Math-1050 (Session 1): Algebra Review #1

-Sets

-Evaluate Functions

-Number Systems

-Domain of an Expression

-Properties of Exponents: x^m, $x^m y^n$, $(x^m)^n$, x^0, x^{-m}

- +/-/* Polynomials

-Special patterns $(x+a)(x-a), x^2-a^2$, $(x-a)^2$, x^3-a^3, $(x-a)^3$

-Long Division

-Factoring

-Common Factors

-Trinomial: Lead Coefficient = 1

-Trinomial: Lead Coefficient ≠ 1

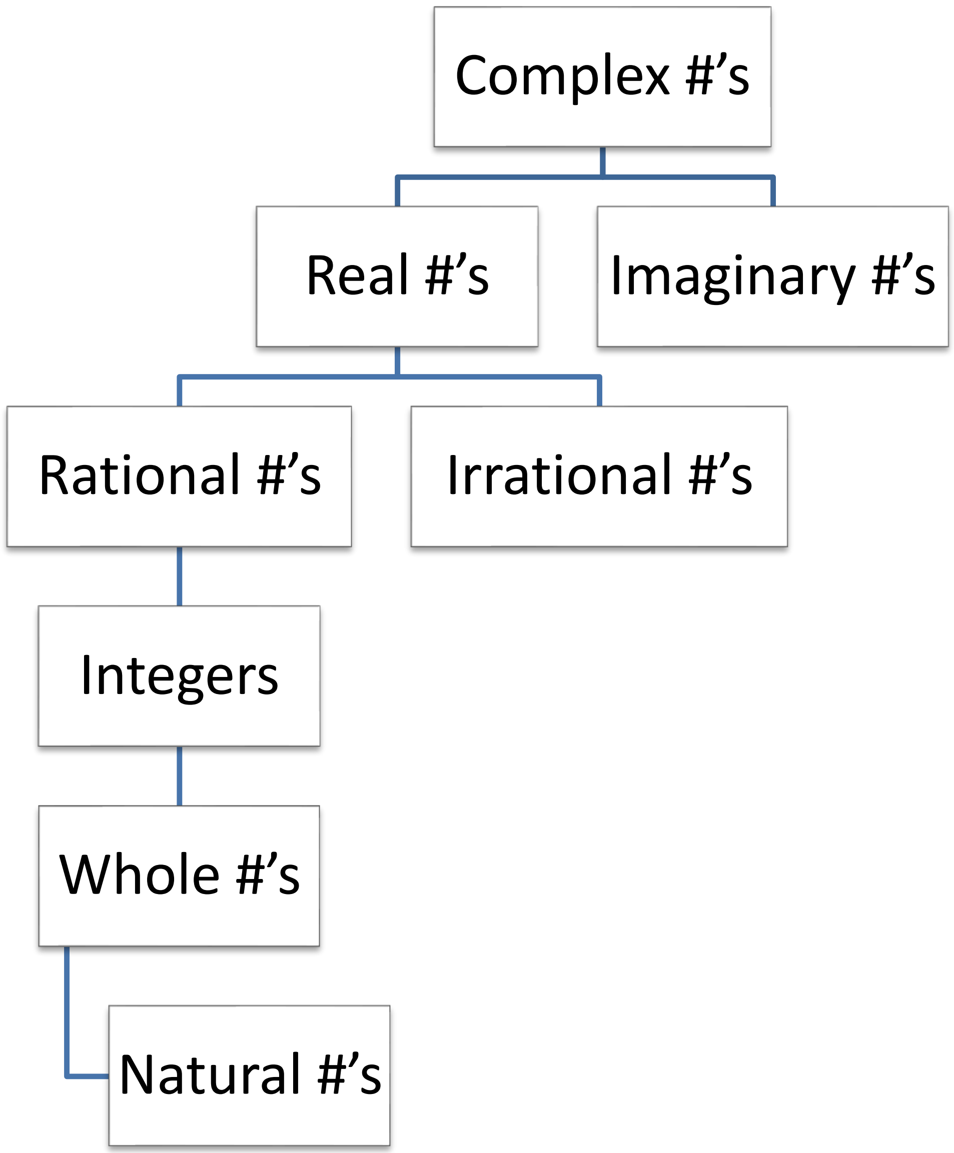
-Difference of two squares

-3rd Degree w/ no constant term

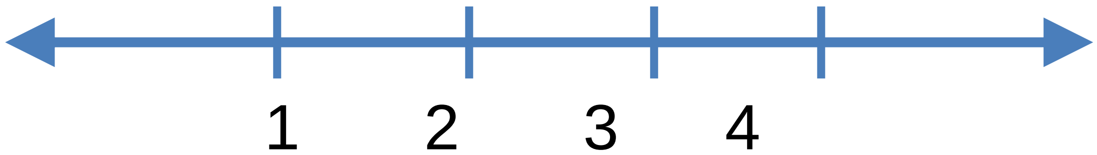
-3rd Degree w/ "nice" pattern

-Sum & Difference of Cubes

