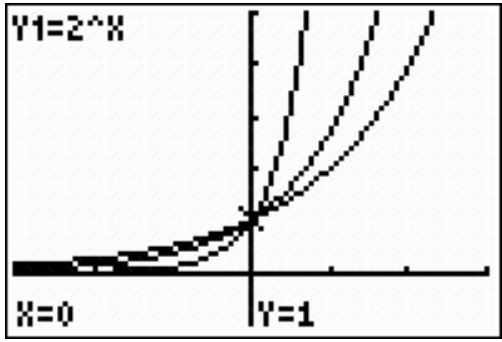
We always write it in this form: $\ln 2.718 = 1$

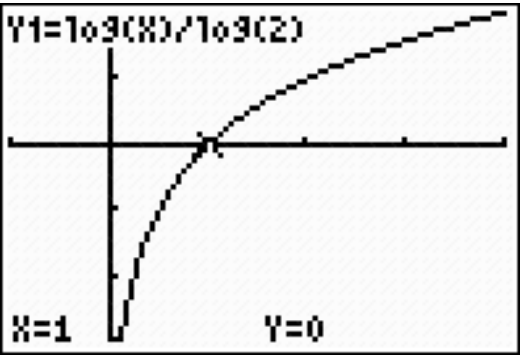


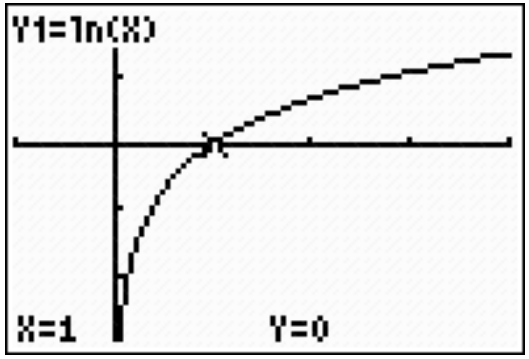


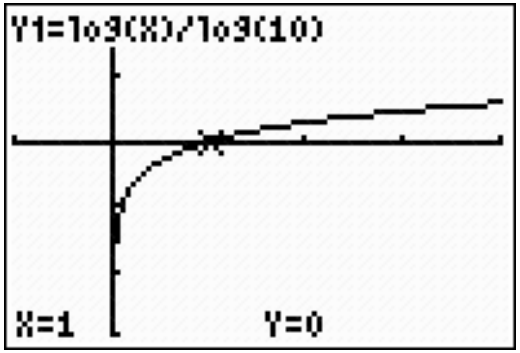


Base increasing: →

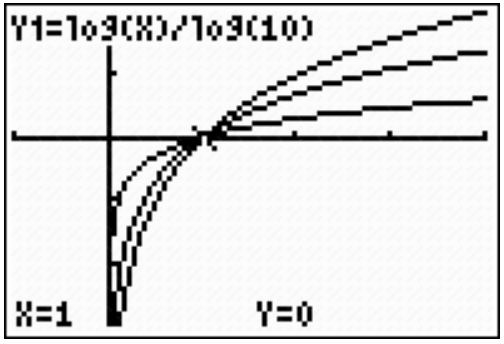








Base increasing: →



Graphing Log Equations

$g(x) = \log(x - 1)$

1) Find the vertical asymptote

VA: x = "the zero of the logarand"

$x - 1 = 0$ $\boxed{x = 1}$

Draw and label

2) Determine some x-y pairs.

