Math-1050
Session #19

Composite Functions

Compositions of Functions $f(x) = 2x$ → $f(3) = ?$

Means: “replace ‘x’ in the function with a 3.

1. Replace the ‘x’ with a set of parentheses.

$$f(3) = 2(\ \ )$$

2. Put the input value ‘3’ into the parentheses.

$$f(3) = 2(3)$$

3. Find the output value.

$$f(3) = 6$$

Function Notation

$f(x) = 2x + 1$

(Input) x	(rule) $2x + 1$	(output) $f(x)$
2	$2(2) + 1$	5
3	$2(3) + 1$	7
$x – 1$	$2(x – 1) + 1$	$2x – 1$
$3x$	$2(3x) + 1$	$6x + 1$

$f(2) = 5$

$f(3) = 7$

$f(x – 1) = 2x – 1$

$f(3x) = 6x + 1$

If your input is an expression instead of a number: replace 'x' with parentheses and "plug in" the expression

→ parentheses, substitute, simplify

Composition of Functions

f(x) = 2x + 1   g(x) = 3x + 2   h(x) = x + 5

$$f(g(x)) = ? \quad = 2(\qquad) + 1 \quad = 2(3x + 2) + 1$$

$$h(g(x)) = ? \quad = (\qquad) + 5 \quad = (3x + 2) + 5$$

$$h(f(x)) = ? \quad = (\qquad) + 5 \quad = (2x + 1) + 5$$

$$g(h(x)) = ? \quad = 3(\qquad) + 2 \quad = 3(x + 5) + 2$$

$$f(f(x)) = ? \quad = 2(\qquad) + 1 \quad = 2(2x + 1) + 1$$

New Notation for the Composition of Functions

$(f \circ g)(x) = f(g(x))$ "g" plugged into rule "f"

$f(x) = 4x - 1 \qquad g(x) = -5x + 3$

$(f \circ g)(x) = ?$ $= 4(\qquad) - 1$ $= 4(-5x + 3) - 1$

"g" plugged into rule "f" $(f \circ g)(x) = -20x + 11$

$(g \circ f)(x) = ?$ $= -5(\qquad) + 3$ $= -5(4x - 1) + 3$

"f" plugged into rule "g" $(g \circ f)(x) = -20x + 8$

$(f \circ f)(x) = ?$ $= 4(\qquad) - 1$ $= 4(4x - 1) - 1$

"f" plugged into rule "f" $(f \circ f)(x) = 16x - 5$

$(g \circ g)(x) = ?$ $= -5(\qquad) + 3$ $= -5(-5x + 3) + 3$

"g" plugged into rule "g" $(g \circ g)(x) = 25x - 12$

One more layer!

$f(x) = 3x \qquad g(x) = x^2$

$f(g(4))$

4

$g(\;) = (\;\;)^2$

$g(4) = (4)^2$

16

$f(\;) = 3(\;\;)$

$f(16) = 3(16)$

48

$f(g(x)) = 3(g(x)) = 3x^2$

$f(g(4)) = 3(g(4)) = 3(4)^2$ = 48

One more layer. $g(x) = x^2$ $f(x) = 3x$

$$(g \circ f)(-1) = ?$$

Rewrite in “old” notation

$$g(f(-1)) = ?$$

The input to f(x) is -1.

$$f(-1) = 3(-1)$$

$$f(-1) = -3$$

The output of f(-1) is -3.

The input to g(x) is -3.