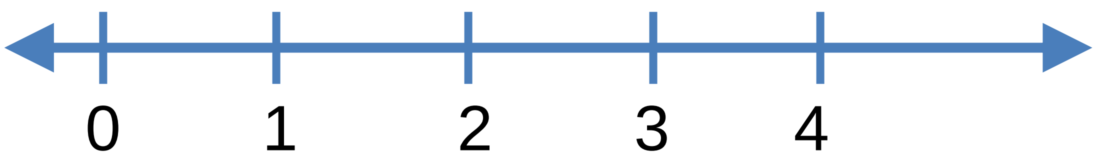
-Higher degree with quadratic form → *WOW!!*


Complex #'s
Real #'s
Imaginary #'s
Rational #'s
Irrational #'s
Integers
Whole #'s
Natural #'s

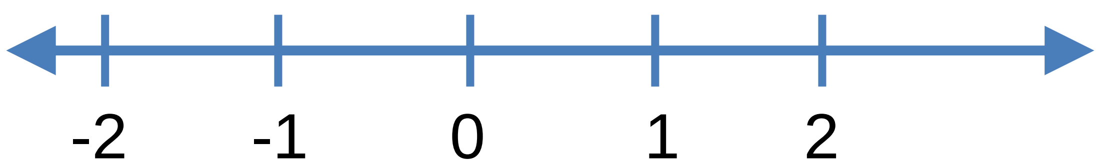
Natural numbers: the positive “counting” numbers that are usually shown on a number line.



Whole numbers: the natural numbers and the number zero.



Integers: the whole numbers and the negative “counting” numbers.



Can anyone interpret what the following means?

$$\text{Rational numbers} = \left\{ R : R = \frac{a}{b};\ \text{a, b} \in \text{integers} \right\}$$