$g(1.5) = \log(1.5 - 1)$

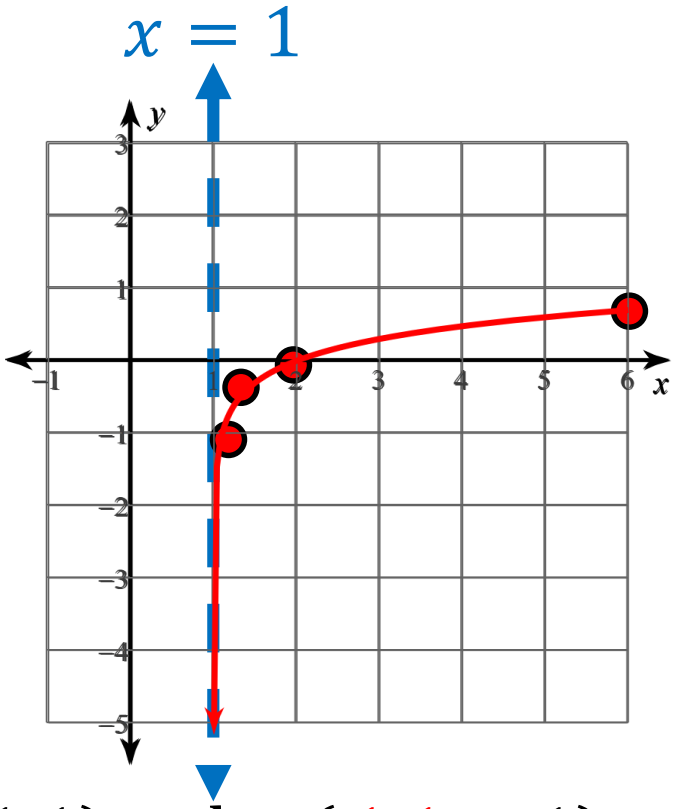
$g(1.5) \approx -0.3$

$g(2) = \log(2 - 1)$

$g(2) = 0$

$g(6) = \log(6 - 1)$

$g(6) \approx 0.7$



$g(1.1) = \log(1.1 - 1)$

$g(1.1) = -1$

Graphing Log Equations

$g(x) = \log(x - 1)$

1) Find the inverse → convert to exponential

$g(x) = log_{10}(x - 1)$

$x = log_{10}(y - 1)$

$10^x = y - 1$

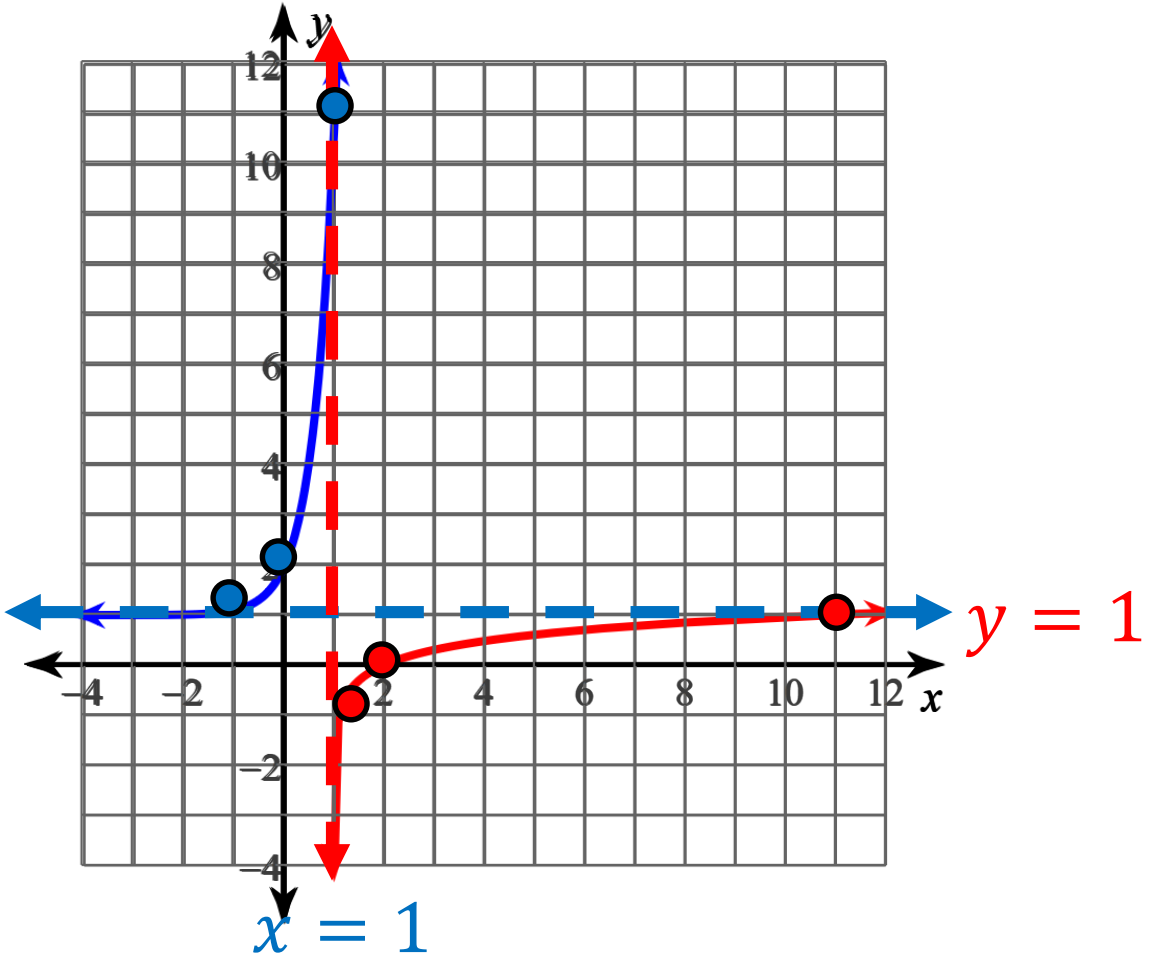
$g^{-1}(x) = 10^x + 1$

$g^{-1}(0) = 2$

$g(2) = 0$

$g^{-1}(1) = 11$

$g(11) = 1$



$g^{-1}(-1) = 1.1$

$g(1.1) = -1$

What is the base?