$$g(-3) = 9$$

$$g(f(-1)) = 9$$

<u>Showing that two composite functions are equal.</u> $f \circ g = g \circ f$

$f(x) = 3x - 2$

$g(x) = \frac{x+2}{3}$

$f \circ g = f(\frac{x+2}{3})$

$g \circ f = g(3x - 2)$

$f\left(\frac{x+2}{3}\right) = 3\left(\frac{\mathrm{x}+2}{3}\right) - 2$

$g(3x-2) = \frac{(3x-2)+2}{3}$

$f\left(\frac{x+2}{3}\right) = \mathrm{x}$

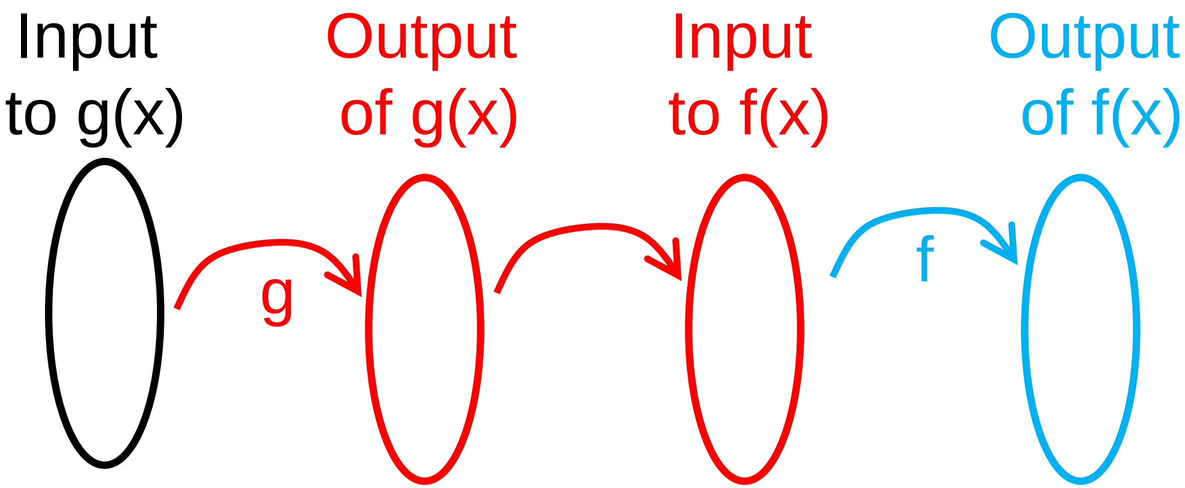
$g(3x-2) = x$

Therefore: $f \circ g = g \circ f$ so the compositions are equal.

This prepares us for 6.2 (Proving that functions are inverses of each other).

The domain of function compositions

Another way to write a composition. $f \circ g = f(g(x))$



The domain of $f \circ g$ are the input values to g(x)

The output of g(x) is the input to f(x)

The output of f(x) is the range of $f \circ g$

Input to 'g' must result in output values that are allowed as input values to 'f'. Sometimes we have to restrict the domain of 'g' to make this work.

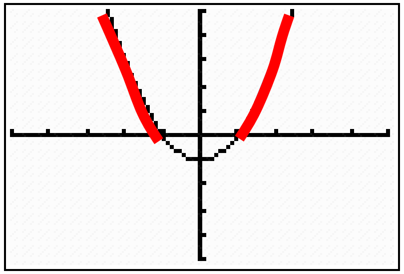
The <u>domain</u> of function compositions

$$f(x) = \sqrt{x} \qquad g(x) = x^2 - 1$$

$$f \circ g = ? = \sqrt{(\qquad)} = \sqrt{(x^2 - 1)}$$

The domain of ‘f’: $x \geq 0$

The input to ‘f’ is the output of ‘g’.



How must the domain of ‘g’ (input values) be restricted so that the output of the ‘g’ is: $y \geq 0$

Domain of ‘g’ must be restricted to

$x \geq 1$ and $x \leq -1$

The domain of function compositions

$$f(x) = \frac{3}{x-2} \qquad g(x) = \frac{2}{x+5}$$

Domain *g(x))*: x ≠ -5

Domain of $f \circ g = ?$

The output of 'g' is the input to 'f'

The input to 'f' *cannot* equal 2 (causes division by 0 in 'f').

The output of 'g' *cannot* equal 2

$$2 \neq \frac{2}{x+5}$$

Domain *f(g(x))*: x ≠ -4, x ≠ -5

$$2(x+5) \neq 2$$

$$2x + 10 \neq 2$$

$$2x \neq -8$$

$$x \neq -4$$

The domain of function compositions

$$f(x) = \frac{3}{x-2} \qquad g(x) = \frac{2}{x+5}$$

Find $f \circ g$

$$f\left(\frac{2}{x+5}\right) = \frac{3}{\frac{2}{x+5} - 2} = \frac{3}{\frac{2}{x+5} - \frac{2(x+5)}{x+5}}$$

$$= \frac{3}{\frac{2-2x-10}{x+5}} = \frac{3}{\frac{-2x-8}{x+5}} = \frac{3(x+5)}{-2x-8} = \boxed{\frac{3(x+5)}{-2(x+4)}}$$

Domain *g(x)*: x ≠ -5

Domain *f(g(x))*: x ≠ -4

Domain *f(g(x))*: {x: x ≠ -5, x ≠ -8}