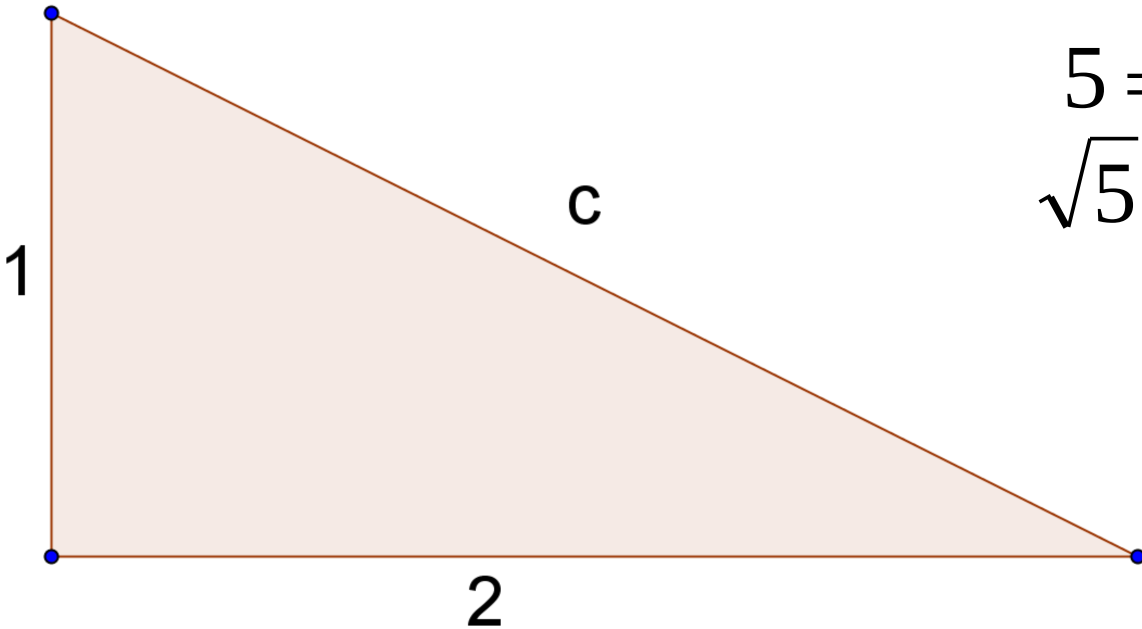
<u>Rational numbers</u>: can be written as a ratio of integers: ½, -⅔, etc.

Pythagorean Theorem: If it's a right triangle, then side lengths can be related by: $a^2 + b^2 = c^2$

$$1^2 + 2^2 = c^2$$

$$5 = c^2$$

$$\sqrt{5} = c$$



What *number system* does SQRT(5) belong to?

Irrational numbers: cannot be written as a ratio of integers: ½, -⅔, etc.

The decimal version of an irrational number never terminates and never repeats. (0 = 5.13257306…).

If we see the radical symbol, the number is usually irrational (unless it is a “perfect square). $\sqrt{3}$

$$\sqrt{4} = 2\,(\text{rational }\#)$$

Identify the number system

(1) $\frac{2}{3}$ (2) $\sqrt{7}$ (3) 5.25 (4) 26 (5) π

(6) $\sqrt{-3}$

N: Natural
W: Whole
Z: Integer
Q: Rational
I: Irrational
R: Real

<u>Domain of an Expression:</u> the values that may be substituted into the expression so that the simplified number is defined (in the real number system)

$$\frac{x-3}{x+2} \qquad Domain: \qquad d = \{x \mid x \neq -2\}$$

The only value that ***does not*** result in the expression being defined is x = -2.   If x = -2, the expression simplifies to: -5/0 which ***is not defined*** in the real number system.

Multiply Powers Property

$$(x^2)(x^3) = (x * x)(x * x * x)$$

This is 'x' used as a factor how many times?

$$(x^2)(x^3) = x^2 x^3 = x^{2+3} = x^5$$

'x' used as a factor five times

When you multiply powers having the same base, you add the exponents.

Exponent of a Power Property $(x^2)^3$

$$(x^2)^3 = (x * x)(x * x)(x * x)$$

This is ‘x’ used as a factor how many times?

$$(x^2)^3 = \quad = x^6$$

‘x’ used as a factor six times

$$(x^2)^3 = x^{2*3} = x^6$$

you multiply the exponents.