$\log_2 8 = x$ $\ln 5 = x$ $\log 20 = x$

What is the Solution?

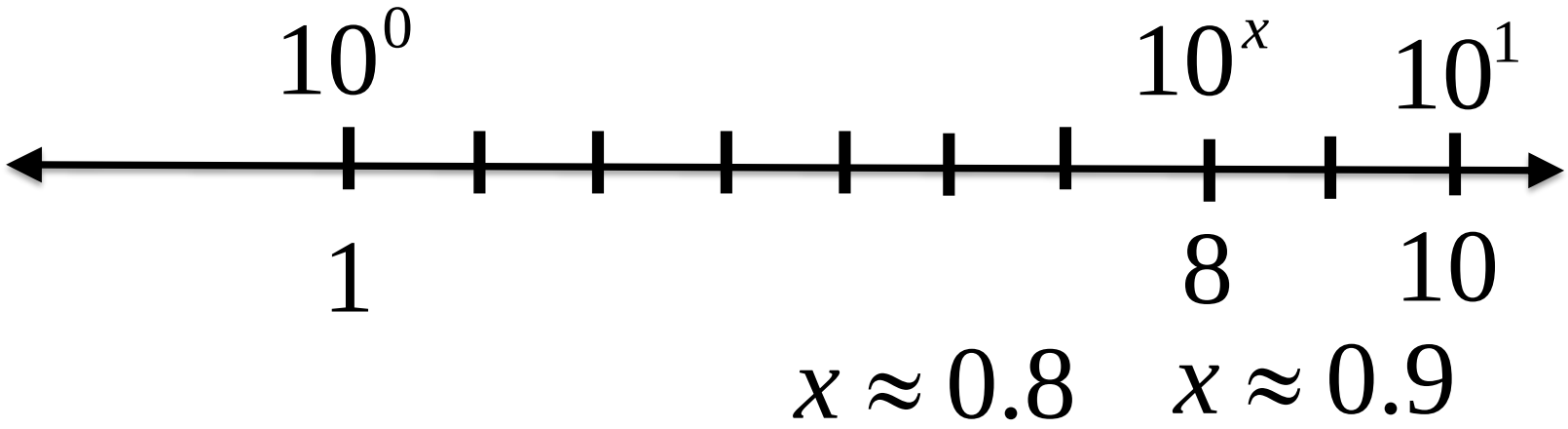
$\frac{1}{100} = log_{10}(x)$ $x = 10^{\frac{1}{100}}$

$x = log_2\sqrt{2}$ $x = \frac{1}{2}$

$x = log_5 \frac{1}{\sqrt[3]{5}}$ $x = -\frac{1}{3}$

Estimate the value of the log: $log(8)$

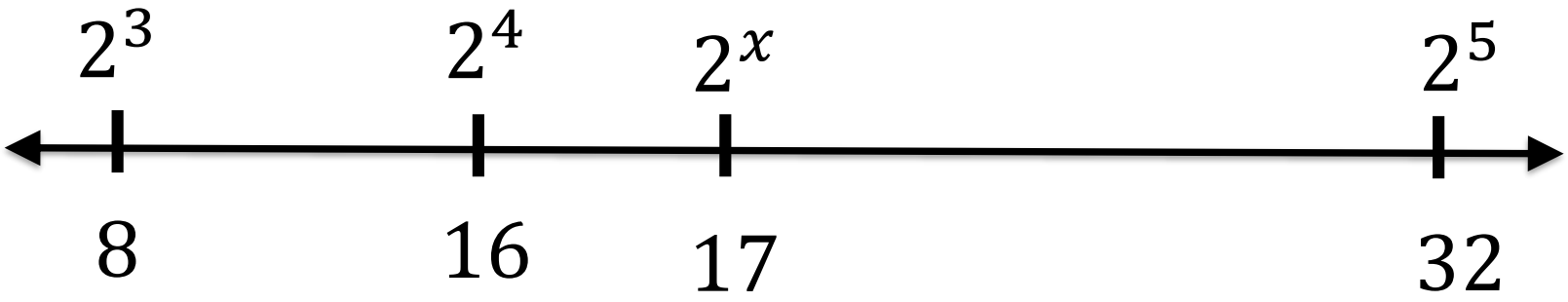
$$\log 8 = x \qquad 10^x = 8$$



Find $\log 8$ on your calculator. $\log 8 = 0.903$

Estimate the value of the log: $\log_2 17$

$$\log_2 17 = x \quad \rightarrow 2^x = 17$$



$$x \approx 4.1 \ ?$$

Find $\log_2 17$ on your calculator. $\log_2 17 = 4.09$

Estimate the value of the log (without using your calculator)

$log_3 30$ $log_5 30$ $log_6 30$