Exponent of a Product Property

$$(xy)^2 = (xy)(xy) = x*y*x*y = x*x*y*y$$

$$= x^2y^2$$

$$(xy)^m = x^m y^m$$

This makes it seem like you can "distribute" in the exponent. This only works with the power of a product!!

$$(x-y)^2 \neq x^2 - y^2$$

$$(x-y)^2 = (x-y)(x-y)$$

$$= x^2 - 2xy + y^2$$

<u>Negative</u> Exponent Property "Grab and drag"

$$x^{-2} = \frac{1 * x^{-2}}{1} = \frac{1}{x^2}$$

When you "Grab and drag" the base and its exponent across the "boundary line" between numerator and denominator, you just change the sign of the exponent.

$$x^2 y^{-2} = \frac{x^2}{y^2}$$

$$\left(\frac{1}{x^3}\right)^{-2} = \frac{1}{x^{-6}} = \frac{1}{x^{-6}} = x^6$$

Negative Exponent Property

Possible errors

$$4x^{-2} = \frac{4 * x^{-2}}{1} = \frac{4}{x^2}$$

When you "Grab and drag" the base and its exponent across the "boundary line" between numerator and denominator, you just change the sign of the exponent.

DO NOT GRAB the coefficient!

$$\frac{4 * x^{-2}}{1} \neq \frac{1}{4x^2}$$

Quotient of Powers Property

$$\frac{x^{5}}{x^{2}} = \frac{x*x*x*\cancel{x}*\cancel{x}}{\cancel{x}*\cancel{x}} = x*x*x \quad = x^{3}$$

$$\frac{x^{5}}{x^{2}} = x^{5}x^{-2} = x^{5-2} = x^{3}$$

This is really a silly property. We don't even need to memorize this as a separate property. It's just the negative exponent property.

$$\frac{x^{m}}{x^{n}} = x^{m-n}$$

Power of a Quotient Property

$$\left(\frac{x}{y}\right)^2 = \left(\frac{x}{y}\right)\left(\frac{x}{y}\right) = \frac{x^2}{y^2}$$

General form of Power of a quotient:

$$\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$$

This is another silly property. Isn't it just exponent of a product?

Zero Exponent Property

Any base raised to the zero power simplifies to one.

$$10^3 = 1000$$

$$10^2 = 100$$

$$10^1 = 10$$

$$10^0 = 1$$

$$2^0 = 1$$

$$(2x)^0 = 1$$

$$2x^0 = 2*1 = 2$$

Combination: (1) Negative Exponent, (2) Product of Powers, (3) Power of a Power, (4) Power of a Quotient

$$\left(\frac{3x^2}{2x^{-4}y}\right)^2 = \left(\frac{3x^2x^4}{2y}\right)^2 = \left(\frac{3x^6}{2y}\right)^2 = \left(\frac{3^1x^6}{2^1y^1}\right)^2$$

$$= \frac{3^{1*2}x^{6*2}}{2^{1*2}y^{1*2}} = \frac{3^2x^{12}}{2^2y^2} = \frac{9x^{12}}{4y^2}$$

To Factor (verb) to break apart a number or an expression into its factors.

distributive property: multiply a term times a sum.

$$2(x + 3) = 2x + 6$$

To factor out the common factor: the “reverse” of the distributive property.

Multiplying simple binomials to see the pattern used to factor trinomials

$$(x+y)(x-y) \quad = x^2 + xy - xy - y^2$$

$$\boxed{= x^2 - y^2}$$

$$(x+y)^2 = (x+y)(x+y) = x^2 + xy + xy + y^2$$

$$\boxed{= x^2 + 2xy + y^2}$$

$$(x-y)^2 = (x-y)(x-y) = x^2 - xy - xy + y^2$$

$$\boxed{= x^2 - 2xy + y^2}$$

Multiplying simple binomials to see the pattern used to factor trinomials

$$(x+y)^3 \quad = (x+y)(x+y)(x+y)$$

$$= (x+y)(x^2+2xy+y^2)$$

$$\boxed{= x^3+3x^2y+3xy^2+y^3)}$$

$$(x-y)^3 \quad = (x-y)(x-y)(x-y)$$

$$= (x-y)(x^2-2xy+y^2)$$

$$\boxed{= x^3-3x^2y+3xy^2-y^3)}$$

Multiplying simple binomials to see the pattern used to factor trinomials

$$(x+2)(x+3)$$

$$= x^2 + (2+3)x + (2*3)$$

$$= x^2 + 5x + 6$$

Left times left is left

Right plus right is middle

Right times right is right

$$x^2 + 5x + 6$$

Left times left is left

$$(\_\_\_ + \_\_\_)(\_\_\_ + \_\_\_)$$

$$(x + \_\_\_)(x + \_\_\_)$$

Right times right is right

$$(x + \_\_\_)(x + \_\_\_)$$

Right plus right is middle

$$(x+2)(x+3)$$

What are the factors of 6 that add up to 5?

Notice a nice pattern when you multiply the binomials

$$(2x+1)(x+3)$$

$$\boxed{2x^2+7x+3}$$

Left times left is left

Right times right is right

"right plus right" *does not* add up to 7, but notice something.

$$\overset{6x}{\overbrace{(2x+1)(x+3)}}$$

$$x$$

$$6x+x=7x$$

$$2x^2+7x+3$$

$$2*3=6$$

Are there any other factors of 6 that add up to 7?

$$1+6=7$$

$$6=1*6$$

$$2 * 15 = 30$$

$$2x^2 + 13x + 15$$

$$\underline{10 + 3 = 13}$$

$$30 = 10 * 3$$

Are there any *other* factors of 30 that add up to 13?

This tells us to break 13x into 10x + 3x

$$2x^2 + 13x + 15$$

$$2x^2 + 10x + 3x + 15$$

These are all of the terms in "the box"

	x	5
2x	$2x^2$	10x
3	3x	15

What is the bottom-left term in the box?

x*(__3__) = 3x

What is the top-right term in the box?

2x*(__5__) = 10x

Final check: 3*5 = 15?

Factored form:

$$2x^2 + 13x + 15$$

$$\rightarrow (2x + 3)(x + 5)$$

“Nice” 3rd Degree Polynomial (with no constant term)

$$y = 3x^3 + 12x^2 - 36x$$

It has no constant term so it can easily be factored into ‘x’ times a quadratic factor.

$$y = 3x(x^2 + 4x - 12)$$

If the *quadratic factor* is “nice” we can factor that into 2 binomials.

$$y = 3x(x + 6)(x - 2)$$

x = 0, -6, 2

This is now “intercept form” so we can “read off” the x-intercepts. What are they?

Convert to standard form:

$$y = (x-3)(x^2+3x+9)$$

$$y = x^3 - 27$$

There are NO x^2 terms
and NO 'x' terms

	x	-3
x^2	x^3	$-3x^2$
$3x$	$3x^2$	$-9x$
9	$9x$	-27

$0x^2$ $0x$

The Difference of cubes: factors as the cubed root of each term multiplied by a 2nd degree polynomial.

$$y = x^3 - 1$$

$$y = (x-1)(ax^2+bx+c)$$

$$y = (x-1)(x^2+1x+1)$$

	x	-1
x^2	x^3	$-1x^2$
$1x$	$1x^2$	$-1x$
1	$1x$	-1

$0x^2$ $0x$

An easier method is "box factoring" (if it has this nice pattern).

$$y = 1x^3 + 2x^2 + 2x + 4$$

These 4 terms are the *numbers in the box*.

Find the *common factor* of the 1st row.

Fill in the rest of the box.

	x	2
x^2	x^3	$2x^2$
2	$2x$	4

Rewrite in intercept form.

$$y = 1x^3 + 2x^2 + 2x + 4$$

$$y = (x^2 + 2)(x + 2)$$

Find the "zeroes."

$$0 = (x^2 + 2)(x + 2)$$

$$0 = x^2 + 2 \qquad 0 = x + 2$$

$$-2 = x^2 \qquad x = -2$$

$$x = \underline{+} i\sqrt{2}$$